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ON HÖLDER ESTIMATES FOR CAUCHY TRANSFORMS ON CONVEX DOMAINS IN $\mathbb{C}^2$

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Abstract. The main purpose of this paper is to establish Hölder estimates for the Cauchy transform in a class of finite/infinite type convex domains in $\mathbb{C}^2$.

1. Introduction

Let Ω be a domain of the complex plane with piecewise smooth boundary $b\Omega$. The Cauchy transform on the complex plane is

$$\mathcal{C}[u](z) = \int_{b\Omega} u(\zeta) \frac{1}{2\pi i} \frac{d\zeta}{\zeta - z}$$

for $z \in \Omega$. It maps L^1-functions on $b\Omega$ to holomorphic functions in Ω. The term $\frac{1}{2\pi i} \frac{d\zeta}{\zeta - z}$ is called the Cauchy kernel on the complex plane, and it is universal. A basic property of the kernel is that: for each $\zeta \in b\Omega$, the kernel is holomorphic in $z \in \bar{\Omega} \setminus \{\zeta\}$. We list some well-known operator-theoretic properties of the Cauchy transform:

(1) Let $\mathcal{O}(\Omega)$ be the space of functions that are holomorphic in Ω, with the topology of uniform convergence on compact subsets of Ω. Let σ be the length measure on $b\Omega$. Then $\mathcal{C}[u] \in \mathcal{O}(\Omega)$ for each $f \in L^1(b\Omega, d\sigma)$. Moreover, $\mathcal{O} : L^p(b\Omega, d\sigma) \to \mathcal{O}(\Omega)$ is continuous for all $1 \leq p \leq \infty$.
(2) Let $E \subset \mathbb{C}$ be a bounded set. For each $0 < \alpha < 1$, $\Lambda_\alpha(E)$ denotes the standard Hölder class of order α on E. Then

$$\mathcal{C} : \Lambda_\alpha(b\Omega) \to \mathcal{O}(\Omega) \cap \Lambda_\alpha(\Omega)$$

is bounded. The boundedness also provides a sufficient condition so that $\mathcal{C}[u]$ extends continuously on the closure $\bar{\Omega}$ when u is at least Hölder continuous of order α.

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The question which we can ask is: *Do the properties above hold in $\mathbb{C}^n$, for $n \geq 2$?* To answer this question, as the first step, we must construct a multi-dimensional version for the Cauchy kernel. The simpliest and oldest answer may be the Cauchy kernel on distinguished boundaries of polydiscs, that is the product of n Cauchy kernels on $\mathbb{C}$. For non-trivial domains, in 1932, A. Weil introduced a Cauchy kernel for polynomial polyhedra in $\mathbb{C}^2$. Since then there were some different versions for the multi-dimensional Cauchy kernel constructed on particular domains. Unfortunately, there is not a canonical Cauchy kernel on natural boundaries of arbitrary smooth domains in $\mathbb{C}^n$.

In 1938 by Martinelli and in 1943 by Bochner, they did construct a differential form kernel which now we called the Bochner-Martinelli kernel $K_0(\zeta, z)$ (see [20, Section 1.2, page 148]). It can be considered as the first multi-dimensional version for the Cauchy kernel on arbitrary smooth domains. However, this kernel is not holomorphic in z. In 1959, in [9], J. Leray introduced the second version for the Cauchy kernel for convex domains in $\mathbb{C}^n$. This significant paper marks the beginning of an interest that today is called Cauchy-Fantappiè Theory. The Cauchy kernel in $\mathbb{C}^n$ was then developed by W. Koppelman in [14, 15], by N. Kerzman and E. Stein in [16], by S. Chen in [3], by L. Lanzani and E. Stein in [5]. E. Ligocka adapted the method by Kerzman-Stein to study the unweighted and weighted Bergman projections in [10–12]. Recently, in the series of perceptive workings [6–8] by Lanazani and Stein, this topic has been advanced to consider different problems. The Cauchy kernel constructed via Cauchy-Fantappiè Theory admits the holomorphicity as itself in $\mathbb{C}$. However, as M. Range wrote “we had to pay a price” in his book ([20]), the kernel is not universal, it strictly depends on the boundary of the considered domains.

Now let Ω be a bounded convex domain in $\mathbb{C}^n$ with smooth boundary $b\Omega$. Let ρ be a defining function for Ω so that $\Omega = \{z \in \mathbb{C}^n : \rho(z) < 0\}$ and $b\Omega = \{z \in \mathbb{C}^n : \rho(z) = 0\}$, $\nabla\rho \neq 0$ on $b\Omega$.

Let us define, for $\zeta, z \in \overline{\Omega}$:

$$\Phi(\zeta, z) = \sum_{j=1}^{n} \frac{\partial\rho}{\partial\zeta_j}(\zeta)(\zeta_j - z_j). \tag{1.1}$$

The convexity of Ω implies

$$\Phi(\zeta, z) = \sum_{j=1}^{n} \frac{\partial\rho}{\partial\zeta_j}(\zeta)(\zeta_j - z_j) \neq 0$$

for all $\zeta \in b\Omega$ and $z \in \Omega$. Now we set

$$C(\zeta, z) = \frac{1}{2\pi i}\left[\sum_{j=1}^{n} \frac{\partial\rho}{\partial\zeta_j}(\zeta) d\zeta_j\right] \frac{1}{\Phi(\zeta, z)}$$

for $\zeta \in b\Omega, z \in \Omega$, which is a $(1,0)$-form of ζ-variables. The Cauchy kernel for the convex domain Ω is

$$\Omega_0\left(C(\zeta,z)\right) = C(\zeta,z) \wedge (\bar{\partial}_\zeta C(\zeta,z))^{n-1},$$

where $(\bar{\partial}_\zeta C(\zeta,z))^{n-1}$ is the $(n-1)$ wedge product $(\bar{\partial}_\zeta C(\zeta,z)) \wedge \cdots \wedge (\bar{\partial}_\zeta C(\zeta,z))$.

Theorem 1.1 ([20, Theorem 3.4, page 171])**.** *For any $u \in \mathcal{O}(\Omega)$ and $z \in \Omega$, we have*

$$u(z) = \int_{b\Omega} u(\zeta)\Omega_0(C(\zeta,z)).$$

By Stoke's Theorem and $\bar{\partial}u = 0$, we have

$$u(z) = \int_{\Omega} u(\zeta)\bar{\partial}_\zeta\Omega_0(C(\zeta,z)).$$

It is clear that when $n = 1$,

$$\Omega_0(C(\zeta,z)) = \frac{1}{2\pi i}\frac{d\zeta}{\zeta - z}.$$

Example 1.2. Let Ω be the unit ball in $\mathbb{C}^n$ $(n \geq 2)$ and let $\rho(z) = \sum_{j=1}^n |z_j|^2 - 1$ be a defining function of Ω. Then, $\frac{\partial\rho}{\partial\zeta_j} = \bar{\zeta}_j$ for $j = 1, \ldots, n$ and

$$\Phi(\zeta,z) = \sum_{j=1}^n \bar{\zeta}_j(\zeta_j - z_j) = |\zeta|^2 - \langle z,\zeta\rangle$$

for $z, \zeta \in \bar{\Omega}$. Then $\Phi(\zeta,z) = 1 - \langle z,\zeta\rangle$ for $\zeta \in b\Omega$. By calculus for differential forms, it follows that

$$\bar{\partial}_\zeta\Omega_0(C(\zeta,z)) = \left(\frac{i}{2}\right)^n \frac{n!}{\pi^n}\frac{1}{(1 - \langle z,\zeta\rangle)^{n+1}} \bigwedge_{j=1}^n d\zeta_j \wedge d\bar{\zeta}_j.$$

It is quite different from the complex plane that a geometric condition on boundaries must be required to obtain Hölder estimates for the Cauchy transform

$$\mathcal{C}[u](z) = \int_{b\Omega} u(\zeta)\Omega_0(C(\zeta,z)),$$

where $u \in L^1(b\Omega, d\sigma)$, and in this case $d\sigma$ is the surface measure on $b\Omega$.

Definition 1.3 ([4])**.** Let $F\colon [0,\infty) \to [0,\infty)$ be a smooth, increasing function such that

(1) $F(0) = 0$;
(2) $\int_0^\delta |\ln F(r^2)|dr < \infty$ for some small $\delta > 0$;
(3) $\frac{F(r)}{r}$ is increasing.

The function F with the properties above is supposed to be used throughout this paper.

Definition 1.4. A domain Ω is said to be *admitting maximal type* F at the boundary point $P \in b\Omega$ if there are positive constants c, c', such that, for all $\zeta \in b\Omega \cap B(P, c')$ we have

$$\rho(z) \gtrsim F(|z-\zeta|^2)$$

for all $z \in B(\zeta, c)$ with $\Phi(\zeta, z) = 0$.

It is to be noted that, by shrinking $c > 0$ if necessary, Corollary 1.13 in [18] implies

$$\rho(z) > 0 \quad \text{for all } z \text{ with } \Phi(\zeta, z) = 0 \text{ and } 0 < |z-\zeta| < c.$$

Here the notation $B(\zeta, r)$ means the Euclidean ball centered at ζ of radius $r > 0$. Also the notations $\lesssim$ and $\gtrsim$ denote inequalities up to a positive constant, and $\approx$ means the combination of $\lesssim$ and $\gtrsim$.

Lemma 1.5 ([4]). *Let Ω be a smoothly bounded, convex domain in $\mathbb{C}^n$ of maximal type F at $P \in b\Omega$. Then there are positive constants c and A such that the support function $\Phi(\zeta, z)$ satisfies the following estimate*

$$|\Phi(\zeta, z)| \geq A(|\rho(z)| + |\operatorname{Im} \Phi(\zeta, z)| + F(|z-\zeta|^2)) \tag{1.2}$$

for every $\zeta \in b\Omega \cap B(P, c)$, and $z \in \overline{\Omega}$, $|z-\zeta| < c$.

The main purpose of this paper is the following theorem.

Theorem 1.6. *Let $\Omega \subset \mathbb{C}^2$ be a smoothly bounded, convex domain. Assume that Ω admits a maximal type F at all boundary points, for some function F. Then*

(1) *$\mathcal{C}[u] \in \mathcal{O}(\Omega)$ for all $f \in L^1(b\Omega, d\sigma)$. Moreover, $\mathcal{O} : L^p(b\Omega, d\sigma) \to \mathcal{O}(\Omega)$ is continuous for all $1 \leq p \leq \infty$.*

(2) *$\mathcal{C} : \Lambda_{t^\alpha}(b\Omega) \to \mathcal{O}(\Omega) \cap \Lambda_f(\Omega)$ is bounded, for $0 < \alpha < 1$, where*

$$f(d^{-1}) := \left(\int_0^d \frac{\left(\sqrt{F^*(t)}\right)^\alpha}{t} dt \right)^{-1},$$

and F^* be the inverse function of F.

Here, the spaces Λ_f are the f-Hölder spaces (first introduced by T. V. Khanh in [13]). That is, let f be an increasing function such that $\lim_{t\to+\infty} f(t) = +\infty$:

- $\Lambda_f(\Omega)$ consists all functions in $L^\infty(\Omega)$ such that

$$||u||_{\Lambda_f(\Omega)} = ||u||_{L^\infty(\Omega, dV)} + \sup_{z, z+h \in \Omega} f(|h|^{-1})|u(z+h) - u(z)|$$

  finite.
- $\Lambda_f(b\Omega)$ consists all functions in $L^\infty(b\Omega)$ such that

$$||u||_{\Lambda_f(b\Omega)} = ||u||_{L^\infty(b\Omega, d\sigma)} + \sup_{\substack{x(\cdot)\in C \\ 0\leq t\leq 1}} f(t^{-1})|u(x(t)) - u(x(0))$$

finite, where C consists of C^1-curves $x(t) : t \in [0,1] \to x(t) \in b\Omega$ and $|x'(t)| \le 1$. That means $\Lambda^f(b\Omega)$ consists all complex-valued functions u such that for each curve $x(\cdot) \in C$, the function $t \mapsto u(x(t)) \in \Lambda^f([0,1])$.

Example 1.7. Every strongly pseudoconvex domain is a domain admitting maximal type $F(t) = t$ (see [20] for details). Then $\mathcal{C} : \Lambda_{t^\alpha} \to \Lambda_{t^{\alpha/2}}$ is bounded for $0 < \alpha < 1$. This was first proved by P. Ahern and R. Schneider in [2].

Example 1.8. On a smooth convex domain of finite type m in the sense of Range ([18, 19]) in $\mathbb{C}^2$, $\mathcal{C} : \Lambda_{t^\alpha} \to \Lambda_{t^{\alpha/m}}$ is bounded for $0 < \alpha < 1$. Moreover, since each convex domain in $\mathbb{C}^2$ with real analytic boundary satisfies the finite Range type m (see [17]), for some $m \ge 1$, this boundedness is also true on such domain ([1]).

Example 1.9. Let us define

$$\Omega^\infty = \{z \in \mathbb{C}^2 : \exp(1 + 2/s) \exp\left(\frac{-1}{|z_1|^s}\right) + |z_2|^2 < 1\}$$

for $0 < s < 1/2$. Then Ω^∞ is a convex domain admitting maximal type $F(t) = \exp\left(-\frac{1}{32t^s}\right)$ (see [21]). Therefore, $\mathcal{C} : \Lambda_{t^\alpha}(\Omega^\infty) \to \Lambda_{g_\alpha}(\Omega^\infty)$ is bounded, for $2s < \alpha \le 1$, where

$$g_\alpha(t) = \frac{1024^s(\alpha - 2s)}{2s} (|\ln t|)^{\frac{\alpha}{2s} - 1}.$$

2. Proof of Theorem 1.6

The assertion (a) is straightforward from the definition of $\mathcal{C}[u]$. In order to prove (b), we need the general Hardy-Lilttewood Lemma which was proved by Khanh in [13].

Lemma 2.1 (General Hardy-Littlewood Lemma). *Let Ω be a smoothly bounded domain in $\mathbb{R}^n$ and let ρ be a defining function of Ω. Let $G : \mathbb{R}^+ \to \mathbb{R}^+$ be an increasing function such that $\frac{G(t)}{t}$ is decreasing and $\int_0^d \frac{G(t)}{t} dt < \infty$ for $d > 0$ small enough. If $u \in C^1(\Omega)$ such that*

$$|\nabla u(x)| \lesssim \frac{G(|\rho(x)|)}{|\rho(x)|} \quad \textit{for every } x \in \Omega,$$

then

$$f(|x-y|^{-1})|u(x) - u(y)| < \infty$$

uniformly in $x, y \in \Omega$, $x \ne y$, and where $f(d^{-1}) := \left(\int_0^d \frac{G(t)}{t} dt\right)^{-1}$.

For each $z \in \Omega$, let $\pi(z) \in b\Omega$ such that $|z - \pi(z)| = \operatorname{dist}(z, b\Omega)$. Applying Theorem 1.1 with $u = 1$,

$$0 = d_z(1) = \int_{b\Omega} d_z\Omega_0(C(\zeta, z)),$$

and so

$$\int_{b\Omega} u(\zeta)d_z\Omega_0(C(\zeta,z)) = \int_{b\Omega}[u(\zeta)-u(\pi(z))]d_z\Omega_0(C(\zeta,z)).$$

Then it follows that

$$\left|d_z\int_{b\Omega} u(\zeta)\Omega_0(\zeta,z)\right| \le \int_{b\Omega}|u(z)-u(\pi(z))||d_z\Omega_0(\zeta,z)|d\sigma(\zeta).$$

Since $u\in\Lambda_\alpha(b\Omega)$ and $|z-\pi(z)|\lesssim|\zeta-z|$, we have

$$\left|d_z\int_{b\Omega} u(\zeta)\Omega_0(\zeta,z)\right| \le ||u||_{\Lambda_\alpha(b\Omega)}\int_{b\Omega}||\zeta-z|^\alpha d_z\Omega_0(\zeta,z)|d\sigma(\zeta).$$

Rewrite the Cauchy kernel implicitly in $\mathbb{C}^2$ (see [3]), we have

$$\begin{aligned}\Omega_0(C(\zeta,z)) &= C(\zeta,z)\wedge(\bar\partial_\zeta C(\zeta,z))\\ &= \frac{\sum_{j=1}^2\frac{\partial\rho}{\partial\zeta_j}(\zeta)d\zeta_j}{\Phi(\zeta,z)}\wedge\bar\partial_\zeta\left(\frac{\sum_{j=1}^2\frac{\partial\rho}{\partial\zeta_j}(\zeta)d\zeta_j}{\Phi(\zeta,z)}\right)\\ &= \frac{\left(\sum_{j=1}^2\frac{\partial\rho}{\partial\zeta_j}(\zeta)d\zeta_j\right)\wedge\left(\sum_{j,k=1}^2\frac{\partial^2\rho}{\partial\bar\zeta_k\partial\zeta_j}(\zeta)d\bar\zeta_k\wedge d\zeta_j\right)}{\Phi^2(\zeta,z)}\\ &= \sum_{j_0\in\{1,2\}}\frac{A_{j_0}(\zeta)}{\Phi^2(\zeta,z)}d\zeta_1\wedge d\zeta_2\wedge d\bar\zeta_{j_0},\end{aligned}$$

where $A_{j_0}(\zeta)$ is a polynomial involving in ζ of $\frac{\partial\rho}{\partial\zeta_1}(\zeta)$ and $\frac{\partial\rho}{\partial\zeta_2}(\zeta)$, and their first derivatives. It is not difficult to show that

$$|d_z\Omega_0(C(\zeta,z))|\lesssim\frac{1}{|\Phi(\zeta,z)|^3}$$

for $\zeta\in b\Omega$, $z\in\Omega$.

Combining these estimates, we obtain

$$\left|d_z\int_{b\Omega} u(\zeta)\Omega_0(\zeta,z)\right| \le ||u||_{\Lambda_\alpha(b\Omega)}\int_{b\Omega}\frac{|\zeta-z|^\alpha}{|\Phi(\zeta,z)|^3}d\sigma(\zeta).$$

By the smoothness of $\Phi(z,\cdot)$ on $b\Omega\setminus B(\pi(z),c)$ (c is the constant in Lemma 1.5), it is sufficient to estimate $\int_{b\Omega\cap B(\pi(z),c)}\frac{|\zeta-z|^\alpha}{|\Phi(\zeta,z)|^3}d\sigma(\zeta)$.

Let F^* be the inverse function of F. Then by Lemma 1.5, $|\zeta-z|\lesssim\sqrt{F^*(|\Phi(\zeta,z)|)}$, so

$$\int_{b\Omega\cap B(\pi(z),c)}\frac{|\zeta-z|^\alpha}{|\Phi(\zeta,z)|^3}d\sigma(\zeta)\lesssim\int_{b\Omega\cap B(\pi(z),c)}\frac{\sqrt{F^*(|\Phi(\zeta,z)|)}^\alpha}{|\Phi(\zeta,z)|^3}d\sigma(\zeta).$$

Since $\frac{F(t)}{t}$ is increasing, for each $0<\alpha<1$, $\frac{\sqrt{F^*(s)}^\alpha}{s}\circ F(t^2)=\frac{t^\alpha}{F(t^2)}=\frac{t^2}{F(t^2)}\frac{1}{t^{2-\alpha}}$ is decreasing. Hence, the chain rule in calculus yields that $\frac{\sqrt{F^*(t)}^\alpha}{t}$

is decreasing, so

$$\int_{b\Omega\cap B(\pi(z),c)}\frac{|\zeta-z|^\alpha}{|\Phi(\zeta,z)|^3}d\sigma(\zeta)\lesssim\frac{\sqrt{F^*(|\rho(z)|)}^\alpha}{|\rho(z)|}\int_{b\Omega\cap B(\pi(z),c)}\frac{d\sigma(\zeta)}{|\Phi(\zeta,z)|^2}.$$

To estimate the last integral, we need a special coordinates, called to be Henkin's coordinates.

Lemma 2.2 ([20, Lemma V.3.4]). *There exist positive constants M, a and $\eta\le\epsilon$, and, for each z with $dist(z,b\Omega)\le a$, there is a smooth local coordinate system $(x_1,x_2,x_3,x_4)=x=x(\zeta,z)$ on the ball $B(z,\eta)$ such that we have*

$$\begin{cases} x_1(\zeta,z)=\rho(\zeta),\\ x(z,z)=(\rho(z),0,0,0),\\ x_2(\zeta,z)=\mathrm{Im}(\Phi(\zeta,z)),\\ |x|<1 \ \ \textit{for } \zeta\in B(z,\eta),\\ |J_{\mathbb{R}}(x(\cdot,z))|\le M \ \textit{ and } |\det J_{\mathbb{R}}(x(\cdot,z))|\ge\frac{1}{M}.\end{cases}$$

Therefore, using $\mathbb{R}^2$ polar coordinates for (x_3,x_4) and integrating in x_2, we have

$$\begin{aligned}&\int_{b\Omega\cap B(\pi(z),c)}\frac{|\zeta-z|^\alpha}{|\Phi(\zeta,z)|^3}d\sigma(\zeta)\\ &\lesssim\frac{\sqrt{F^*(|\rho(z)|)}^\alpha}{|\rho(z)|}\int_{|(x_2,x_3,x_4)|}\frac{dx_2dx_3dx_4}{(|\rho(z)|+x_2+F(|(x_3,x_4)|)^2)|(x_3,x_4)|}\\ &\lesssim\frac{\sqrt{F^*(|\rho(z)|)}^\alpha}{|\rho(z)|}\int_{|(x_2,r)|<c}\frac{rdx_2dr}{(|\rho(z)|+x_2+F(r^2))r}\\ &\lesssim\frac{\sqrt{F^*(|\rho(z)|)}^\alpha}{|\rho(z)|}\underbrace{\int_0^c|\ln F(r^2)|dr}_{\text{finite since Definition 1.3}}\\ &\lesssim\frac{\sqrt{F^*(|\rho(z)|)}^\alpha}{|\rho(z)|}.\end{aligned}$$

The function $\frac{\sqrt{F^*(t)}^\alpha}{t}$ satisfies all conditions in the general Hardy-Littlewood Lemma, see [13, page 527] for $\alpha=1$. For general $0<\alpha<1$, it is proved similarly and we shall omit the details. Thus the proof of Theorem 1.6 is complete.

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