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THE PROPERTIES OF JORDAN DERIVATIONS OF SEMIPRIME RINGS AND BANACH ALGEBRAS, II

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Abstract. Let A be a Banach algebra with $\text{rad}(A)$. We show that if there exists a continuous linear Jordan derivation D on A, then

$$[D(x), x]D(x)^2 \in \text{rad}(A)$$

if and only if $D(x)[D(x), x]D(x) \in \text{rad}(A)$ for all $x \in A$.

1. Introduction

Throughout, R represents an associative ring and A will be a complex Banach algebra. We write $[x, y]$ for the commutator $xy - yx$ for x, y in a ring. Let $\text{rad}(R)$ denote the (*Jacobson*) *radical* of a ring R. And a ring R is said to be (*Jacobson*) *semisimple* if its Jacobson radical $\text{rad}(R)$ is zero.

A ring R is called *n-torsion free* if $nx = 0$ implies $x = 0$. Recall that R is *prime* if $aRb = (0)$ implies that either $a = 0$ or $b = 0$, and is *semiprime* if $aRa = (0)$ implies $a = 0$. On the other hand, let X be an element of a normed algebra. Then for every $a \in X$ the *spectral radius* of a, denoted by $r(a)$, is defined by $r(a) = \inf\{||a^n||^{\frac{1}{n}} : n \in \mathbb{N}\}$. It is well-known that the following theorem holds: if a is an element of a normed algebra, then $r(a) = \lim_{n\to\infty} ||a^n||^{\frac{1}{n}}$ (see Bonsall and Duncan [1]).

An additive mapping D from R to R is called a *derivation* if $D(xy) = D(x)y + xD(y)$ holds for all $x, y \in R$. And an additive mapping D from R to R is called a *Jordan derivation* if $D(x^2) = D(x)x + xD(x)$ holds for all $x \in R$.

Johnson and Sinclair [5] have proved that any linear derivation on a semisimple Banach algebra is continuous. A result of Singer and Wermer [14] states that every continuous linear derivation on a commutative Banach algebra maps the algebra into its radical. From these two results, we can conclude that there are no nonzero linear derivations on a commutative semisimple Banach algebra. Thomas [15] has proved that any linear derivation on a commutative Banach algebra maps the algebra into its radical.

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Vukman [16] has proved the following: Let R be a 2-torsion free prime ring. If $D : R \longrightarrow R$ is a derivation such that $[D(x), x]D(x) = 0$ for all $x \in R$, then $D = 0$.

Moreover, using the above result, he has proved that the following holds: Let A be a noncommutative semisimple Banach algebra. Suppose that

$$[D(x), x]D(x) = 0$$

holds for all $x \in A$. In this case, $D = 0$.

In this paper, our first aim is to prove the following result in the ring theory in order to apply it to the Banach algebra theory:

Let R be a 3!-torsion free semiprime ring, and let $D : R \to R$ be a Jordan derivation on a semiprime ring R. In this case, we show that $[D(x), x]D(x) = 0$ if and only if $D(x)[D(x), x] = 0$ for every $x \in R$. In particular, let A be a noncommutative Banach algebra with rad(A) and D is a continuous linear Jordan derivation on A, then we see that $[D(x), x]D(x) \in \text{rad}(A)$ if and only if $D(x)[D(x), x] \in \text{rad}(A)$ for all $x \in A$.

2. Preliminaries

The following lemma is due to Chung and Luh [4].

Lemma 2.1. *Let R be an $n!$-torsion free ring. Suppose there exist elements $y_1, y_2, \ldots, y_{n-1}, y_n$ in R such that $\sum_{k=1}^{n} t^k y_k = 0$ for all $t = 1, 2, \ldots, n$. Then we have $y_k = 0$ for every positive integer k with $1 \le k \le n$.*

The following theorem is due to Brešar [2].

Theorem 2.2. *Let R be a 2-torsion free semiprime ring and let $D : R \longrightarrow R$ be a Jordan derivation. In this case, D is a derivation.*

We denote by $Q(A)$ the set of all quasinilpotent elements in a Banach algebra.

Bresar [3] has also proved the following theorem.

Theorem 2.3. *Let D be a bounded derivation of a Banach algebra A. Suppose that $[D(x), x] \in Q(A)$ for every $x \in A$. Then D maps A into rad(A).*

The following theorems and lemma are due to Kim [11].

Theorem 2.4. *Let R be a 3!-torsionfree semiprime ring. Suppose there exists a Jordan derivation $D : R \longrightarrow R$ such that*

$$[[D(x), x], x]D(x) = 0$$

for all $x \in R$. Then we have $3f(x)D(x)^2 - D(x)f(x)D(x) = 0$ for all $x \in R$.

Lemma 2.5. *Let R be a 5!-torsionfree semiprime ring. Let $D : R \longrightarrow R$ be a Jordan derivation on R. And assume that*

$$g(x)D(x)yD^2(x)D(x) = [f(x), x]D(x)yD^2(x)D(x) = 0$$

for all $x, y \in R$. Then we have $g(x)D(x) = [f(x), x]D(x) = 0$ for all $x \in R$.

Theorem 2.6. *Let R be a 5!-torsionfree semiprime ring. Let $D : R \longrightarrow R$ be a Jordan derivation on R. In this case, it follows that*

$$[D(x), x]D(x)^2 = 0 \ \Leftrightarrow \ D(x)^2[D(x), x] = 0$$

for every $x \in R$.

And see [6–10, 12, 13] for the further results.

3. Main results

We need the following notations. After this, by S_m we denote the set $\{k \in \mathbb{N} \mid 1 \leq k \leq m\}$ where m is a positive integer. When R is a ring, we shall denote the maps $B : R \times R \longrightarrow R$, $f, g : R \longrightarrow R$ by $B(x, y) \equiv [D(x), y] + [D(y), x]$, $f(x) \equiv [D(x), x]$, $g(x) \equiv [f(x), x]$ for all $x, y \in R$ respectively. And we have the basic properties:

$$B(x, y) = B(y, x),\ B(x, yz) = B(x, y)z + yB(x, z) + D(y)[z, x] + [y, x]D(z),$$
$$B(x, x) = 2f(x),\ B(x, x^2) = 2(f(x)x + xf(x)),\ x, y, z \in R.$$

Theorem 3.1. *Let R be a 5!-torsionfree semiprime ring. Let $D : R \longrightarrow R$ be a Jordan derivation on R. In this case, we show that if $f(x)D(x)^2 = 0$, then $D(x)f(x)D(x) = 0$ for all $x \in R$.*

Proof. When R is commutative, it is easy to check that the statements holds. Thus it suffices to prove the case that R is noncommutative.

Assume that

$$[D(x), x]D(x)^2 = f(x)D(x)^2 = 0,\ x \in R. \tag{1}$$

Then by (211) in the proof of Theorem 2.6, we have

$$g(x)D(x)yD^2(x)D(x) = 0,\ x, y \in R. \tag{2}$$

And by Lemma 2.5, it follows from (2) that

$$g(x)D(x) = 0,\ x \in R. \tag{3}$$

And again, by Theorem 2.4, the relation (3) yields

$$3f(x)D(x)^2 - D(x)f(x)D(x) = 0,\ x \in R. \tag{4}$$

Then by the assumption that $f(x)D(x)^2 = 0$ for all $x \in R$, we obtain from (4)

$$D(x)f(x)D(x) = 0,\ x \in R. \tag{5}$$

□

Theorem 3.2. *Let R be a 5!-torsionfree semiprime ring. Let $D : R \longrightarrow R$ be a Jordan derivation on R. In this case, we show that if $D(x)f(x)D(x) = 0$, then $f(x)^9 = 0$ for all $x \in R$.*

Proof. When R is commutative, it is easy to check that the statement holds. Thus it suffices to prove the case that R is noncommutative.

Suppose

$$D(x)[D(x),x]D(x) = D(x)f(x)D(x) = 0,\ x \in R. \tag{6}$$

Replacing $x+ty$ for x in (6), we have

$$\begin{aligned} & D(x+ty)[D(x+ty),x+ty]D(x+ty) \\ &\equiv D(x)f(x)D(x)+t\{D(y)f(x)D(x)+D(x)B(x,y)D(x)+D(x)f(x)D(y)\} \\ &\quad + t^2H_1(x,y) + t^3H_2(x,y) + t^4D(y)f(y)D(y) = 0,\ x,y \in R,\ t \in S_3, \end{aligned} \tag{7}$$

where H_1 and H_2 denote the terms satisfying the identity (7).

From (6) and (7), we obtain

$$\begin{aligned} & t\{D(y)f(x)D(x) + D(x)B(x,y)D(x) + D(x)f(x)D(y)\} \\ &\quad + t^2H_1(x,y) + t^3H_2(x,y) = 0,\ x,y \in R,\ t \in S_3. \end{aligned} \tag{8}$$

Since R is 3!-torsionfree, by Lemma 2.1 the relation (8) yields

$$D(y)f(x)D(x) + D(x)B(x,y)D(x) + D(x)f(x)D(y) = 0,\ x,y \in R. \tag{9}$$

Let $y = x$ in (9). Then we get

$$\begin{aligned} 0 &= \{D(x)x + xD(x)\}f(x)D(x) + 2D(x)\{f(x)x + xf(x)\}D(x) \\ &\quad + D(x)f(x)\{D(x)x + xD(x)\} \\ &= 3D(x)xf(x)D(x) + xD(x)f(x)D(x) + 3D(x)f(x)xD(x) \\ &\quad + D(x)f(x)D(x)x,\ x \in R. \end{aligned} \tag{10}$$

From (6) and (10), we obtain

$$0 = 3D(x)xf(x)D(x) + 3D(x)f(x)xD(x),\ x \in R. \tag{11}$$

From (6) and (11), we arrive at

$$\begin{aligned} 0 &= 3D(x)xf(x)D(x) + 3D(x)f(x)xD(x) \\ &= 3D(x)xf(x)D(x) - 3xD(x)f(x)D(x) + 3D(x)f(x)xD(x) \\ &\quad - 3xD(x)f(x)D(x) \\ &= 3[D(x),x]f(x)D(x) + 3[D(x)f(x),x]D(x) \\ &= 6f(x)^2D(x) + 3D(x)g(x)D(x),\ x \in R. \end{aligned} \tag{12}$$

Since R is 5!-torsion free, (12) yields

$$2f(x)^2D(x) + D(x)g(x)D(x) = 0,\ x \in R. \tag{13}$$

On the other hand, we have

$$\begin{aligned} 0 &= [D(x)f(x)D(x),x] \\ &= f(x)^2D(x) + D(x)g(x)D(x) + D(x)f(x)^2,\ x \in R. \end{aligned} \tag{14}$$

From (13) and (14), we obtain

$$f(x)^2D(x) - D(x)f(x)^2 = 0,\ x \in R. \tag{15}$$

Writing xy for y in (9), we have

$$\begin{aligned} &xD(y)f(x)D(x) + D(x)yf(x)D(x) + D(x)xB(x,y)D(x) \\ &+ D(x)f(x)yD(x) + D(x)^2[y,x]D(x) + D(x)f(x)xD(y) \\ &+ D(x)f(x)D(x)y = 0,\ x,y \in R. \end{aligned} \tag{16}$$

Left multiplication of (9) by x leads to

$$xD(y)f(x)D(x) + xD(x)B(x,y)D(x) + xD(x)f(x)D(y) = 0,\ x,y \in R. \tag{17}$$

From (16) and (17), we obtain

$$\begin{aligned} &D(x)yf(x)D(x) + f(x)B(x,y)D(x) \\ &+ D(x)f(x)yD(x) + D(x)^2[y,x]D(x) \\ &+ \{f(x)^2 + D(x)g(x)\}D(y) + D(x)f(x)D(x)y = 0,\ x,y \in R. \end{aligned} \tag{18}$$

From (6) and (18), we get

$$\begin{aligned} &D(x)yf(x)D(x) + f(x)B(x,y)D(x) + D(x)f(x)yD(x) \\ &+ D(x)^2[y,x]D(x) + \{f(x)^2 + D(x)g(x)\}D(y) = 0,\ x,y \in R. \end{aligned} \tag{19}$$

Right multiplication of (19) by x leads to

$$\begin{aligned} &D(x)yf(x)D(x)x + f(x)B(x,y)D(x)x + D(x)f(x)yD(x)x \\ &+ D(x)^2[y,x]D(x)x + \{f(x)^2 + D(x)g(x)\}D(y)x = 0,\ x,y \in R. \end{aligned} \tag{20}$$

Writing yx for y in (19), we have

$$\begin{aligned} &D(x)yxf(x)D(x) + f(x)B(x,y)xD(x) + 2f(x)yf(x)D(x) \\ &+ f(x)[y,x]D(x)^2 + D(x)f(x)yxD(x) + D(x)^2[y,x]xD(x) \\ &+ \{f(x)^2+D(x)g(x)\}D(y)x+\{f(x)^2+D(x)g(x)\}yD(x) = 0,\ x,y \in R. \end{aligned} \tag{21}$$

From (20) and (21), we obtain

$$\begin{aligned} &D(x)y\{g(x)D(x) + f(x)^2\} + f(x)B(x,y)f(x) + D(x)f(x)yf(x) \\ &- 2f(x)yf(x)D(x) - f(x)[y,x]D(x)^2 + D(x)^2[y,x]f(x) \\ &- \{f(x)^2 + D(x)g(x)\}yD(x) = 0,\ x,y \in R. \end{aligned} \tag{22}$$

Let $y = x$ in (22). Then we get

$$\begin{aligned} 0 &= D(x)x\{g(x)D(x) + f(x)^2\} + 2f(x)^3 + D(x)f(x)xf(x) \\ &\quad - 2f(x)xf(x)D(x) - \{f(x)^2 + D(x)g(x)\}xD(x) \\ &= D(x)x\{g(x)D(x) + f(x)^2\} + 2f(x)^3 + D(x)f(x)xf(x) \\ &\quad - 2f(x)xf(x)D(x) - \{f(x)^2 + D(x)g(x)\}xD(x),\ x \in R. \end{aligned} \tag{23}$$

Left multiplication of (23) by $D(x)f(x)$ leads to

$$\begin{aligned}0 = {} & D(x)f(x)D(x)x\{g(x)D(x) + f(x)^2\} + 2D(x)f(x)^4 \\ & + D(x)f(x)D(x)f(x)xf(x) - 2D(x)f(x)f(x)xf(x)D(x) \\ & - D(x)f(x)\{f(x)^2 + D(x)g(x)\}xD(x), \ x \in R. \end{aligned} \tag{24}$$

From (6) and (24), we obtain

$$\begin{aligned}0 &= 2D(x)f(x)^4 - 2D(x)f(x)^2xf(x)D(x) - D(x)f(x)^3xD(x) \\ &= 2D(x)f(x)^4 - 2D(x)f(x)^2xf(x)D(x) + 2D(x)f(x)^3D(x)x \\ &\quad - D(x)f(x)^3xD(x) + D(x)f(x)^3D(x)x \\ &= 2D(x)f(x)^4 + 2D(x)f(x)^2\{g(x)D(x) + f(x)^2\} + D(x)f(x)^4 \\ &= 5D(x)f(x)^4 + 5D(x)f(x)^2g(x)D(x), \ x \in R. \end{aligned} \tag{25}$$

From (15) and (25), we have

$$5D(x)f(x)^4 + 5f(x)^2D(x)g(x)D(x) = 0, \ x \in R. \tag{26}$$

From (13), (15) and (26), we obtain

$$\begin{aligned}0 &= 5D(x)f(x)^4 + 5f(x)^2\{-2f(x)^2D(x)\} \\ &= 5D(x)f(x)^4 - 10f(x)^2\{f(x)^2D(x)\} \\ &= 5D(x)f(x)^4 - 10f(x)^2\{D(x)f(x)^2\} \\ &= 5D(x)f(x)^4 - 10D(x)f(x)^4 \\ &= -5D(x)f(x)^4, \ x \in R. \end{aligned} \tag{27}$$

Since R is 5!-torsion free, it follows from (27) that

$$D(x)f(x)^4 = 0, \ x \in R. \tag{28}$$

From (15) and (28), we get

$$f(x)^4D(x) = f(x)^2D(x)f(x)^2 = D(x)f(x)^4 = 0, \ x \in R. \tag{29}$$

From (28), we obtain

$$\begin{aligned}0 &= [D(x)f(x)^4, x] \\ &= f(x)^5 + D(x)g(x)f(x)^3 + D(x)f(x)g(x)f(x)^2 + D(x)f(x)^2g(x)f(x) \\ &\quad + D(x)f(x)^3g(x), \ x \in R. \end{aligned} \tag{30}$$

Left multiplication of (30) by $f(x)^4$ leads to

$$\begin{aligned}0 = {} & f(x)^9 + f(x)^4D(x)g(x)f(x)^3 + f(x)^4D(x)f(x)g(x)f(x)^2 \\ & + f(x)^4D(x)f(x)^2g(x)f(x) + f(x)^4D(x)f(x)^3g(x), \ x \in R. \end{aligned} \tag{31}$$

From (29) and (31), we arrive at

$$f(x)^9 = 0, \ x \in R. \tag{32}$$

□

We have the following statement by applying Theorems 3.1 and 2.3 to it, and its proof is partly proved by the same arguments as in the proof of J. Vukman's theorem [17].

Theorem 3.3. *Let A be a Banach algebra with rad(A). Let $D : A \longrightarrow A$ be a continuous linear Jordan derivation. Then we obtain that*

$$[D(x), x]D(x)^2 \in rad(A) \iff D(x)[D(x), x]D(x) \in rad(A)$$

for every $x \in A$.

Proof. It suffices to prove the case that A is noncommutative. By the result of B. E. Johnson and A. M. Sinclair [5] any linear derivation on a semisimple Banach algebra is continuous. Sinclair [13] has proved that every continuous linear Jordan derivation on a Banach algebra leaves the primitive ideals of A invariant. Hence for any primitive ideal $P \subseteq A$ one can introduce a derivation $D_P : A/P \longrightarrow A/P$, where A/P is a prime and factor Banach algebra, by $D_P(\hat{x}) = D(x) + P$, $\hat{x} = x + P$. By the assumption that $[D(x), x]D(x)^2 \in \text{rad}(A)$, $x \in A$. we obtain $[D_P(\hat{x}), \hat{x}](D_P(\hat{x}))^2 = \hat{0}$ for all $\hat{x} = x + P$ and primitive ideals P of A. By (211) in the proof of Theorem 2.6, $[[D_P(\hat{x}), \hat{x}], \hat{x}]D_P(\hat{x})\hat{y}((D_P)^2(\hat{x})D_P(\hat{x}) = \hat{0}$ for all $\hat{y} \in A/P$ and $\hat{x} = x + P$. Then by Lemma 2.5, we get $[[D_P(\hat{x}), \hat{x}], \hat{x}]D_P(\hat{x}) = \hat{0}$ for all $\hat{x} = x + P$ and primitive ideals P of A. Hence from Theorem 2.6, we get $3[D_P(\hat{x}), \hat{x}](D_P(\hat{x}))^2 - D_P(\hat{x})[D_P(\hat{x}), \hat{x}]D_P(\hat{x}) = \hat{0}$ for all $\hat{x} = x + P$ and primitive ideals P of A. And by the assumption that $[D_P(\hat{x}), \hat{x}](D_P(\hat{x}))^2 = \hat{0}$ for all $\hat{x} = x + P$ and primitive ideals P of A, we arrive at $D_P(\hat{x})[D_P(\hat{x}), \hat{x}]D_P(\hat{x}) = \hat{0}$ for all $\hat{x} = x + P$ and primitive ideals P of A. Thus we have $D(x)[D(x), x]D(x) \in P$ for all $x \in A$ and primitive ideals P of A. That is, it follows that $D(x)[D(x), x]D(x) \in \cap P = \text{rad}(A)$ for all $x \in A$ and primitive ideals P of A. Hence the necessity is proved.

On the other hand, assume that $D(x)[D(x), x]D(x) \in \text{rad}(A)$ for all $x \in A$. Then we get $(D_P(\hat{x})[D_P(\hat{x}), \hat{x}]D_P(\hat{x}) = \hat{0}$ for all $\hat{x} = x + P$ and primitive ideals P of A. By Theorem 3,3, we obtain $[D_P(\hat{x}), \hat{x}]^9 = \hat{0}$ for all $x \in A$. Then by Theorem 3.3, we get $(r_P([D_P(\hat{x}), \hat{x}])^9 = r_P([D_P(\hat{x}), \hat{x}]^9) = \hat{0}$ for all $x \in A$ and all primitive ideals P of A where r_P denotes the spectral radius in A/P. Hence we have $r_P([D_P(\hat{x}), \hat{x}]) = \hat{0}$ for all $x \in A$ and all primitive ideals P of A. Thus by Theorem 2.3 we obtain $D_P(\hat{x}) \in \text{rad}(A/P) = \hat{0}$ for all $x \in A$ and all primitive ideals P of A. Hence we get $D(x) \in \cap P = \text{rad}(A)$ for all $x \in A$ and all primitive ideals P of A. Thus since rad(A) is a closed two sided ideal of A, it follows that $[D(x), x]D(x)^2 \in [D(x), x]\text{rad}(A) \subseteq \text{rad}(A)$ for all $x \in A$. □

As a special case of Theorem 3.3 we get the following result which characterizes commutative semisimple Banach algebras.

Corollary 3.4. *Let A be a semisimple Banach algebra. Suppose*

$$[y, x][[y, x], x]][y, x] = 0 \iff [[y, x], x][y, x]^2 = 0$$

for every $x, y \in A$.

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