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CHEMICAL HYPERSTRUCTURES FOR OZONE DEPLETION

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Abstract. In this paper, we study the mathematical hyperstructures of ozone(O_3) depletion by Freon gas.

1. Introduction

In 1934, F. Marty[11] introduced algebraic hyperstructures as an extension of traditional algebra.

Half a century later, several mathematicians[6, 7] studied about algebraic hyperstructures, and in 1994, T. Vougiouklis[18] generalized the concept of algebraic hyperstructures and studied H_v-groups. Later, in 2013 B. Davvaz[8] summarized the basic algebraic hyperstructures.

Until recently, many researchers[2, 3, 4, 5, 10, 9, 13, 14, 15, 16, 17] have studied the mathematical hyperstructures of biological, chemical and physical reactions.

Due to environmental pollution, the Earth's Ozone destruction is underway. In this paper, we give the mathematical hyperstructures of chemical reactions for a set of ozone(O_3) depletion by Freon gas.

2. Hyperalgebraic structures

Let H be a non-empty set and a function $\cdot : H \times H \longrightarrow \wp^*(H)$ be a *hyperoperation*, where $\wp^*(H)$ is the set of all non-empty subset of H. The couple $(H, \cdot)$ is called a *hypergroupoid*. For the subset A, B of H, we define $A \cdot B = \cup_{a\in A, b\in B} a \cdot b$, and for a singleton $\{a\}$ we denote $\{a\} \cdot B = a \cdot B$ and $B \cdot \{a\} = B \cdot a$.

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Definition 2.1. The hypergroupoid $(H, \cdot)$ is called a *semihypergroup* if

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z, \text{ for all } x, y, z \in H.$$

In this case, the hyperoperation $(\cdot)$ is called *associate*.

The hypergroupoid $(H, \cdot)$ is called an H_v*-semigroup* if

$$x \cdot (y \cdot z) \cap (x \cdot y) \cdot z \neq \emptyset, \text{ for all } x, y, z \in H.$$

In this case, the hyperoperation $(\cdot)$ is called *weak associate*.

The hypergroupoid $(H, \cdot)$ is called a *quasihypergroup* if

$$x \cdot H = H \cdot x = H, \text{ for all } x \in H.$$

The hyperoperation $(\cdot)$ is called *commutative* if

$$x \cdot y = y \cdot x, \text{ for all } x, y \in H.$$

The hypergroupoid $(H, \cdot)$ is called a *hypergroup* if it is a semihypergroup and a quasihypergroup. If a non-empty subset B of H is a hypergroup, then $(B, \cdot)$ is called a *subhypergroup* of H.

The hypergroupoid $(H, \cdot)$ is called an H_v*-group* if it is an H_v-semigroup and a quasihypergroup. If a non-empty subset B of H is an H_v-group, then $(B, \cdot)$ is called a H_v*-subgroup* of H.

The hypergroupoid $(H, \cdot)$ is called a *commutative hypergroup* if it is a hypergroup with a commutative hyperoperation $(\cdot)$.

The hypergroupoid $(H, \cdot)$ is called a *commutative* H_v*-group* if it is an H_v-group with a commutative hyperoperation $(\cdot)$.

Let $(H_1, \cdot)$ and $(H_2, *)$ be two H_v-groups. A map $f : H_1 \longrightarrow H_2$ is called an H_v*-homomorphism* or *weak homomorphism* if

$$f(x \cdot y) \cap f(x) * f(y) \neq \emptyset, \text{ for all } x, y \in H_1.$$

f is called an *inclusion homomorphism* if

$$f(x \cdot y) \subset f(x) * f(y), \text{ for all } x, y \in H_1.$$

Finally, f is called a *strong homomorphism* if

$$f(x \cdot y) = f(x) * f(y), \text{ for all } x, y \in H_1.$$

If f is onto, one to one and strong homomorphism, then it is called an *isomorphism*. In this case, H_1 and H_2 are called *isomorphic* and we write $H_1 \cong H_2$.

3. Chemical reactions in ozone depletion

The destruction of ozone is due to the freon gas used as the refrigerant in the refrigerator and the halon gas used as the extinguisher in the fire extinguishers.

There are many kinds of freon gas(CFCs). For example, $CFCl_3, CF_2Cl_2, C_2F_3Cl_3, C_2F_4Cl_2, C_2F_5Cl, CHF_2Cl, C_2HF_3Cl_2, C_2HF_4Cl, C_2H_3FCl_2$ and $C_2H_3F_2Cl$ are freon gases.

When the freon gas reaches the stratosphere, it is decomposed by ultraviolet rays to form chlorine atoms.

$$CF_xCl_y + UV\text{-}C \quad \longrightarrow \quad CF_xCl_{y-1} + Cl$$

In this paper, we study some algebraic hyperstructures for ozone depletion by freon gas $CFCl_3$. In ozone decomposition, the chemical reactions take the following 3 steps[1, 12, 19]:

$$CFCl_3 \longrightarrow CFCl_2 + Cl \tag{1}$$
$$Cl + O_3 \longrightarrow ClO + O_2 \tag{2}$$
$$ClO + O_3 \longrightarrow Cl + 2O_2 \tag{3}$$

The above chemical reaction shows the loss of ozone. Other freon gases also show similar reactions.

We give a hyperoperation table for the set $\{CFCl_3, CFCl_2, Cl, ClO, O_2, O_3\}$ of the chemical elements obtained from the above chemical reactions.

Definition 3.1. Let G be the set $\{CFCl_3, CFCl_2, Cl, ClO, O_2, O_3\}$ of the chemical elements and a *hyperoperation* $\oplus_1$ on G is defined as follows:

$$\oplus_1 : G \times G \to \wp^*(G)$$

where $\wp^*(G)$ is the set of all non-empty subset of G. For all $x, y \in G$, $x \oplus_1 y$ is defined as the union of all the possible chemical reactions that appear in the above primary reactions (1) $\sim$ (3). If $x \oplus_1 y$ does not appear in the above reactions (1) $\sim$ (3), then we define as follows:

$$x \oplus_1 y = \{x, y\} \tag{4}$$

Then we can define $x \oplus_1 y$ for elements $x, y \in \{CFCl_3, CFCl_2, Cl, ClO, O_2, O_3\}$ as follows.

First for the $x = CFCl_3$,

$$\begin{aligned}
CFCl_3 \oplus_1 CFCl_3 &= CFCl_3 \oplus_1 CFCl_3 &&= \{CFCl_2, Cl\} \text{ by (1).}\\
CFCl_3 \oplus_1 CFCl_2 &= CFCl_2 \oplus_1 CFCl_3 &&= \{CFCl_3, CFCl_2\} \text{ by (4).}\\
CFCl_3 \oplus_1 Cl &= Cl \oplus_1 CFCl_3 &&= \{CFCl_3, Cl\} \text{ by (4).}\\
CFCl_3 \oplus_1 ClO &= ClO \oplus_1 CFCl_3 &&= \{CFCl_3, ClO\} \text{ by (4).}\\
CFCl_3 \oplus_1 O_2 &= O_2 \oplus_1 CFCl_3 &&= \{CFCl_3, O_2\} \text{ by (4).}\\
CFCl_3 \oplus_1 O_3 &= O_3 \oplus_1 CFCl_3 &&= \{CFCl_3, O_3\} \text{ by (4).}
\end{aligned}$$

Next for the $x = CFCl_2$,

$$\begin{aligned}
CFCl_2 \oplus_1 CFCl_2 &= CFCl_2 \oplus_1 CFCl_2 &&= \{CFCl_2\} \text{ by (4).}\\
CFCl_2 \oplus_1 Cl &= Cl \oplus_1 CFCl_2 &&= \{CFCl_2, Cl\} \text{ by (4).}\\
CFCl_2 \oplus_1 ClO &= ClO \oplus_1 CFCl_2 &&= \{CFCl_2, ClO\} \text{ by (4).}\\
CFCl_2 \oplus_1 O_2 &= O_2 \oplus_1 CFCl_2 &&= \{CFCl_2, O_2\} \text{ by (4).}\\
CFCl_2 \oplus_1 O_3 &= O_3 \oplus_1 CFCl_2 &&= \{CFCl_2, O_3\} \text{ by (4).}
\end{aligned}$$

For the $x = Cl$,

$$\begin{aligned}
Cl \oplus_1 Cl &= Cl \oplus_1 Cl &&= \{Cl\} \text{ by (4).}\\
Cl \oplus_1 ClO &= ClO \oplus_1 Cl &&= \{Cl, ClO\} \text{ by (4).}\\
Cl \oplus_1 O_2 &= O_2 \oplus_1 Cl &&= \{Cl, O_2\} \text{ by (4).}\\
Cl \oplus_1 O_3 &= O_3 \oplus_1 Cl &&= \{ClO, O_2\} \text{ by (2).}
\end{aligned}$$

For the $x = ClO$,

$$\begin{aligned}
ClO \oplus_1 ClO &= ClO \oplus_1 ClO &&= \{ClO\} \text{ by (4).}\\
ClO \oplus_1 O_2 &= O_2 \oplus_1 ClO &&= \{ClO, O_2\} \text{ by (4).}\\
ClO \oplus_1 O_3 &= O_3 \oplus_1 ClO &&= \{Cl, O_2\} \text{ by (3).}
\end{aligned}$$

For the $x = O_2$,

$$\begin{aligned}
O_2 \oplus_1 O_2 &= O_2 \oplus_1 O_2 &&= \{O_2\} \text{ by (4).}\\
O_2 \oplus_1 O_3 &= O_3 \oplus_1 O_2 &&= \{O_2, O_3\}) \text{ by (4).}
\end{aligned}$$

For the $x = O_3$,

$$O_3 \oplus_1 O_3 = O_3 \oplus_1 O_3 = \{O_3\} \text{ by (4).}$$

We have a hyperoperation table for the set $\{CFCl_3, CFCl_2, Cl, ClO, O_2, O_3\}$ in the primary reactions (1) $\sim$ (4).

$\oplus_1$	$CFCl_3$	$CFCl_2$	Cl	ClO	O_2	O_3
$CFCl_3$	$CFCl_2$ Cl	$CFCl_3$ $CFCl_2$	$CFCl_3$ Cl	$CFCl_3$ ClO	$CFCl_3$ O_2	$CFCl_3$ O_3
$CFCl_2$	$CFCl_3$ $CFCl_2$	$CFCl_2$	$CFCl_2$ Cl	$CFCl_2$ ClO	$CFCl_2$ O_2	$CFCl_2$ O_3
Cl	$CFCl_3$ Cl	$CFCl_2$ Cl	Cl	Cl ClO	Cl O_2	ClO O_2
ClO	$CFCl_3$ ClO	$CFCl_2$ ClO	Cl ClO	ClO	ClO O_2	Cl O_2
O_2	$CFCl_3$ O_2	$CFCl_2$ O_2	Cl O_2	ClO O_2	O_2	O_2 O_3
O_3	$CFCl_3$ O_3	$CFCl_2$ O_3	ClO O_2	Cl O_2	O_2 O_3	O_3

TABLE 1. Hyperoperation table for the operation $\oplus_1$

From the Table 1, if we change the name from $CFCl_3$, $CFCl_2$, Cl, ClO, O_2 and O_3 to a, b, c, d, e and f, respectively, then the following Theorem 3.2 holds.

THEOREM 3.2. *Let $G = \{a, b, c, d, e, f\}$ be the set of the chemical elements obtained from the chemical reactions in ozone depletion and $\oplus_1$ be the hyperoperation on G. Consider the following hyperoperation table:*

$\oplus_1$	a	b	c	d	e	f
a	$\{b,c\}$	$\{a,b\}$	$\{a,c\}$	$\{a,d\}$	$\{a,e\}$	$\{a,f\}$
b	$\{a,b\}$	$\{b\}$	$\{b,c\}$	$\{b,d\}$	$\{b,e\}$	$\{b,f\}$
c	$\{a,c\}$	$\{b,c\}$	$\{c\}$	$\{c,d\}$	$\{c,e\}$	$\{d,e\}$
d	$\{a,d\}$	$\{b,d\}$	$\{c,d\}$	$\{d\}$	$\{d,e\}$	$\{c,e\}$
e	$\{a,e\}$	$\{b,e\}$	$\{c,e\}$	$\{d,e\}$	$\{e\}$	$\{e,f\}$
f	$\{a,f\}$	$\{b,f\}$	$\{d,e\}$	$\{c,e\}$	$\{e,f\}$	$\{f\}$

TABLE 2. Hyperoperation table for the operation $\oplus_1$

Then we have the following.

(1) *The hyperstructures* $(\{b,c\},\oplus_1)$, $(\{b,d\},\oplus_1)$, $(\{b,e\},\oplus_1)$, $(\{b,f\},\oplus_1)$, $(\{c,d\},\oplus_1)$, $(\{c,e\},\oplus_1)$, $(\{d,e\},\oplus_1)$, $(\{e,f\},\oplus_1)$ *are commutative subhypergroups of* G *and isomorphic. The isomorphic hyperoperation table is the following:*

$\oplus_1$	x	y
x	$\{x\}$	$\{x,y\}$
y	$\{x,y\}$	$\{y\}$

(2) *The hyperstructure* $(\{a,b,c\},\oplus_1)$ *is a commutative* H_v*-subgroups of* G*. The hyperoperation table is the following:*

$\oplus_1$	a	b	c
a	$\{b,c\}$	$\{a,b\}$	$\{a,c\}$
b	$\{a,b\}$	$\{b\}$	$\{b,c\}$
c	$\{a,c\}$	$\{b,c\}$	$\{c\}$

The hyperstructures $(\{b,c,d\},\oplus_1)$, $(\{b,c,e\},\oplus_1)$, $(\{b,d,e\},\oplus_1)$, $(\{b,e,f\},\oplus_1)$ *and* $(\{c,d,e\},\oplus_1)$ *are commutative subhypergroups of* G *and isomorphic. The isomorphic hyperoperation table is the following:*

$\oplus_1$	x	y	z
x	$\{x\}$	$\{x,y\}$	$\{x,z\}$
y	$\{x,y\}$	$\{y\}$	$\{y,z\}$
z	$\{x,z\}$	$\{y,z\}$	$\{z\}$

(3) *The hyperstructures* $(\{a,b,c,d\},\oplus_1)$ *and* $(\{a,b,c,e\},\oplus_1)$ *are commutative* H_v*-subgroups of* G *and isomorphic. The isomorphic hyperoperation table is the following:*

$\oplus_1$	x	y	z	w
x	$\{y,z\}$	$\{x,y\}$	$\{x,z\}$	$\{x,w\}$
y	$\{x,y\}$	$\{y\}$	$\{y,z\}$	$\{y,w\}$
z	$\{x,z\}$	$\{y,z\}$	$\{z\}$	$\{z,w\}$
w	$\{x,w\}$	$\{y,w\}$	$\{z,w\}$	$\{w\}$

The hyperstructure $(\{b,c,d,e\},\oplus_1)$ *is a commutative subhypergroup of* G*. The hyperoperation table is the following:*

$\oplus_1$	b	c	d	e
b	$\{b\}$	$\{b,c\}$	$\{b,d\}$	$\{b,e\}$
c	$\{b,c\}$	$\{c\}$	$\{c,d\}$	$\{c,e\}$
d	$\{b,d\}$	$\{c,d\}$	$\{d\}$	$\{d,e\}$
e	$\{b,e\}$	$\{c,e\}$	$\{d,e\}$	$\{e\}$

The hyperstructure $(\{c,d,e,f\},\oplus_1)$ *is a commutative* H_v*-subsemigroup of* G*. The hyperoperation table is the following:*

$\oplus_1$	c	d	e	f
c	$\{c\}$	$\{c,d\}$	$\{c,e\}$	$\{d,e\}$
d	$\{c,d\}$	$\{d\}$	$\{d,e\}$	$\{c,e\}$
e	$\{c,e\}$	$\{d,e\}$	$\{e\}$	$\{e,f\}$
f	$\{d,e\}$	$\{c,e\}$	$\{e,f\}$	$\{f\}$

(4) *The hyperstructure* $(\{a,b,c,d,e\},\oplus_1)$ *is a commutative* H_v*-subgroup of* G*. The hyperoperation table is the following:*

$\oplus_1$	a	b	c	d	e
a	$\{b,c\}$	$\{a,b\}$	$\{a,c\}$	$\{a,d\}$	$\{a,e\}$
b	$\{a,b\}$	$\{b\}$	$\{b,c\}$	$\{b,d\}$	$\{b,e\}$
c	$\{a,c\}$	$\{b,c\}$	$\{c\}$	$\{c,d\}$	$\{c,e\}$
d	$\{a,d\}$	$\{b,d\}$	$\{c,d\}$	$\{d\}$	$\{d,e\}$
e	$\{a,e\}$	$\{b,e\}$	$\{c,e\}$	$\{d,e\}$	$\{e\}$

The hyperstructure $(\{b,c,d,e,f\},\oplus_1)$ *is a commutative* H_v*-subsemigroup of* G*. The hyperoperation table is the following:*

$\oplus_1$	b	c	d	e	f
b	$\{b\}$	$\{b,c\}$	$\{b,d\}$	$\{b,e\}$	$\{b,f\}$
c	$\{b,c\}$	$\{c\}$	$\{c,d\}$	$\{c,e\}$	$\{d,e\}$
d	$\{b,d\}$	$\{c,d\}$	$\{d\}$	$\{d,e\}$	$\{c,e\}$
e	$\{b,e\}$	$\{c,e\}$	$\{d,e\}$	$\{e\}$	$\{e,f\}$
f	$\{b,f\}$	$\{d,e\}$	$\{c,e\}$	$\{e,f\}$	$\{f\}$

(5) *The hyperstructure* $(G,\oplus_1)$ *is a commutative* H_v*-semigroup.*

Proof. In order to show that hyperoperation $\oplus_1$ satisfies weak associative law, we must identify the following properties: for all elements x', y', z', we show that $x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \neq \emptyset$.

(1) It is obvious.

(2) In the case $(\{a,b,c\}, \oplus_1)$, we have the following: for $x', y', z' \in \{a,b,c\}$

$$\begin{cases} x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni c, & \text{if } \{x', y', z'\} \ni c; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni b, & \text{otherwise.} \end{cases}$$

Thus the hypergroupoid $(\{a,b,c\}, \oplus_1)$ is an H_v-semigroup. However, the hyperoperation $\oplus_1$ is not associative; for example, since $a \oplus_1 (a \oplus_1 b) = \{a,b,c\}$ and $(a \oplus_1 a) \oplus_1 b = \{b,c\}$, we have

$$a \oplus_1 (a \oplus_1 b) \neq (a \oplus_1 a) \oplus_1 b.$$

Therefore $(\{a,b,c\}, \oplus_1)$ is not a hypergroup, but it is a commutative quasihypergroup. Hence $(\{a,b,c\}, \oplus_1)$ is a commutative H_v-subgroup of G.

In the cases $(\{b,c,d\}, \oplus_1)$, $(\{b,c,e\}, \oplus_1)$, $(\{b,d,e\}, \oplus_1)$, $(\{b,e,f\}, \oplus_1)$ and $(\{c,d,e\}, \oplus_1)$, they all have the same type of hyperoperation tables with $\{x,y,z\}$ and are therefore isomorphic. On the other hand, we have the following: for $x', y', z' \in \{x,y,z\}$

$$x' \oplus_1 (y' \oplus_1 z') = (x' \oplus_1 y') \oplus_1 z' = \{x', y', z'\}$$

Thus the hypergroupoid $(\{x,y,z\}, \oplus_1)$ is a semihypergroup. Obviously it is a commutative quasihypergroup. Hence $(\{x,y,z\}, \oplus_1)$ is a commutative subhypergroup of G.

(3) In the cases $(\{a,b,c,d\}, \oplus_1)$ and $(\{a,b,c,e\}, \oplus_1)$, since they have the same type of hyperoperation tables with $\{x,y,z,w\}$, they are isomorphic. On the other hand, we have the following: for $x', y', z' \in \{x,y,z,w\}$

$$\begin{cases} x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni z, & \text{if } \{x', y', z'\} \ni z; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni w, & \text{if } \{x', y', z'\} \ni w; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni y, & \text{otherwise.} \end{cases}$$

Thus the hypergroupoid $(\{x,y,z,w\}, \oplus_1)$ is an H_v-semigroup. But the hyperoperation $\oplus_1$ is not associative; for example, since $x \oplus_1 (x \oplus_1 y) = \{x,y,z\}$ and $(x \oplus_1 x) \oplus_1 y = \{y,z\}$, we have

$$x \oplus_1 (x \oplus_1 y) \neq (x \oplus_1 x) \oplus_1 y.$$

Therefore $(\{x,y,z,w\}, \oplus_1)$ is not a hypergroup. Obviously it is a commutative quasihypergroup. Hence $(\{x,y,z,w\}, \oplus_1)$ is a commutative H_v-subgroup of G.

In the case $(\{b,c,d,e\}, \oplus_1)$, we have the following: for $x', y', z' \in \{b,c,d,e\}$

$$x' \oplus_1 (y' \oplus_1 z') = (x' \oplus_1 y') \oplus_1 z' = \{x', y', z'\}$$

Thus the hypergroupoid $(\{b, c, d, e\}, \oplus_1)$ is a semihypergroup. Obviously it is a commutative quasihypergroup. Hence $(\{b, c, d, e\}, \oplus_1)$ is a commutative subhypergroup of G.

In the case $(\{c, d, e, f\}, \oplus_1)$, we have the following: for $x', y', z', r \in \{c, d, e, f\}$

$$\begin{cases} x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' = \{r\}, & \text{if } \{x', y', z'\} = \{r\}; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni c \text{ or } e, & \text{otherwise.} \end{cases}$$

Thus the hypergroupoid $(\{c, d, e, f\}, \oplus_1)$ is an H_v-semigroup. But the hyperoperation $\oplus_1$ is not associative; for example, since $c \oplus_1 (c \oplus_1 f) = \{c, d, e\}$ and $(c \oplus_1 c) \oplus_1 f = \{d, e\}$, we have

$$c \oplus_1 (c \oplus_1 f) \neq (c \oplus_1 c) \oplus_1 f.$$

Therefore $(\{c, d, e, f\}, \oplus_1)$ is not a hypergroup. Moreover, since $c \oplus_1 \{c, d, e, f\} = \{c, d, e\} \neq \{c, d, e, f\}$, it is not a quasihypergroup. Hence $(\{c, d, e, f\}, \oplus_1)$ is a commutative H_v-subsemigroup of G.

(4) In the case $(\{a, b, c, d, e\}, \oplus_1)$, we have the following: for $x', y', z' \in \{a, b, c, d, e\}$

$$\begin{cases} x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni c, & \text{if } \{x', y', z'\} \ni c; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni d, & \text{if } \{x', y', z'\} \ni d; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni e, & \text{if } \{x', y', z'\} \ni e; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni b, & \text{otherwise.} \end{cases}$$

Thus the hypergroupoid $(\{a, b, c, d, e\}, \oplus_1)$ is an H_v-semigroup. But the hyperoperation $\oplus_1$ is not associative; for example, since $a \oplus_1 (a \oplus_1 b) = \{a, b, c\}$ and $(a \oplus_1 a) \oplus_1 b = \{b, c\}$, we have

$$a \oplus_1 (a \oplus_1 b) \neq (a \oplus_1 a) \oplus_1 b.$$

Therefore $(\{a, b, c, d, e\}, \oplus_1)$ is not a hypergroup. Obviously it is a commutative quasihypergroup. Hence $(\{a, b, c, d, e\}, \oplus_1)$ is a commutative H_v-subgroup of G.

In the case $(\{b, c, d, e, f\}, \oplus_1)$, we have the following: for $x', y', z', r \in \{b, c, d, e, f\}$

$$\begin{cases} x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' = \{r\}, & \text{if } \{x', y', z'\} = \{r\}; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni b, & \text{if } \{x', y', z'\} \ni b; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni e, & \text{if } \{x', y', z'\} \ni e; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' = \{c, d\}, & \text{if } \{x', y', z'\} = \{c, d\}; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni e, & \text{otherwise.} \end{cases}$$

Thus the hypergroupoid $(\{b, c, d, e, f\}, \oplus_1)$ is an H_v-semigroup. But the hyperoperation $\oplus_1$ is not associative; for example, since $b \oplus_1 (c \oplus_1 f) = \{b, d, e\}$ and $(b \oplus_1 c) \oplus_1 f = \{b, d, e, f\}$, we have

$$b \oplus_1 (c \oplus_1 f) \neq (b \oplus_1 c) \oplus_1 f.$$

Therefore $(\{b,c,d,e,f\},\oplus_1)$ is not a hypergroup. Moreover, since $c\oplus_1\{b,c,d,e,f\} = \{b,c,d,e\} \neq \{b,c,d,e,f\}$, it is not a quasihypergroup. Hence $(\{b,c,d,e,f\},\oplus_1)$ is a commutative H_v-subsemigroup of G.

(5) We show the following: for $x',y',z' \in G$

$$\begin{cases} x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni a \text{ or } b, & \text{if } \{x',y',z'\} \ni a; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni b, & \text{if } \{x',y',z'\} \ni b; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni c \text{ or } e, & \text{if } \{x',y',z'\} \ni c; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni d \text{ or } e, & \text{if } \{x',y',z'\} \ni d; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni e, & \text{if } \{x',y',z'\} \ni e; \\ x' \oplus_1 (y' \oplus_1 z') \cap (x' \oplus_1 y') \oplus_1 z' \ni e \text{ or } f, & \text{if } \{x',y',z'\} \ni f. \end{cases}$$

Thus the hypergroupoid $(G,\oplus_1)$ is an H_v-semigroup. But the hyperoperation $\oplus_1$ is not associative; for example, since $a \oplus_1 (a \oplus_1 b) = \{a,b,c\}$ and $(a \oplus_1 a) \oplus_1 b = \{b,c\}$, we have

$$a \oplus_1 (a \oplus_1 b) \neq (a \oplus_1 a) \oplus_1 b.$$

Therefore $(G,\oplus_1)$ is not a hypergroup. Moreover, since $c\oplus_1 G = \{a,b,c,d,e\} \neq G$, it is not a quasihypergroup. Hence $(G,\oplus_1)$ is a commutative H_v-subsemigroup of G.

□

The operation $\oplus_1$ is the definition of the reaction for each chemical element, but the following operation $\oplus_2$ is the definition of the reaction in the chemical element set.

Definition 3.3. Let G be the set $\{CFCl_3, CFCl_2, Cl, ClO, O_2, O_3\}$ of the chemical elements and a *hyperoperation* $\oplus_2$ on G is defined as follows:

$$\oplus_2 : G \times G \to \wp^*(G)$$

where $\wp^*(G)$ is the set of all non-empty subset of G. For all $x,y \in G$, $x \oplus_2 y$ is defined as follows:

$$(x \oplus_1 x) \cup (x \oplus_1 y) \cup (y \oplus_1 y)$$

For example, in the case $CFCl_3 \oplus_2 CFCl_2$,

$$\begin{aligned} & CFCl_3 \oplus_2 CFCl_2 \\ = {} & (CFCl_3 \oplus_1 CFCl_3) \cup (CFCl_3 \oplus_1 CFCl_2) \cup (CFCl_2 \oplus_1 CFCl_2) \\ = {} & \{CFCl_2, Cl\} \cup \{CFCl_3, CFCl_2\} \cup \{CFCl_2\} \\ = {} & \{CFCl_3, CFCl_2, Cl\} \end{aligned}$$

Then we can define $x\oplus_2 y$ for elements $x,y \in \{CFCl_3, CFCl_2, Cl, ClO, O_2, O_3\}$ as follows.

First for the $x = CFCl_3$,

$$\begin{aligned}
CFCl_3 \oplus_2 CFCl_3 &= CFCl_3 \oplus_2 CFCl_3 &&= \{CFCl_2, Cl\}.\\
CFCl_3 \oplus_2 CFCl_2 &= CFCl_2 \oplus_2 CFCl_3 &&= \{CFCl_3, CFCl_2, Cl\}.\\
CFCl_3 \oplus_2 Cl &= Cl \oplus_2 CFCl_3 &&= \{CFCl_3, CFCl_2, Cl\}.\\
CFCl_3 \oplus_2 ClO &= ClO \oplus_2 CFCl_3 &&= \{CFCl_3, CFCl_2, Cl, ClO\}.\\
CFCl_3 \oplus_2 O_2 &= O_2 \oplus_2 CFCl_3 &&= \{CFCl_3, CFCl_2, Cl, O_2\}.\\
CFCl_3 \oplus_2 O_3 &= O_3 \oplus_2 CFCl_3 &&= \{CFCl_3, CFCl_2, Cl, O_3\}.
\end{aligned}$$

Next for the $x = CFCl_2$,

$$\begin{aligned}
CFCl_2 \oplus_2 CFCl_2 &= CFCl_2 \oplus_2 CFCl_2 &&= \{CFCl_2\}.\\
CFCl_2 \oplus_2 Cl &= Cl \oplus_2 CFCl_2 &&= \{CFCl_2, Cl\}.\\
CFCl_2 \oplus_2 ClO &= ClO \oplus_2 CFCl_2 &&= \{CFCl_2, ClO\}.\\
CFCl_2 \oplus_2 O_2 &= O_2 \oplus_2 CFCl_2 &&= \{CFCl_2, O_2\}.\\
CFCl_2 \oplus_2 O_3 &= O_3 \oplus_2 CFCl_2 &&= \{CFCl_2, O_3\}.
\end{aligned}$$

For the $x = Cl$,

$$\begin{aligned}
Cl \oplus_2 Cl &= Cl \oplus_2 Cl &&= \{Cl\}.\\
Cl \oplus_2 ClO &= ClO \oplus_2 Cl &&= \{Cl, ClO\}.\\
Cl \oplus_2 O_2 &= O_2 \oplus_2 Cl &&= \{Cl, O_2\}.\\
Cl \oplus_2 O_3 &= O_3 \oplus_2 Cl &&= \{Cl, ClO, O_2, O_3\}.
\end{aligned}$$

For the $x = ClO$,

$$\begin{aligned}
ClO \oplus_2 ClO &= ClO \oplus_2 ClO &&= \{ClO\}.\\
ClO \oplus_2 O_2 &= O_2 \oplus_2 ClO &&= \{ClO, O_2\}.\\
ClO \oplus_2 O_3 &= O_3 \oplus_2 ClO &&= \{Cl, ClO, O_2, O_3\}.
\end{aligned}$$

For the $x = O_2$,

$$\begin{aligned}
O_2 \oplus_2 O_2 &= O_2 \oplus_2 O_2 &&= \{O_2\}.\\
O_2 \oplus_2 O_3 &= O_3 \oplus_2 O_2 &&= \{O_2, O_3\}.
\end{aligned}$$

For the $x = O_3$,

$$O_3 \oplus_2 O_3 = O_3 \oplus_2 O_3 = \{O_3\}.$$

We have a hyperoperation $\oplus_2$ table for the set $\{CFCl_3, CFCl_2, Cl, ClO, O_2, O_3\}$ by definition 3.3.

$\oplus_2$	$CFCl_3$	$CFCl_2$	Cl	ClO	O_2	O_3
$CFCl_3$	$CFCl_2$ Cl	$CFCl_3$ $CFCl_2$ Cl	$CFCl_3$ $CFCl_2$ Cl	$CFCl_3$ $CFCl_2$ Cl, ClO	$CFCl_3$ $CFCl_2$ Cl, O_2	$CFCl_3$ $CFCl_2$ Cl, O_3
$CFCl_2$	$CFCl_3$ $CFCl_2$ Cl	$CFCl_2$	$CFCl_2$ Cl	$CFCl_2$ ClO	$CFCl_2$ O_2	$CFCl_2$ O_3
Cl	$CFCl_3$ $CFCl_2$ Cl	$CFCl_2$ Cl	Cl	Cl ClO	Cl O_2	Cl ClO $O_2,\ O_3$
ClO	$CFCl_3$ $CFCl_2$ Cl, ClO	$CFCl_2$ ClO	Cl ClO	ClO	ClO O_2	Cl ClO O_2, O_3
O_2	$CFCl_3$ $CFCl_2$ Cl, O_2	$CFCl_2$ O_2	Cl O_2	ClO O_2	O_2	O_2 O_3
O_3	$CFCl_3$ $CFCl_2$ Cl, O_3	$CFCl_2$ O_3	Cl ClO O_2, O_3	Cl ClO O_2, O_3	O_2 O_3	O_3

TABLE 3. Hyperoperation table for the operation $\oplus_2$

THEOREM 3.4. *Let $G = \{a, b, c, d, e, f\}$ be the set of the chemical elements obtained from the chemical reactions in ozone depletion and $\oplus_2$ be the hyperoperation on G. Consider the following hyperoperation table:*

$\oplus_2$	a	b	c	d	e	f
a	$\{b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c,d\}$	$\{a,b,c,e\}$	$\{a,b,c,f\}$
b	$\{a,b,c\}$	$\{b\}$	$\{b,c\}$	$\{b,d\}$	$\{b,e\}$	$\{b,f\}$
c	$\{a,b,c\}$	$\{b,c\}$	$\{c\}$	$\{c,d\}$	$\{c,e\}$	$\{c,d,e,f\}$
d	$\{a,b,c,d\}$	$\{b,d\}$	$\{c,d\}$	$\{d\}$	$\{d,e\}$	$\{c,d,e,f\}$
e	$\{a,b,c,e\}$	$\{b,e\}$	$\{c,e\}$	$\{d,e\}$	$\{e\}$	$\{e,f\}$
f	$\{a,b,c,f\}$	$\{b,f\}$	$\{c,d,e,f\}$	$\{c,d,e,f\}$	$\{e,f\}$	$\{f\}$

TABLE 4. Hyperoperation table for the operation $\oplus_2$

Then we have the following.

(1) *The hyperstructures* $(\{b,c\},\oplus_2)$, $(\{b,d\},\oplus_2)$, $(\{b,e\},\oplus_2)$, $(\{b,f\},\oplus_2)$, $(\{c,d\},\oplus_2)$, $(\{c,e\},\oplus_2)$, $(\{d,e\},\oplus_2)$, $(\{e,f\},\oplus_2)$ *are commutative subhypergroups of* G *and isomorphic. The isomorphic hyperoperation table is the following:*

$\oplus_2$	x	y
x	$\{x\}$	$\{x,y\}$
y	$\{x,y\}$	$\{y\}$

(2) *The hyperstructure* $(\{a,b,c\},\oplus_2)$ *is commutative* H_v*-subsemigroups of* G*. The hyperoperation table is the following:*

$\oplus_2$	a	b	c
a	$\{b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$
b	$\{a,b,c\}$	$\{b\}$	$\{b,c\}$
c	$\{a,b,c\}$	$\{b,c\}$	$\{c\}$

The hyperstructures $(\{b,c,d\},\oplus_2)$, $(\{b,c,e\},\oplus_2)$, $(\{b,d,e\},\oplus_2)$, $(\{b,e,f\},\oplus_2)$ *and* $(\{c,d,e\},\oplus_2)$ *are commutative* H_v*-subsemigroups of* G *and isomorphic. The isomorphic hyperoperation table is the following:*

$\oplus_2$	x	y	z
x	$\{x\}$	$\{x,y\}$	$\{x,z\}$
y	$\{x,y\}$	$\{y\}$	$\{y,z\}$
z	$\{x,z\}$	$\{y,z\}$	$\{z\}$

(3) *The hyperstructures* $(\{a,b,c,d\},\oplus_2)$ *and* $(\{a,b,c,e\},\oplus_2)$ *are commutative* H_v*-subsemigroups of* G *and isomorphic. The isomorphic hyperoperation table is the following:*

$\oplus_2$	x	y	z	w
x	$\{y,z\}$	$\{x,y,z\}$	$\{x,y,z\}$	$\{x,y,z,w\}$
y	$\{x,y,z\}$	$\{y\}$	$\{y,z\}$	$\{y,w\}$
z	$\{x,y,z\}$	$\{y,z\}$	$\{z\}$	$\{z,w\}$
w	$\{x,y,z,w\}$	$\{y,w\}$	$\{z,w\}$	$\{w\}$

The hyperstructure $(\{b, c, d, e\}, \oplus_2)$ *is a commutative* H_v*-subsemigroup of* G*. The hyperoperation table is the following:*

$\oplus_2$	b	c	d	e
b	$\{b\}$	$\{b,c\}$	$\{b,d\}$	$\{b,e\}$
c	$\{b,c\}$	$\{c\}$	$\{c,d\}$	$\{c,e\}$
d	$\{b,d\}$	$\{c,d\}$	$\{d\}$	$\{d,e\}$
e	$\{b,e\}$	$\{c,e\}$	$\{d,e\}$	$\{e\}$

The hyperstructure $(\{c, d, e, f\}, \oplus_2)$ *is a commutative* H_v*-subsemigroup of* G*. The hyperoperation table is the following:*

$\oplus_2$	c	d	e	f
c	$\{c\}$	$\{c,d\}$	$\{c,e\}$	$\{c,d,e,f\}$
d	$\{c,d\}$	$\{d\}$	$\{d,e\}$	$\{c,d,e,f\}$
e	$\{c,e\}$	$\{d,e\}$	$\{e\}$	$\{e,f\}$
f	$\{c,d,e,f\}$	$\{c,d,e,f\}$	$\{e,f\}$	$\{f\}$

(4) *The hyperstructure* $(\{a, b, c, d, e\}, \oplus_2)$ *is a commutative* H_v*-subsemigroup of* G*. The hyperoperation table is the following:*

$\oplus_2$	a	b	c	d	e
a	$\{b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c,d\}$	$\{a,b,c,e\}$
b	$\{a,b,c\}$	$\{b\}$	$\{b,c\}$	$\{b,d\}$	$\{b,e\}$
c	$\{a,b,c\}$	$\{b,c\}$	$\{c\}$	$\{c,d\}$	$\{c,e\}$
d	$\{a,b,c,d\}$	$\{b,d\}$	$\{c,d\}$	$\{d\}$	$\{d,e\}$
e	$\{a,b,c,e\}$	$\{b,e\}$	$\{c,e\}$	$\{d,e\}$	$\{e\}$

The hyperstructure $(\{b, c, d, e, f\}, \oplus_2)$ *is a commutative* H_v*-subsemigroup of* G*. The hyperoperation table is the following:*

$\oplus_2$	b	c	d	e	f
b	$\{b\}$	$\{b,c\}$	$\{b,d\}$	$\{b,e\}$	$\{b,f\}$
c	$\{b,c\}$	$\{c\}$	$\{c,d\}$	$\{c,e\}$	$\{c,d,e,f\}$
d	$\{b,d\}$	$\{c,d\}$	$\{d\}$	$\{d,e\}$	$\{c,d,e,f\}$
e	$\{b,e\}$	$\{c,e\}$	$\{d,e\}$	$\{e\}$	$\{e,f\}$
f	$\{b,f\}$	$\{c,d,e,f\}$	$\{c,d,e,f\}$	$\{e,f\}$	$\{f\}$

(5) *The hyperstructure* $(G, \oplus_2)$ *is a commutative* H_v*-semigroup.*

Proof. In order to show that hyperoperation $\oplus_2$ satisfies weak associative law, we must identify the following properties: for all elements x', y', z', we show that $x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \neq \emptyset$.

(1) It is obvious.

(2) In the case $(\{a, b, c\}, \oplus_2)$, we have the following: for $x', y', z' \in \{a, b, c\}$

$$\begin{cases} x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni c, & \text{if } \{x', y', z'\} \ni c; \\ x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni b, & \text{otherwise.} \end{cases}$$

Thus the hypergroupoid $(\{a, b, c\}, \oplus_2)$ is an H_v-semigroup. However, the hyperoperation $\oplus_2$ is not associative; for example, since $a \oplus_2 (a \oplus_2 b) = \{a, b, c\}$ and $(a \oplus_2 a) \oplus_2 b = \{b, c\}$, we have

$$a \oplus_2 (a \oplus_2 b) \neq (a \oplus_2 a) \oplus_2 b.$$

Therefore $(\{a, b, c\}, \oplus_2)$ is not a hypergroup, but it is a commutative quasihypergroup. Hence $(\{a, b, c\}, \oplus_2)$ is a commutative H_v-subgroup of G.

In the cases $(\{b, c, d\}, \oplus_2)$, $(\{b, c, e\}, \oplus_2)$, $(\{b, d, e\}, \oplus_2)$, $(\{b, e, f\}, \oplus_2)$ and $(\{c, d, e\}, \oplus_2)$, they all have the same type of hyperoperation tables with $\{x, y, z\}$ and are therefore isomorphic. On the other hand, we have the following: for $x', y', z' \in \{x, y, z\}$

$$x' \oplus_2 (y' \oplus_2 z') = (x' \oplus_2 y') \oplus_2 z' = \{x', y', z'\}$$

Thus the hypergroupoid $(\{x, y, z\}, \oplus_2)$ is a semihypergroup. Obviously it is a commutative quasihypergroup. Hence $(\{x, y, z\}, \oplus_2)$ is a commutative subhypergroup of G.

(3) In the cases $(\{a, b, c, d\}, \oplus_2)$ and $(\{a, b, c, e\}, \oplus_2)$, since they have the same type of hyperoperation tables with $\{x, y, z, w\}$, they are isomorphic. On the other hand, we have the following: for $x', y', z' \in \{x, y, z, w\}$

$$\begin{cases} x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni z, & \text{if } \{x', y', z'\} \ni z; \\ x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni w, & \text{if } \{x', y', z'\} \ni w; \\ x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni y, & \text{otherwise.} \end{cases}$$

Thus the hypergroupoid $(\{x, y, z, w\}, \oplus_2)$ is an H_v-semigroup. But the hyperoperation $\oplus_2$ is not associative; for example, since $x \oplus_2 (x \oplus_2 y) = \{x, y, z\}$ and $(x \oplus_2 x) \oplus_2 y = \{y, z\}$, we have

$$x \oplus_2 (x \oplus_2 y) \neq (x \oplus_2 x) \oplus_2 y.$$

Therefore $(\{x, y, z, w\}, \oplus_2)$ is not a hypergroup, but it is a commutative quasihypergroup. Hence $(\{x, y, z, w\}, \oplus_2)$ is a commutative H_v-subgroup of G.

In the case $(\{b, c, d, e\}, \oplus_2)$, we have the following: for $x', y', z' \in \{b, c, d, e\}$

$$x' \oplus_2 (y' \oplus_2 z') = (x' \oplus_2 y') \oplus_2 z' = \{x', y', z'\}$$

Thus the hypergroupoid $(\{b,c,d,e\},\oplus_2)$ is a semihypergroup and it is a commutative quasihypergroup. Hence $(\{b,c,d,e\},\oplus_2)$ is a commutative subhypergroup of G.

In the case $(\{c,d,e,f\},\oplus_2)$, we have the following: for $x',y',z',r,s \in \{c,d,e,f\}$

$$\begin{cases} x' \oplus_2 (y' \oplus_2 z') = (x' \oplus_2 y') \oplus_2 z' = \{c,d,e,f\}, & \text{if } \{x',y',z'\} \supset \{c,f\}; \\ x' \oplus_2 (y' \oplus_2 z') = (x' \oplus_2 y') \oplus_2 z' = \{c,d,e,f\}, & \text{if } \{x',y',z'\} \supset \{d,f\}; \\ x' \oplus_2 (y' \oplus_2 z') = (x' \oplus_2 y') \oplus_2 z' = \{x',y',z'\}, & \text{otherwise.} \end{cases}$$

Thus the hypergroupoid $(\{b,c,d,e\},\oplus_2)$ is a semihypergroup and it is a commutative quasihypergroup. Hence $(\{b,c,d,e\},\oplus_2)$ is a commutative subhypergroup of G.

(4) In the case $(\{a,b,c,d,e\},\oplus_2)$, we have the following: for $x',y',z' \in \{a,b,c,d,e\}$

$$\begin{cases} x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni b, & \text{if } \{x',y',z'\} \ni a \text{ or } b; \\ x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni c, & \text{if } \{x',y',z'\} \ni c; \\ x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni d, & \text{if } \{x',y',z'\} \ni d; \\ x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni e, & \text{if } \{x',y',z'\} \ni e. \end{cases}$$

Thus the hypergroupoid $(\{a,b,c,d,e\},\oplus_2)$ is an H_v-semigroup. But the hyperoperation $\oplus_2$ is not associative; for example, since $a \oplus_2 (a \oplus_2 b) = \{a,b,c\}$ and $(a \oplus_2 a) \oplus_2 b = \{b,c\}$, we have

$$a \oplus_2 (a \oplus_2 b) \neq (a \oplus_2 a) \oplus_2 b.$$

Therefore $(\{a,b,c,d,e\},\oplus_2)$ is not a hypergroup, but it is a commutative quasihypergroup. Hence $(\{a,b,c,d,e\},\oplus_2)$ is a commutative H_v-subgroup of G.

In the case $(\{b,c,d,e,f\},\oplus_2)$, we have the following: for $x',y',z' \in \{b,c,d,e,f\}$

$$\begin{cases} x' \oplus_2 (y' \oplus_2 z') = (x' \oplus_2 y') \oplus_2 z' = \{c,d,e,f\}, \\ \qquad \text{if } \{x',y',z'\} = \{c,f\} \text{ or } \{d,f\}; \\ x' \oplus_2 (y' \oplus_2 z') = (x' \oplus_2 y') \oplus_2 z' = \{c,d,e,f\}, \\ \qquad \text{if } \{x',y',z'\} = \{c,d,f\},\{c,e,f\} \text{ or } \{d,e,f\}; \\ x' \oplus_2 (y' \oplus_2 z') = (x' \oplus_2 y') \oplus_2 z' = \{b,c,d,e,f\}, \\ \qquad \text{if } \{x',y',z'\} = \{b,c,f\} \text{ or } \{b,d,f\}; \\ x' \oplus_2 (y' \oplus_2 z') = (x' \oplus_2 y') \oplus_2 z' = \{x',y',z'\}, \qquad \text{otherwise.} \end{cases}$$

Thus the hypergroupoid $(\{b,c,d,e,f\},\oplus_2)$ is a semihypergroup and it is a commutative quasihypergroup. Hence $(\{b,c,d,e,f\},\oplus_2)$ is a commutative subhypergroup of G.

(5) We show the following: for $x',y',z',r \in G$

$$\begin{cases} x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni b, & \text{if } \{x', y', z'\} \ni a \text{ or } b; \\ x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni c, & \text{if } \{x', y', z'\} \ni c; \\ x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni d, & \text{if } \{x', y', z'\} \ni d; \\ x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni e, & \text{if } \{x', y', z'\} \ni e; \\ x' \oplus_2 (y' \oplus_2 z') \cap (x' \oplus_2 y') \oplus_2 z' \ni f, & \text{if } \{x', y', z'\} \ni f. \end{cases}$$

Thus the hypergroupoid $(G, \oplus_2)$ is an H_v-semigroup. However, the hyperoperation $\oplus_2$ is not associative; for example, since $a \oplus_2 (a \oplus_2 b) = \{a, b, c\}$ and $(a \oplus_2 a) \oplus_2 b = \{b, c\}$, we have

$$a \oplus_2 (a \oplus_2 b) \neq (a \oplus_2 a) \oplus_2 b.$$

Therefore $(G, \oplus_2)$ is not a hypergroup, but it is a commutative quasihypergroup. Hence $(G, \oplus_2)$ is a commutative H_v-group.

□

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References

[1] U. Christopher, *Depletion of the Stratospheric Ozone Layer (Ozone Depletion)*, *https://www.academia.edu/14791332/Depletion_of_the_Stratospheric_Ozone_Layer_Ozone_Depletion_*

[2] K. M. Chun, *Chemical hyperstructures of chemical reactions for iron and indium*, J. Chungcheong Math. Soc., **27** (2014), 319-325.

[3] K. M. Chun, *Chemical hyperstructures for Titatium*, J. Chungcheong Math. Soc., **30** (2017), no. 4, 459-466.

[4] S. Chung, *Chemical hyperstructures for Vanadium*, J. Chungcheong Math. Soc., **27** (2014), no. 2, 309-317.

[5] S.-C. Chung, K. M. Chun, N. J. Kim, S. Y. Jeong, H. Sim, J. Lee and H. Maeng, *Chemical hyperalgebras for three consecutive oxidation states of elements*, MATCH Communications in Mathematical and in Computer Chemistry, **72** (2014), 389–402.

[6] P. Corsini, *Prolegomena of Hypergroup Theory*, Aviani Editore, Tricesimo, Italian, 1993.

[7] P. Corsini and V. Leoreanu, *Applications of Hyperstructure Theory*, Springer, 2003.

[8] B. Davvaz, *Polygroup Theory and Related Systems*, World Scientific Publishing Co., 2013.

[9] B. Davvaz, *Weak Algebraic Hyperstructures as a Model for Interpretation of Chemical Reactions*, Iranian Journal of Mathematical Chemistry, **7** (2016), no. 2, 267–283.

[10] B. Davvaz, A. D. Nezhad, and A. Benvidi, *Chemical hyperalgebra: Dismutation reactions*, MATCH Communications in Mathematical and in Computer Chemistry, **67** (2012), 55-63.

[11] F. Marty, *Sur une generalization de la notion de groupe*, 8th Congress Math. Scandenaves, Stockholm 1934, 45-49.

[12] P. A. Newman, *Stratospheric Ozone*, An Electronic Textbook, Chapter 5, 4.2.8 Cl_x Catalytic Reactions, NASA's Goddard Space Flight Center Atmospheric Chemistry and Dynamics Branch. 2017. *(http://www.ccpo.odu.edu/SEES/ozone/oz_class.htm)*

[13] A. D. Nezhad, S. M. M. Nejad, M. Nadjafikhah, and B. Davvaz, *A physical example of algebraic hyperstructures: Leptons*, Indian J. Phys., **86** (2012), no. 11, 1027-1032.
[14] M. Norouzia, A. Mohammadib, and V. LeoreanuFotea, *Hypergroups Obtained from Formation Reaction of Simple Gas Hydrates*, MATCH Commun. Math. Comput. Chem., **80** (2018), 383-392.
[15] M. Al Tahan and B. Davvaz, *Algebraic hyperstructures associated to biological inheritance*, Mathematical Biosciences, **285** (2017), 112-118.
[16] M. A. Al Tahan and B. Davvaz, *Weak Chemical Hyperstructures Associated to Electrochemical Cells*, Iranian J. Math. Chem., **9** (2018), no. 1, 65-75.
[17] M. Al Tahan and B. Davvaz, *Electrochemical Cells as Experimental Verifications of n-ary Hyperstructures*, MATEMATIKA , **35** (2019), no. 1, 13-24.
[18] T. Vougiouklis, *Hyperstructures and their representations*, Hadronic press, Inc., 1994.
[19] *https://en.m.wikipedia.org/wiki/Ozone_depletion*

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