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PAIRWISE PREOPEN AND PRECLOSED MAPPINGS IN THE INTUITIONISTIC SMOOTH BITOPOLOGICAL SPACES

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Abstract. We introduce the intuitionistic fuzzy pairwise preopen and preclosed mappings in the intuitionistic smooth bitopological spaces, and obtain the characterizations for the mappings.

1. Introduction and Preliminaries

Many researchers [3, 2, 1, 7, 8, 10, 9] studied to obtain the properties of the intuitionistic fuzzy topological spaces and their generalized spaces. The authors also introduced the concepts of the intuitionistic smooth bitopological spaces and some kinds of continuity of the spaces in the previous papers [4, 5, 6].

In this paper, we introduce the intuitionistic fuzzy pairwise preopen and preclosed mappings in the intuitionistic smooth bitopological spaces, and obtain the characterizations for the mappings.

Throughout this paper, the symbol I denotes the unit interval $[0, 1]$ of the real line and $I_0 = (0, 1]$. A member μ of I^X is called a *fuzzy set* in X. For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with value of 0 and 1, respectively.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A +$

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$\gamma_A \leq 1$. Obviously, every fuzzy set μ in X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$. $I(X)$ denotes a family of all intuitionistic fuzzy sets in X and "IF" stands for intuitionistic fuzzy.

For any intuitionistic fuzzy sets $A = (\mu_A, \gamma_A)$, $B = (\mu_B, \gamma_B)$ and $A_i = (\mu_{A_i}, \gamma_{A_i})$ for $i \in \Gamma$, the union and intersection are defined as follows:

$$A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B), \quad A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B),$$

$$\bigcap_i A_i = (\bigwedge_i \mu_{A_i}, \bigvee_i \gamma_{A_i}), \quad \bigcup_i A_i = (\bigvee_i \mu_{A_i}, \bigwedge_i \gamma_{A_i}).$$

All the definitions and notations which are not mentioned in this paper, we refer to [4, 5].

Definition 1.1 ([4]). An *intuitionistic smooth topology* on X is a mapping $\mathcal{T} : I(X) \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\underline{0}) = \mathcal{T}(\underline{1}) = 1$.
(2) $\mathcal{T}(A \cap B) \geq \mathcal{T}(A) \wedge \mathcal{T}(B)$.
(3) $\mathcal{T}(\bigcup A_i) \geq \bigwedge \mathcal{T}(A_i)$.

The pair $(X, \mathcal{T})$ is called an *intuitionistic smooth topological space*. An intuitionistic fuzzy set A is called *IF $\mathcal{T}$-r-open* if $\mathcal{T}(A) \geq r$ and *IF $\mathcal{T}$-r-closed* if $\mathcal{T}(A^c) \geq r$.

Definition 1.2 ([4]). A system $(X, \mathcal{T}_1, \mathcal{T}_2)$ consisting of a set X with two intuitionistic smooth topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X is called a *intuitionistic smooth bitopological space*(ISBTS for short).

Throughout this paper the indices i, j take the value in $\{1, 2\}$ and $i \neq j$.

Let us define open, closed, preopen, preclosed sets in the intuitionistic smooth bitopological spaces for their continuity.

Definition 1.3. Let A be an intuitionistic fuzzy set in an ISBTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then A is said to be

1) *IF* $(\mathcal{T}_i, \mathcal{T}_j)$-$(r, s)$-*open* if $\mathcal{T}_i(A) \geq r$ and $\mathcal{T}_j(A) \geq s$,
2) *IF* $(\mathcal{T}_i, \mathcal{T}_j)$-$(r, s)$-*closed* if $\mathcal{T}_i(A^c) \geq r$ and $\mathcal{T}_j(A^c) \geq s$.

Definition 1.4 ([6]). Let A be an intuitionistic fuzzy set in an ISBTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then A is said to be

1) *IF* $(\mathcal{T}_i, \mathcal{T}_j)$-$(r, s)$-*preopen* if $A \subseteq \mathcal{T}_i\text{-int}(\mathcal{T}_j\text{-cl}(A, s), r)$,
2) *IF* $(\mathcal{T}_i, \mathcal{T}_j)$-$(r, s)$-*preclosed* if $\mathcal{T}_i\text{-cl}(\mathcal{T}_j\text{-int}(A, s), r) \subseteq A$.

Definition 1.5. [6] Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an ISBTS and $r, s \in I_0$. For each $A \in I(X)$, the *IF* $(\mathcal{T}_i, \mathcal{T}_j)$*-*$(r, s)$*-preinterior* is defined by

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s) = \bigcup\{B \in I(X) \mid B \subseteq A,\ B \text{ is IF } (\mathcal{T}_i, \mathcal{T}_j)\text{-}(r, s)\text{-preopen}\},$$

and the *IF* $(\mathcal{T}_i, \mathcal{T}_j)$*-*$(r, s)$*-preclosure* is defined by

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A, r, s) = \bigcap\{B \in I(X) \mid A \subseteq B,\ B \text{ is IF } (\mathcal{T}_i, \mathcal{T}_j)\text{-}(r, s)\text{-preclosed}\}.$$

Definition 1.6 ([5]). Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be

(1) *IF pairwise* (r, s)*-open* if the induced mapping $f : (X, \mathcal{T}_1) \to (Y, \mathcal{U}_1)$ is an IF r-open mapping and the induced mapping $f : (X, \mathcal{T}_2) \to (Y, \mathcal{U}_2)$ is an IF s-open mapping,
(2) *IF pairwise* (r, s)*-closed* if the induced mapping $f : (X, \mathcal{T}_1) \to (Y, \mathcal{U}_1)$ is an IF r-closed mapping and the induced mapping $f : (X, \mathcal{T}_2) \to (Y, \mathcal{U}_2)$ is an IF s-closed mapping.

2. Intuitionistic fuzzy pairwise (r, s)-preopen and preclosed mappings

Now we define the notions of the intuitionistic fuzzy pairwise (r, s)-preopen and preclosed mappings in intuitionistic smooth bitopological spaces, and investigate some of their properties.

Definition 2.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be

(1) *IF pairwise* (r, s)*-preopen* if $f(A)$ is IF $(\mathcal{U}_1, \mathcal{U}_2)$-$(r, s)$-preopen in Y for each IF $\mathcal{T}_1$-r-open set A in X and $f(B)$ is IF $(\mathcal{U}_2, \mathcal{U}_1)$-$(s, r)$-preopen in Y for each IF $\mathcal{T}_2$-s-open set B in X,
(2) *IF pairwise* (r, s)*-preclosed* if $f(A)$ is IF $(\mathcal{U}_1, \mathcal{U}_2)$-$(r, s)$-preclosed in Y for each IF $\mathcal{T}_1$-r-closed set A in X and $f(B)$ is IF $(\mathcal{U}_2, \mathcal{U}_1)$-$(s, r)$-preclosed set in Y for each IF $\mathcal{T}_2$-s-closed set B in X.

Remark 2.2. It is clear that every IF pairwise (r, s)-open mapping is IF pairwise (r, s)-preopen. But the following example shows that the converse need not be true.

EXAMPLE 2.3. *Let $X = \{x, y\}$ and let A_1, A_2, A_3, and A_4 be intuitionistic fuzzy sets in X defined as*

$$A_1(x) = (0.2, 0.7),\ A_1(y) = (0.4, 0.5);$$

$$A_2(x) = (0.6, 0.3),\ A_2(y) = (0.2, 0.7);$$

$$A_3(x) = (0.2, 0.7),\ A_3(y) = (0.2, 0.7);$$

and

$$A_4(x) = (0.5, 0.5),\ A_4(y) = (0.1, 0.8).$$

Define $\mathcal{T}_1 : I(X) \to I$ and $\mathcal{T}_2 : I(X) \to I$ by

$$\mathcal{T}_1(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{2} & \text{if } A = A_1, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{T}_2(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{3} & \text{if } A = A_2, \\ 0 & \text{otherwise.} \end{cases}$$

Then $(\mathcal{T}_1, \mathcal{T}_2)$ is an ISBT on X.

Also, define $\mathcal{U}_1 : I(X) \to I$ and $\mathcal{U}_2 : I(X) \to I$ by

$$\mathcal{U}_1(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{2} & \text{if } A = A_3, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}_2(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ 0 & \text{otherwise.} \end{cases}$$

Then $(\mathcal{U}_1, \mathcal{U}_2)$ is an ISBT on X.

Consider a mapping $h : (X, \mathcal{U}_1, \mathcal{U}_2) \to (X, \mathcal{T}_1, \mathcal{T}_2)$ defined by $h(x) = x$ and $h(y) = y$. Then h is an IF pairwise $(\frac{1}{2}, \frac{1}{3})$-preopen mapping. But h is not IF pairwise $(\frac{1}{2}, \frac{1}{3})$-open.

THEOREM 2.4. *Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then the following statements are equivalent:*

(1) *f is IF pairwise (r, s)-preopen.*
(2) *For any intuitionistic fuzzy set A in X,*

$$f(\mathcal{T}_1\text{-int}(A, r)) \subseteq (\mathcal{U}_1, \mathcal{U}_2)\text{-pint}(f(A), r, s)$$

and

$$f(\mathcal{T}_2\text{-int}(A, s)) \subseteq (\mathcal{U}_2, \mathcal{U}_1)\text{-pint}(f(A), s, r).$$

(3) *For any intuitionistic fuzzy set B in Y,*

$$\mathcal{T}_1\text{-int}(f^{-1}(B), r) \subseteq f^{-1}((\mathcal{U}_1, \mathcal{U}_2)\text{-pint}(B, r, s))$$

and

$$\mathcal{T}_2\text{-int}(f^{-1}(B), s) \subseteq f^{-1}((\mathcal{U}_2, \mathcal{U}_1)\text{-pint}(B, s, r)).$$

Proof. (1) $\Rightarrow$ (2) Let A be an intuitionistic fuzzy set in X. Then $\mathcal{T}_1$-int(A, r) is an IF $\mathcal{T}_1$-r-open set and $\mathcal{T}_2$-int(A, s) is an IF $\mathcal{T}_2$-s-open set in X. Since f is IF pairwise (r, s)-preopen, $f(\mathcal{T}_1\text{-int}(A, r))$ is IF $(\mathcal{U}_1, \mathcal{U}_2)$-$(r, s)$-preopen and $f(\mathcal{T}_2\text{-int}(A, s))$ is IF $(\mathcal{U}_2, \mathcal{U}_1)$-$(s, r)$-preopen in Y. Thus

$$\begin{aligned} f(\mathcal{T}_1\text{-int}(A, r)) &= (\mathcal{U}_1, \mathcal{U}_2)\text{-pint}(f(\mathcal{T}_1\text{-int}(A, r)), r, s) \\ &\subseteq (\mathcal{U}_1, \mathcal{U}_2)\text{-pint}(f(A), r, s) \end{aligned}$$

and

$$\begin{aligned} f(\mathcal{T}_2\text{-int}(A, s)) &= (\mathcal{U}_2, \mathcal{U}_1)\text{-pint}(f(\mathcal{T}_2\text{-int}(A, s)), s, r) \\ &\subseteq (\mathcal{U}_2, \mathcal{U}_1)\text{-pint}(f(A), s, r). \end{aligned}$$

(2) $\Rightarrow$ (3) Let B be an intuitionistic fuzzy set in Y. Then by (2), we have

$$\begin{aligned} f(\mathcal{T}_1\text{-int}(f^{-1}(B), r)) &\subseteq (\mathcal{U}_1, \mathcal{U}_2)\text{-pint}(f(f^{-1}(B)), r, s) \\ &\subseteq (\mathcal{U}_1, \mathcal{U}_2)\text{-pint}(B, r, s) \end{aligned}$$

and

$$\begin{aligned} f(\mathcal{T}_2\text{-int}(f^{-1}(B), s)) &\subseteq (\mathcal{U}_2, \mathcal{U}_1)\text{-pint}(f(f^{-1}(B)), s, r) \\ &\subseteq (\mathcal{U}_2, \mathcal{U}_1)\text{-pint}(B, s, r). \end{aligned}$$

Hence

$$\mathcal{T}_1\text{-int}(f^{-1}(B), r) \subseteq f^{-1}((\mathcal{U}_1, \mathcal{U}_2)\text{-pint}(B, r, s))$$

and

$$\mathcal{T}_2\text{-int}(f^{-1}(B), s) \subseteq f^{-1}((\mathcal{U}_2, \mathcal{U}_1)\text{-pint}(B, s, r)).$$

(3) $\Rightarrow$ (1) Let A be any IF $\mathcal{T}_1$-r-open set and B any IF $\mathcal{T}_2$-s-open set in X. Then $A = \mathcal{T}_1$-int(A, r) and $B = \mathcal{T}_2$-int(B, s). By (3), we obtain

$$\begin{aligned} A = \mathcal{T}_1\text{-int}(A, r) &\subseteq \mathcal{T}_1\text{-int}(f^{-1}(f(A)), r) \\ &\subseteq f^{-1}((\mathcal{U}_1, \mathcal{U}_2)\text{-pint}(f(A), r, s)) \end{aligned}$$

and

$$\begin{aligned} B = \mathcal{T}_1\text{-int}(B, s) &\subseteq \mathcal{T}_2\text{-int}(f^{-1}(f(B)), s) \\ &\subseteq f^{-1}((\mathcal{U}_2, \mathcal{U}_1)\text{-pint}(f(B), s, r)). \end{aligned}$$

Thus

$$f(A) \subseteq (\mathcal{U}_1,\mathcal{U}_2)\text{-pint}(f(A), r, s)$$

and

$$f(B) \subseteq (\mathcal{U}_2,\mathcal{U}_1)\text{-pint}(f(B), s, r).$$

Hence

$$f(A) = (\mathcal{U}_1,\mathcal{U}_2)\text{-pint}(f(A), r, s)$$

and

$$f(B) = (\mathcal{U}_2,\mathcal{U}_1)\text{-pint}(f(B), s, r).$$

Thus, $f(A)$ is an IF $(\mathcal{U}_1,\mathcal{U}_2)$-$(r, s)$-preopen set and $f(B)$ is an IF $(\mathcal{U}_2,\mathcal{U}_1)$-$(s, r)$-preopen set in Y. Therefore, f is an IF pairwise (r, s)-preopen mapping. □

THEOREM 2.5. *Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then the following statements are equivalent:*

(1) *f is IF pairwise (r, s)-preclosed.*

(2) *For any intuitionistic fuzzy set A in X,*

$$(\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(f(A), r, s) \subseteq f(\mathcal{T}_1\text{-cl}(A, r))$$

and

$$(\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(f(A), s, r) \subseteq f(\mathcal{T}_2\text{-cl}(A, s)).$$

Proof. (1) ⇒ (2) Let A be an intuitionistic fuzzy set in X. Then $\mathcal{T}_1$-cl(A, r) is IF $\mathcal{T}_1$-r-closed and $\mathcal{T}_2$-cl(A, s) is IF $\mathcal{T}_2$-s-closed in X. Since f is IF pairwise (r, s)-preclosed, $f(\mathcal{T}_1\text{-cl}(A, r))$ is IF $(\mathcal{U}_1,\mathcal{U}_2)$-$(r, s)$-preclosed and $f(\mathcal{T}_2\text{-cl}(A, s))$ is IF $(\mathcal{U}_2,\mathcal{U}_1)$-$(s, r)$-preopen in Y. Hence

$$\begin{aligned}(\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(f(A), r, s) &\subseteq (\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(f(\mathcal{T}_1\text{-cl}(A, r)), r, s)\\ &= f(\mathcal{T}_1\text{-cl}(A, r))\end{aligned}$$

and

$$\begin{aligned}(\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(f(A), s, r) &\subseteq (\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(f(\mathcal{T}_2\text{-cl}(A, s)), s, r)\\ &= f(\mathcal{T}_2\text{-cl}(A, s)).\end{aligned}$$

(2) ⇒ (1) Let A be any IF $\mathcal{T}_1$-r-closed set and B any IF $\mathcal{T}_2$-s-closed set in X. Then $A = \mathcal{T}_1\text{-cl}(A, r)$ and $B = \mathcal{T}_2\text{-cl}(A, s)$. Hence by (2), we have

$$\begin{aligned}(\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(f(A), r, s) &\subseteq f(\mathcal{T}_1\text{-cl}(A, r))\\ &= f(A)\\ &\subseteq (\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(f(A), r, s)\end{aligned}$$

and

$$\begin{aligned}(\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(f(B),s,r) &\subseteq f(\mathcal{T}_2\text{-cl}(B,s))\\ &= f(B)\\ &\subseteq (\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(f(B),s,r).\end{aligned}$$

Thus, $f(A)$ is IF $(\mathcal{U}_1,\mathcal{U}_2)$-$(r,s)$-preclosed and $f(B)$ is IF $(\mathcal{U}_2,\mathcal{U}_1)$-$(s,r)$-preclosed in Y. Therefore f is an IF pairwise (r,s)-preclosed mapping. □

THEOREM 2.6. *Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a bijective mapping from an ISBTS X to an ISBTS Y and $r,s\in I_0$. Then f is IF pairwise (r,s)-preclosed if and only if*

$$f^{-1}((\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(B,r,s))\subseteq \mathcal{T}_1\text{-cl}(f^{-1}(B),r)$$

and

$$f^{-1}((\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(B,s,r))\subseteq \mathcal{T}_2\text{-cl}(f^{-1}(B),s)$$

for any intuitionistic fuzzy set B in Y.

Proof. Let B be an intuitionistic fuzzy set in Y. Since f is onto, by Theorem 2.5, we have

$$\begin{aligned}(\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(B,r,s) &= (\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(f(f^{-1}(B)),r,s)\\ &\subseteq f(\mathcal{T}_1\text{-cl}(f^{-1}(B),r))\end{aligned}$$

and

$$\begin{aligned}(\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(B,s,r) &= (\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(f(f^{-1}(B)),s,r)\\ &\subseteq f(\mathcal{T}_2\text{-cl}(f^{-1}(B),s)).\end{aligned}$$

As f is one-to-one, we obtain

$$\begin{aligned}f^{-1}((\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(B,r,s)) &\subseteq f^{-1}(f(\mathcal{T}_1\text{-cl}(f^{-1}(B),r)))\\ &= \mathcal{T}_1\text{-cl}(f^{-1}(B),r)\end{aligned}$$

and

$$\begin{aligned}f^{-1}((\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(B,s,r)) &\subseteq f^{-1}(f(\mathcal{T}_2\text{-cl}(f^{-1}(B),s)))\\ &= \mathcal{T}_2\text{-cl}(f^{-1}(B),s).\end{aligned}$$

Conversely, let A be an intuitionistic fuzzy set in X. Since f is one-to-one, we have

$$\begin{aligned}f^{-1}((\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(f(A),r,s)) &\subseteq \mathcal{T}_1\text{-cl}(f^{-1}(f(A)),r)\\ &= \mathcal{T}_1\text{-cl}(A,r)\end{aligned}$$

and

$$\begin{aligned} f^{-1}((\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(f(A),s,r)) &\subseteq \mathcal{T}_2\text{-cl}(f^{-1}(f(A)),s) \\ &= \mathcal{T}_2\text{-cl}(A,s). \end{aligned}$$

Since f is onto, we obtain

$$\begin{aligned} (\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(f(A),r,s) &= f(f^{-1}((\mathcal{U}_1,\mathcal{U}_2)\text{-pcl}(f(A),r,s))) \\ &\subseteq f(\mathcal{T}_1\text{-cl}(A,r)) \end{aligned}$$

and

$$\begin{aligned} (\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(f(A),s,r) &= f(f^{-1}((\mathcal{U}_2,\mathcal{U}_1)\text{-pcl}(f(A),s,r))) \\ &\subseteq f(\mathcal{T}_2\text{-cl}(A,s)). \end{aligned}$$

Thus, by Theorem 2.5, f is an IF pairwise (r,s)-preclosed mapping. □

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