

# Simplified Numerical Load–transfer Finite Element Modelling of Tunnelling Effects on Piles



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## Abstract

Tunnelling in urban environments is very common nowadays as large cities are expanding and transportation demands require the use of the underground space for creating extra capacity. Inevitably, any such new construction may have significant effects on existing nearby infrastructure and therefore relevant assessment of structural integrity and soil–structure interaction is required. Foundation piles can be rather sensitive to nearby tunnel construction and therefore their response needs to be evaluated carefully. Although detailed three–dimensional continuum finite element analysis can provide a wealth of information about this behaviour of piles, such analyses are generally very computationally demanding and may require a number of material and other model parameters to be properly calibrated. Therefore, relevant simplified approaches are used to provide a practical way for such an assessment. This paper presents a simple method where the pile is modelled with beam finite elements, pile–soil interaction is modelled with soil springs and tunnelling–induced displacements are introduced as an input boundary condition at the end of the soil springs. The performance of this approach is assessed through some examples of applications.

**Keywords:** tunnelling, piles, finite element modelling, soil–structure interaction.

## 1. Introduction

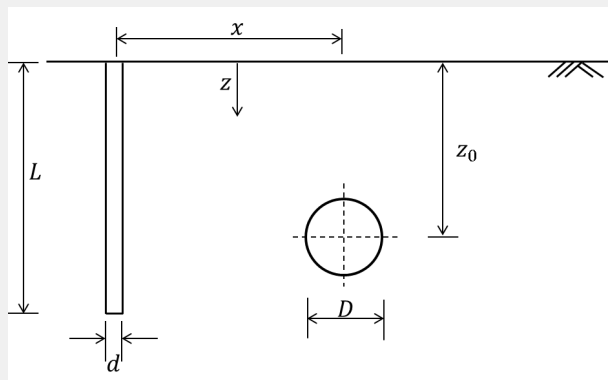
It is now well established that any tunnelling construction processes inevitably results in ground deformations, both settlements and lateral displacements. The impact of these ground deformations becomes even more significant in urban areas, where new tunnels are excavated in close proximity to other existing civil infrastructure, such as underground services, buildings, foundations, other tunnels, etc. The case of tunnelling-induced pile deformations is of high engineering interest as it may result in, probably uneven, building settlements, pile bending, cracking etc.

Tunnelling settlements, or ground deformations in general, have been studied widely by a number of researchers (Sagaseta, 1987; Verruijt & Booker, 1998; Mair et al., 1993; Loganathan & Poulos, 1998; Chen et al., 1999; Puzrin et al., 2012) adopting various different approaches, including experimental, field, numerical or analytical techniques. Likewise, the effects of tunnelling on building structures and more specifically on piled foundations has been a central research effort for many years (Potts & Addenbrooke, 1997; Loganathan et al., 2000, 2001; Jacobsz et al., 2004; Kitiyodom et al., 2005; Lee & Jacobsz, 2006; Surjadinata, 2006; Marshall, 2012; Bym et al., 2013; Franza et al., 2017; Franza & Marshall, 2017; Williamson et al., 2017a, 2017b). It should be noted that significant progress has been made to date regarding the prediction of pile deformations to tunnelling activities. The use of three-dimensional (3D) solid non-linear finite element (FE) analysis now presents a wide range of opportunities for analysing complex geometries and advanced soil material behaviour. However, these techniques rely on a detailed and reliable set of parameters, which may require some costly experimental techniques to be calibrated. Finally, reliable analyses are usually very complicated, and therefore require a great deal of time and computational power.

This paper presents a simplified and approximate approach to model tunnelling-induced deformations on existing piles. The pile is modelled with linear beam elements whereas green-field ground deformations due to tunnelling from relevant established analytical relations are imposed on the pile. The model is compared with previous results from the literature exhibiting an excellent agreement. Further aspects of the problem, such as different input analytical/empirical relations for ground deformations and the effects of any applied mechanical load on the pile are evaluated. It is showed that this simplified approach may be adopted to provide at least an initial estimate of the effects of tunnelling on existing piles.

## 2. Problem Statement

The problem under study is shown graphically in Figure 1: a circular tunnel is being excavated in close proximity to an existing pile. It is well known that tunnelling results in ground displacements, in both the vertical and horizontal directions. The aim of this work is to evaluate the effects of the tunnel excavation on the existing pile, in particular the total vertical and horizontal displacements, shortening/elongation and bending.



(Figure 1) Graphical problem description of tunnelling effects on piles

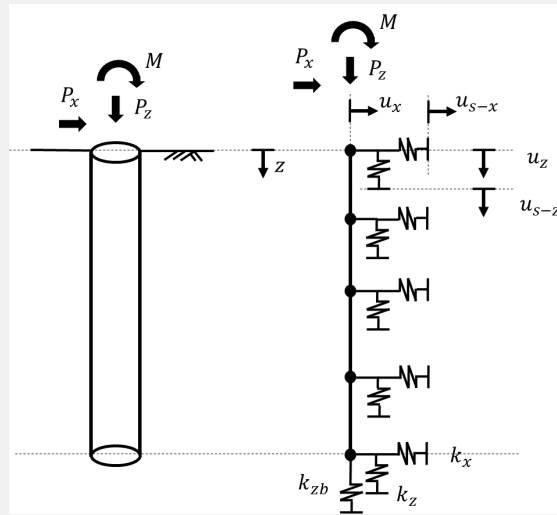
Pile–soil–tunnel interaction is a complicated problem: (i) the presence of the new tunnel causes deformations to the existing pile, and (ii) the existing pile affects the tunnelling–induced ground deformations, i.e. the ground displacements are different than those in green–field conditions (i.e. in the absence of any piles).

This problem, considering both (i) and (ii) aspects mentioned above, can be well described and analysed using three–dimensional (3D) solid continuum finite elements (Potts & Zdravković, 1999, 2001; Lee, 2012), this approach requires detailed geometry discretisation and thus a large number of elements, making the problem very computationally demanding and potentially time consuming.

However, in many cases the problem can be simplified and the second aspect (i.e. (ii) mentioned above) can be ignored, therefore one may concentrate only on the first aspect (i.e. (i) mentioned above) and hence may assume that green–field ground deformations can be directly imposed on the pile. This work is concerned with this latter, simplified and approximate approach in which it is assumed that the presence of the pile has negligible effects on the green–field tunnelling–induced ground deformations.

### 3. Simplified Soil–structure Interaction Model

The numerical model adopted in this study is shown in Figure 2. The pile is modelled with 2-noded beam elements, and each node has 3 degrees-of-freedom (DOF): horizontal and vertical displacement and rotation. The surrounding soil and the associated soil–pile interaction is modelled with a series of two linear translational springs attached at each node and which represent horizontal and vertical shaft friction, while at the base of the pile an additional vertical translational spring represents base reaction. Any mechanical applied load (i.e. horizontal or vertical applied force or pile head moment) can be represented by equivalent forces or moment at the top node of the pile. This combination of beam elements for the pile and springs for the soil is usually termed “load transfer”, or “t–z” or “p–y” approach in the literature (Kitiyodom, et al., 2005; Pelecanos et al., 2017, 2018) and has been used in many cases of piled foundations (Soga et al., 2015, 2017; Kechavarzi et al., 2016).



<Figure 2> Overview of the considered numerical model

The governing equation for global force equilibrium (Potts & Zdravković, 1999) is given by:

$$[K_p + K_s] \cdot \{u_p - u_s\} = \{F\} \quad (1)$$

where,  $[K_p]$  and  $[K_s]$  are the global pile and soil stiffness matrices respectively,  $\{u_p\}$  and  $\{u_s\}$  are vectors containing the pile and soil degrees of freedom (displacements and rotations) respectively and  $\{F\}$  is the global external load (forces and bending moments) vector.

Rearranging the previous equation one can obtain the vector of unknown pile displacements and rotations,  $\{u_p\}$ , as follows:

$$u_p = [K_p + K_s]^{-1} \cdot (\{F\} + [K_p + K_s] \cdot \{u_s\}) \quad (2)$$

The global external load vector,  $\{F\}$ , contains information about the applied “mechanical” loads on the pile, i.e. applied horizontal or vertical forces or moments, which are usually applied at the top. In the absence of any such external applied loads this vector contains null values.

The global pile stiffness matrix,  $[K_p]$ , is formed by an element assembly procedure and consists of the local pile “beam” element stiffness matrices,  $[K_{EL}]$ , which are given by:

$$K_{EL} = \begin{bmatrix} \frac{EA}{L_{EL}} & 0 & 0 & -\frac{EA}{L_{EL}} & 0 & 0 \\ 0 & \frac{12EI}{L_{EL}^3} & \frac{6EI}{L_{EL}^2} & 0 & \frac{-12EI}{L_{EL}^3} & \frac{6EI}{L_{EL}^2} \\ 0 & \frac{6EI}{L_{EL}^2} & \frac{4EI}{L_{EL}} & 0 & \frac{-6EI}{L_{EL}^2} & \frac{2EI}{L_{EL}} \\ -\frac{EA}{L_{EL}} & 0 & 0 & \frac{EA}{L_{EL}} & 0 & 0 \\ 0 & \frac{-12EI}{L_{EL}^3} & \frac{-6EI}{L_{EL}^2} & 0 & \frac{12EI}{L_{EL}^3} & \frac{-6EI}{L_{EL}^2} \\ 0 & \frac{6EI}{L_{EL}^2} & \frac{2EI}{L_{EL}} & 0 & \frac{-6EI}{L_{EL}^2} & \frac{4EI}{L_{EL}} \end{bmatrix} \quad (3)$$

where,  $E$ ,  $A$ ,  $I$  and  $L_{EL}$  are the pile Young’s Modulus, cross-sectional area, second moment of area and element length, respectively.

The global soil stiffness matrix,  $[K_s]$ , is formed by an element assembly procedure and consists of the local soil “spring” stiffness values,  $k_x$  and  $k_z$  for the horizontal and vertical shaft resistance respectively and  $k_{zb}$  for the end bearing resistance, which are given by (Boussinesq, 1885; Vesic, 1961; Randolph & Wroth, 1978; Kitiyodom et al., 2005):

$$k_x = 0.65 \cdot \frac{E_s}{d \cdot (1 - \nu_s^2)} \cdot \sqrt[12]{\frac{d^4 \cdot E_s}{E \cdot I}} \quad (4)$$

$$k_z = \frac{2\pi \cdot G_s}{\ln\left(2.5 \cdot L \cdot \frac{(1 - \nu_s)}{r}\right)} \quad (5)$$

$$k_{zb} = \frac{d \cdot E}{(1 - \nu_s^2)} \quad (6)$$

where,  $E_s$ ,  $G_s$  and  $\nu_s$ , are the Young's Modulus, Shear Modulus and Poisson's ratio of the soil, respectively, whereas  $r$  is the radius of the pile.

The vector of soil DOF (displacements and rotations),  $\{u_s\}$ , contains information about the effects of tunnelling on the pile. These are the "expected" values of soil displacement (only displacements, no external rotations are imposed) at the location of the pile due to the presence of the tunnel. Practically, considering the above-mentioned numerical model, these are the values of the displacements that the soil springs are "pulled" vertically and horizontally. A number of analytical or empirical relations of different complexity exist that can predict the expected soil vertical and horizontal displacements due to tunnelling (Sagaseta, 1987; Mair et al., 1993; Verruijt & Booker, 1998; Loganathan & Poulos, 1998; Puzrin et al., 2012). In this work the relations of Loganathan & Poulos (1998) for vertical,  $u_z$ , and horizontal soil displacements,  $u_x$ , are adopted which are very comprehensive and are given by:

$$u_{s-x}(z) = -R^2 x \left[ \frac{1}{(x^2 + (z_0 - z)^2)} + \frac{3 - 4\nu_s}{x^2 + (z_0 + z)^2} - \frac{4z(z + z_0)}{(x^2 + (z_0 + z)^2)^2} \right] \frac{4gR + g^2}{4R^2} \exp\left\{ - \left[ \frac{1.38x^2}{(z_0 + R)^2} + \frac{0.69z^2}{z_0^2} \right] \right\} \quad (7)$$

$$u_{s-z}(z) = R^2 \left\{ - \frac{z - z_0}{x^2 + (z - z_0)^2} + (3 - 4\nu_s) \frac{z + z_0}{x^2 + (z + z_0)^2} - \frac{2z[x^2 - (z + z_0)^2]}{[x^2 + (z + z_0)^2]^2} \right\} \frac{4Rg + g^2}{4R^2} \exp\left\{ - \left[ \frac{1.38x^2}{(z_0 + R)^2} + \frac{0.69z^2}{z_0^2} \right] \right\} \quad (8)$$

where,  $x$  and  $z$  are the horizontal and vertical distance from the centre of the tunnel, whereas  $R$  and  $z_0$  are the radius and the depth of the centre of the tunnel, respectively (Figure 1). Finally,  $g$  is the so-called "gap parameter" which is related to the "volume loss",  $V_L$ , and, for undrained conditions, may be obtained from the following relation (Loganathan & Poulos, 1998):

$$V_L = \frac{4gR + g^2}{4R^2} \quad (9)$$

## 4. Applications

### 4.1 Load-transfer Vs. continuum formulation

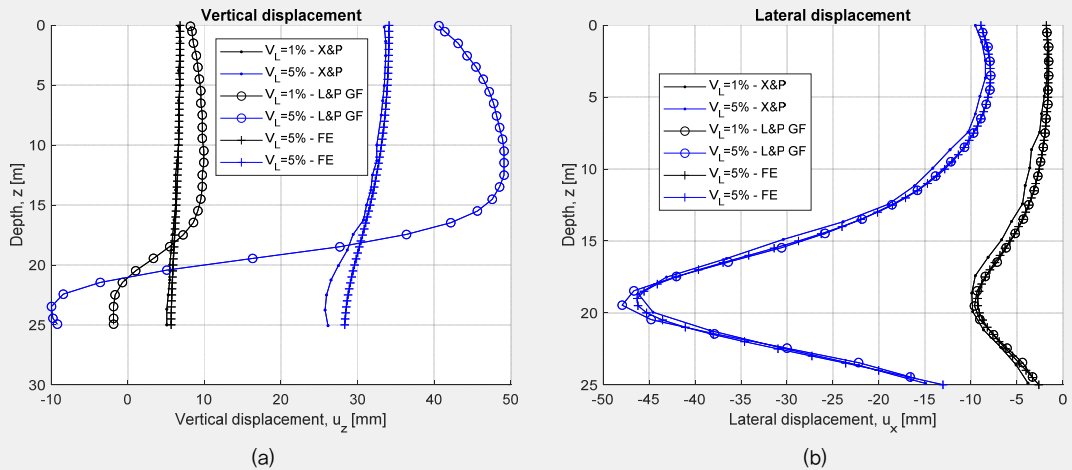
The numerical model introduced above is used in an example application problem to examine its behaviour. This application is taken from Xu & Poulos (2001) who have also analysed this using a slightly different approach. In their approach a similar pile model was used but instead of using discrete soil springs defined using the relations presented above, the soil response is described using elastic continuum relations. The complete introduction of that model is beyond the scope of this work and therefore, for brevity, it is not included here; the interested reader may consult the original publication (Xu & Poulos, 2001) for further details.

The problem under examination is a simple vertical pile in uniform homogeneous soil which deforms due to the excavation of a circular tunnel. This is similar to the problem introduced earlier and shown graphically in Figure 1 and the relevant problem parameters are listed in Table 1. The length of the beam elements,  $L_{EL} = 0.5 \text{ m}$  and two cases are considered for the volume loss,  $V_L = 1\%$  and  $V_L = 5\%$ .

(Table 1) Parameters adopted for the model

Pile diameter, d	Pile length, L	Pile Young Modulus, E	Soil Young Modulus, E <sub>s</sub>	Soil Poisson's ratio, ν <sub>s</sub>	Tunnel depth, z <sub>0</sub>	Tunnel diameter, D	Horizontal pile distance to tunnel, x	Volume loss, V <sub>L</sub>
m	m	MPa	MPa	–	m	m	m	%
0.5	25	30000	24	0.5	20	6	4.5	1, 5

The predicted pile response is shown in Figure 3, including both vertical and horizontal displacements (denoted as “FE”), and this is compared to the results of Xu & Poulos (2001), denoted as “X&P”. It is shown that a very good agreement is obtained between the two approaches which confirms the validity of this approach. Finally, on the same plot, the input green field ground deformations from the relations of Loganathan & Poulos (1998) are also plotted for comparison; the latter are denoted as “L&P GF”.



〈Figure 3〉 Predicted pile response: (a) vertical displacements, (b) lateral displacements

#### 4.2 Free-field tunnelling displacement field input

To examine the sensitivity of this simplified modelling approach a different input of tunnelling-induced green-field ground deformations are adopted. Although a plethora of relevant solutions exist (Sagaseta, 1987; Verruijt & Booker, 1998; Mair et al., 1993; Loganathan & Poulos, 1998; Chen et al., 1999; Puzrin et al., 2012), for the purposes of comparison, the relations of Mair et al. (1993) are used, which are very well established. These solutions are based on a thorough study of small-scale physical modelling centrifuge tests (Schofield, 1980; Mair, 1979) and provide relations for the distribution of sub-surface settlements, i.e. only vertical ground displacements, and no information on horizontal displacements.

The vertical ground displacements (settlements) are given by:

$$u_{s-z}(z) = u_{z-max} \exp\left(-\frac{x^2}{2i}\right) \quad (10)$$

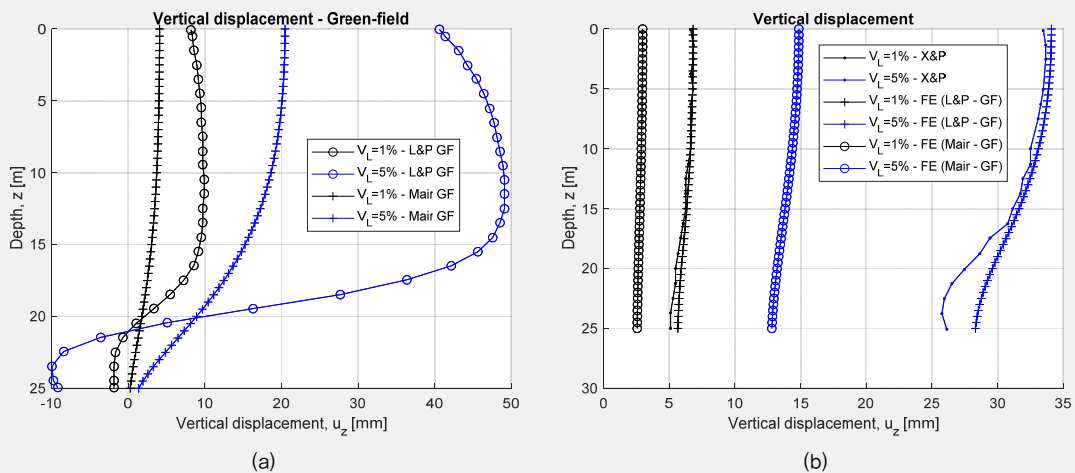
where,  $u_{z-max}$  is the maximum value of vertical tunnel settlement, whereas  $i$  is the horizontal distance from the centre of the tunnel to the point of inflection of ground settlements.



$$u_{z-\max} = 0.313 \frac{V_L D^2}{i} \quad (11)$$

$$i = z_0 \left( 0.175 + 0.325 \left( 1 - \frac{z}{z_0} \right) \right) \quad (12)$$

So, in this section, the relations of Mair et al. (1993) are used instead of those of Loganathan & Poulos (1998) for the vertical ground displacements and zero values are used for the corresponding horizontal displacements. The results of this exercise are shown graphically in Figure 4.



(Figure 4) Input tunnelling-induced displacements: (a) green-field settlements, (b) predicted pile vertical displacements

Considering green field tunnelling-induced ground deformations, it may be observed that the Mair et al. (1993) relations provide smaller values than those from Loganathan & Poulos (1998). Regarding vertical pile displacements, it is shown that these relations are indeed predicting some pile displacements of which the magnitude is comparable to those from Xu & Poulos (2011), but approximately half of their value. However, this is only a qualitative comparative evaluation and not an exercise to suggest a more appropriate input relation, as the latter would require comparison (or verification) of both predictions against some available field or lab experimental data.

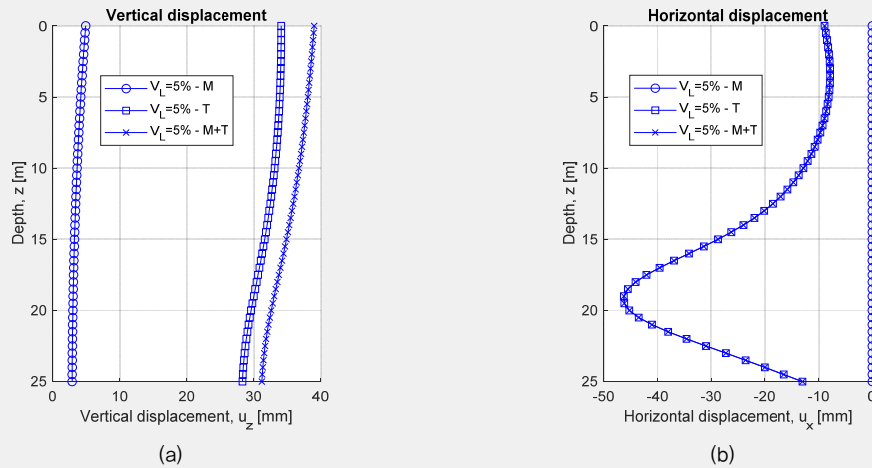
### 4.3 Applied pile mechanical load

In this section, the effect of applied mechanical load at the pile head is evaluated. The aim is to understand what the relative effect of an applied mechanical load (M) is, compared to tunnelling-induced deformations (T) and a combination of both “loading” mechanisms (M+T).

In this section the analyses presented initially with the Loganathan & Poulos (1998) relations are repeated (i.e. Section 4.1), but in addition a vertical applied force at the pile head of  $F_V = 1MN$  is included too. Three cases are considered:

- (a) Pure (only) mechanical load (M)
- (b) Tunnelling-induced deformations only (T) – this is what was analysed in Section 4.1.
- (c) Combination of the two previous cases, i.e. both mechanical load and tunnelling deformations (M+T)

The results of this exercise are shown in Figure 5, where both vertical and horizontal pile displacements are plotted for each of the 3 cases mentioned above for  $V_L = 5\%$ . It is shown that as expected, the case of pure mechanical vertical load (M), results in vertical displacements only. Secondly, the cases of tunnelling-induced deformations only (T) results in both horizontal and vertical displacements which are related to those discussed earlier in Section 4.1. Finally, the third case of both mechanical load and tunnelling-induced deformations (M+T) appears to provide a combination of both mechanisms. It should be noted that, for simplicity, linear soil behaviour has been assumed in this study, whereas in reality, nonlinear soil springs should be used (Pelecanos et al., 2017). In that case the combined effect of the two mechanisms will not be a superposition of the values. Nevertheless, this study shows that tunnelling construction may result in additional pile displacements which may be a risk for the safety of both the pile and the superstructure and therefore should be evaluated carefully. Finally, tunnelling may induce some significant lateral pile displacements (and hence bending moments) which should be appropriately evaluated, especially in the cases where the pile was not designed to sustain any bending moments and hence potentially tensile stresses (that may lead to concrete cracking).



(Figure 5) Predicted pile response (a) vertical displacements, (b) lateral displacements

## 5. Conclusions

This paper presents a simplified approach for estimating the effects of tunnelling on piled foundations. The adopted model is simplified, which means that it makes relevant and appropriate assumptions for the geometry and loading conditions in order to reduce significantly the size and thus the computational demand of the problem. The pile is modelled with linear beam elements with axial and lateral displacement degrees of freedom along with rotational degrees of freedom. Pile–soil interaction is modelled with equivalent linear soil springs attached at all the nodes of the pile beam elements and which represent both shaft and base resistance. Tunnelling–induced displacements are introduced as boundary conditions at the end of the soil springs. The latter are determined using established solutions from the literature.

The adopted model is compared to previous solutions from the literature and appears to exhibit an excellent agreement. Moreover, the effect of the input analytical/empirical relation for the green–field settlements is evaluated by comparing two well–established relations. Finally, it was shown that tunnelling may induced additional settlements on piles that are mechanically loaded and especially lateral displacements that are associated with bending moments and thus potentially tensile stresses. It is therefore of high importance that these lateral displacements are appropriately evaluated so that any cracking of the concrete pile is avoided.

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[This article is composed entirely based on the authors' opinion and does not have any relation to do with the Korean tunnelling and underground space association's official stance.]