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NEW SUBCLASS OF BI-UNIVALENT FUNCTIONS BY (p, q)-DERIVATIVE OPERATOR

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Abstract. In this paper, we introduce interesting subclasses $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta,\alpha}$ and $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta}(\gamma)$ of bi-univalent functions by (p, q)-derivative operator. Furthermore, we find upper bounds for the second and third coefficients for functions in these subclasses. The results presented in this paper would generalize and improve some recent works of several earlier authors.

1. Introduction

Let $\mathcal{A}$ be a family of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. Also we let $\mathcal{S}$ to denote the class of functions $f \in \mathcal{A}$ which are univalent in $\mathbb{U}$. Every function $f \in \mathcal{S}$ has an inverse f^{-1}, which is defined by

$$f^{-1}(f(z)) = z \ (z \in \mathbb{U})$$

and

$$f(f^{-1}(w)) = w \ \left(|w| < r_0(f),\ r_0(f) \geq \frac{1}{4}\right).$$

In fact, the inverse function f^{-1} is given by

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots. \tag{2}$$

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A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$, if both f and f^{-1} are univalent in $\mathbb{U}$. Let $\sigma_{\mathfrak{B}}$ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1)

In recent years, various subclasses of the bi-univalent functions $\sigma_{\mathfrak{B}}$ were introduced by researcher and obtained non-sharp estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in these subclasses of the function class $\sigma_{\mathfrak{B}}$. For a brief history and interesting examples of functions in the class $\sigma_{\mathfrak{B}}$, see [12]. In fact that this widely-cited work by Srivastava et al. [12] actually revived the study of analytic and bi-univalent functions in recent years and that it has led to a flood of papers on the subject by (for example) Srivastava et al.[2, 3, 1, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24], and others [9, 26, 27, 28].

In the field of Geometric Function Theory, various subclasses of the normalized analytic function class $\mathcal{A}$ have been studied from different viewpoints. The q-calculus as well as the fractional q-calculus provide important tools that have been used in order to investigate various subclasses of $\mathcal{A}$. Historically speaking, a firm footing of the usage of the q-calculus in the context of Geometric Function Theory was actually provided and the basic (or q-) hypergeometric functions were first used in Geometric Function Theory in a book chapter by Srivastava (see, for details, [10, pp. 347 et seq.]). In fact, the theory of univalent functions can be described by using the theory of the q-calculus. We begin by providing some basic definitions and concept details of the q-calculus which are used in this paper. We shall follow the notation and terminology in [10, 11]. We first recall the definition of fractional p,q-calculus operators of a complex-valued function $f(z)$:

Definition 1.1. *For a function $f \in \mathcal{A}$ given by (1) and $0 < q < p \leq 1$, the (p,q)-derivative of function f is defined by*

$$D_{p,q}f(z) = \begin{cases} \frac{f(pz)-f(qz)}{(p-q)z} \ ; \ z \neq 0 \\ \\ f'(0) \ ; \ z = 0. \end{cases}$$

For $f(z) = z + \sum\limits_{n=2}^{\infty} a_n z^n$, according to the definition, we have

$$D_{p,q}f(z) = 1 + \sum_{n=2}^{\infty} [n]_{p,q} a_n z^{n-1}, \tag{3}$$

where the symbol denotes the so-called (p,q)-bracket or twin-basic number

$$[n]_{p,q}=\frac{p^n-q^n}{p-q}. \tag{4}$$

Note that, by putting $p=1$, the (p,q)-derivative reduces to the q-derivative.

$$D_qf(z)=\begin{cases}\frac{f(z)-f(qz)}{(1-q)z}\ ;\ z\neq 0\\[2ex] f'(0)\ ;\ z=0.\end{cases}$$

So, for $f\in\mathcal{A}$, we have

$$D_qf(z)=1+\sum_{n=2}^{\infty}[n]_q a_n z^{n-1}, \tag{5}$$

where

$$[n]_q=\frac{1-q^n}{1-q}. \tag{6}$$

Also, by taking $q\to 1^-$, $D_qf(z)$ reduces to $f'(z)$, for $f\in\mathcal{A}$.

In this paper, we present new subclasses of the bi-univalent functions $\sigma_{\mathfrak{B}}$ by using the (p,q)-derivative operator. Furthermore, we obtain estimates on the initial coefficients $|a_2|$ and $|a_3|$ for functions in these new subclasses.

2. Coefficient estimates for the function class $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta,\alpha}$

In this section, we present and investigate the subclass $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta,\alpha}$.

Definition 2.1. *A function $f(z)$ given by (1) is said to be in the class $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta,\alpha}$ $(0<p<q\leq 1,\ 0<\alpha\leq 1,\ \beta>0)$, if the following conditions are satisfied:*

$$f\in\sigma_{\mathfrak{B}}\ ,\ \left|arg\left(D_{p,q}f(z)+\beta z(D_{p,q}f(z))'\right)\right|<\frac{\alpha\pi}{2}\ (z\in\mathbb{U}) \tag{7}$$

and

$$\left|arg\left(D_{p,q}g(w)+\beta w(D_{p,q}g(w))'\right)\right|<\frac{\alpha\pi}{2}\ (w\in\mathbb{U}), \tag{8}$$

where $2(1-\alpha)\sum\limits_{m=1}^{\infty}\frac{(-1)^{m-1}}{\beta m+1}\leq 1$ and g is the extension of f^{-1} to $\mathbb{U}$.

Remark 2.2. *By taking $q\to 1^-$ and $p=1$ in Definition 2.1, the class $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta,\alpha}$ reduces to the class $\mathcal{H}_{\Sigma}(\alpha,\beta)$, wad defined by Frasin [5].*

Now, we obtain the estimates on the coefficients $|a_2|$ and $|a_3|$ for function class $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta,\alpha}$. For this purpose we need the following lemma.

Lemma 2.3. [4] *If $p(z) \in P$, then $|p_k| \leq 2$ for each k, where P is the family of all functions $p(z)$ analytic in $\mathbb{U}$ for which*

$$Re\ p(z) > 0,\ p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots\ for\ z \in \mathbb{U}.$$

Theorem 2.4. *Let the $f(z)$ given by (1) is said to be in the class $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta,\alpha}$. Then*

$$|a_2| \leq\ min \left\{ \frac{2\alpha}{(1+\beta)[2]_{p,q}}, \frac{2\alpha}{\sqrt{|2\alpha(1+2\beta)[3]_{p,q} + (1-\alpha)(1+\beta)^2[2]_{p,q}^2|}} \right\}$$

and

$$|a_3| \leq \frac{4\alpha^2}{(1+\beta)^2[2]_{p,q}^2} + \frac{2\alpha}{(1+2\beta)[3]_{p,q}}.$$

Proof. At the first, we write the argument inequalities in (7) and (8) as follows:

$$D_q f(z) + \beta z(D_q f(z))' = (k(z))^\alpha\ (z \in \mathbb{U}) \tag{9}$$

and

$$D_q g(w) + \beta w(D_q g(w))' = (h(w))^\alpha\ (w \in \mathbb{U}), \tag{10}$$

Respectively, where $k(z)$ and $h(w)$ satisfy the following inequalities

$$Re(k(z)) > 0\ and\ Re(h(w)) > 0\ (z, w \in \mathbb{U}).$$

Furthermore, the functions $k(z)$ and $h(w)$ have the forms

$$k(z) = 1 + k_1 z + k_2 z^2 + k_3 z^3 + \cdots \tag{11}$$

and

$$h(w) = 1 + h_1 w + h_2 w^2 + h_3 w^2 + \cdots, \tag{12}$$

Now, equating the coefficients in (9) and (10), we get

$$(1+\beta)[2]_{p,q} a_2 = \alpha k_1, \tag{13}$$

$$(1+2\beta)[3]_{p,q} a_3 = \alpha k_2 + \frac{\alpha(\alpha-1)}{2} k_1^2, \tag{14}$$

$$-(1+\beta)[2]_{p,q} a_2 = \alpha h_1 \tag{15}$$

and

$$(1+2\beta)[3]_{p,q}(2a_2^2 - a_3) = \alpha h_2 + \frac{\alpha(\alpha-1)}{2} h_1^2. \tag{16}$$

From (13) and (15), we get

$$h_1 = -k_1 \tag{17}$$

and

$$2(1+\beta)^2[2]_{p,q}^2 a_2^2 = \alpha^2(k_1^2 + h_1^2). \tag{18}$$

Now from (14), (16) and (18), we obtain

$$\begin{aligned} 2(1+2\beta)[3]_{p,q}a_2^2 &= \alpha(k_2+h_2) + \frac{\alpha(\alpha-1)}{2}(k_1^2+h_1^2) \\ &= \alpha(k_2+h_2) + \frac{\alpha-1}{\alpha}(1+\beta)^2[2]_{p,q}^2 a_2^2. \end{aligned}$$

So, we have

$$a_2^2 = \frac{\alpha^2(k_2+h_2)}{2\alpha(1+2\beta)[3]_{p,q} + (1-\alpha)(1+\beta)^2[2]_{p,q}^2}. \tag{19}$$

Now getting the absolute values of the Eqs. (18) and (19) and using Lemma 2.3, we have

$$|a_2|^2 \leq \frac{\alpha^2(|k_1|^2+|h_1|^2)}{2(1+\beta)^2[2]_{p,q}^2} \leq \frac{4\alpha^2}{(1+\beta)^2[2]_{p,q}^2}$$

and

$$\begin{aligned} |a_2|^2 &\leq \frac{\alpha^2(|k_2|+|h_2|)}{|2\alpha(1+2\beta)[3]_{p,q} + (1-\alpha)(1+\beta)^2[2]_{p,q}^2|} \\ &\leq \frac{4\alpha^2}{|2\alpha(1+2\beta)[3]_q + (1-\alpha)(1+\beta)^2[2]_{p,q}^2|}. \end{aligned}$$

respectively. Next, in order to find the bound on the coefficient $|a_3|$, we subtract (16) from (14), we thus get

$$a_3 = a_2^2 + \frac{\alpha(k_2-h_2)}{2(1+2\beta)[3]_{p,q}}. \tag{20}$$

Upon substituting the value of a_2^2 from (18) into (20), it follows that

$$a_3 = \frac{\alpha^2(k_1^2+h_1^2)}{2(1+\beta)^2[2]_{p,q}^2} + \frac{\alpha(k_2-h_2)}{2(1+2\beta)[3]_{p,q}}. \tag{21}$$

So, by using Lemma 2.3, we get

$$\begin{aligned} |a_3| &\leq \frac{\alpha^2(|k_1|^2+|h_1|^2)}{2(1+\beta)^2[2]_{p,q}^2} + \frac{\alpha(|k_2|+|h_2|)}{2(1+2\beta)[3]_{p,q}} \\ &\leq \frac{4\alpha^2}{(1+\beta)^2[2]_{p,q}^2} + \frac{2\alpha}{(1+2\beta)[3]_{p,q}}. \end{aligned}$$

This evidently completes the proof of Theorem 2.4. □

3. Coefficient estimates for the function class $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta}(\gamma)$

We begin this section by introducing the function class $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta}(\gamma)$ by means of the following definition.

Definition 3.1. *A function $f(z)$ given by* (1) *is said to be in the class $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta}(\gamma)$ $(0 < q < p \leq 1,\ 0 \leq \gamma < 1,\ \beta \geq 0)$, if the following conditions are satisfied:*

(22) $$f \in \sigma_{\mathfrak{B}}\ ,\ Re\left(D_{p,q}f(z) + \beta z(D_{p,q}f(z))'\right) > \gamma\ (z \in \mathbb{U})$$

and

(23) $$Re\left(D_{p,q}g(w) + \beta w(D_{p,q}g(w))'\right) > \gamma\ (w \in \mathbb{U}),$$

where $2(1-\gamma)\sum\limits_{m=1}^{\infty}\frac{(-1)^{m-1}}{\beta m+1} \leq 1$ *and g is the extension of f^{-1} to $\mathbb{U}$.*

Remark 3.2. *By taking $q \to 1^-$ and $p = 1$ in Definition 3.1, the class $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta}(\gamma)$ reduces to the class $\mathcal{H}_{\Sigma}(\gamma,\beta)$, introduced and studied by Frasin* [5].

Now, we find the estimates on the initial coefficients for class $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta}(\gamma)$.

Theorem 3.3. *Let the $f(z)$ given by* (1) *is said to be in the class $\mathcal{H}_{\sigma_{\mathfrak{B}}}^{p,q,\beta}(\gamma)$. Then*

$$|a_2| \leq \min\left\{\frac{2(1-\gamma)}{(1+\beta)[2]_{p,q}}, \sqrt{\frac{2(1-\gamma)}{(1+2\beta)[3]_{p,q}}}\right\}$$

and

$$|a_3| \leq \min\left\{\frac{4(1-\gamma)^2}{(1+\beta)^2[2]_{p,q}^2} + \frac{2(1-\gamma)}{(1+2\beta)[3]_{p,q}}, \frac{2(1-\gamma)}{(1+2\beta)[3]_{p,q}}\right\}.$$

Proof. At the first, we write the argument inequalities in (22) and (23) as follows:

(24) $$D_q f(z) + \beta z(D_q f(z))' = \gamma + (1-\gamma)k(z)\ (z \in \mathbb{U})$$

and

(25) $$D_q g(w) + \beta w(D_q g(w))' = \gamma + (1-\gamma)h(w)\ (w \in \mathbb{U}),$$

where $k(z)$ and $h(w)$ have the forms (11) and (12), respectively. Now, equating the coefficients in (24) and (25), we get

$$(1+\beta)[2]_{p,q}a_2 = (1-\gamma)k_1, \tag{26}$$

$$(1+2\beta)[3]_{p,q}a_3 = (1-\gamma)k_2, \tag{27}$$

$$-(1+\beta)[2]_{p,q}a_2 = (1-\gamma)h_1 \tag{28}$$

and

$$(1+2\beta)[3]_{p,q}(2a_2^2 - a_3) = (1-\gamma)h_2. \tag{29}$$

From (26) and (28), we get

$$h_1 = -k_1 \tag{30}$$

and

$$2(1+\beta)^2[2]_{p,q}^2 a_2^2 = (1-\gamma)^2(k_1^2 + h_1^2). \tag{31}$$

Now from (27) and (29), we obtain

$$2(1+2\beta)[3]_{p,q}a_2^2 = (1-\gamma)(k_2 + h_2). \tag{32}$$

Now getting the absolute values of the Eqs. (31) and (32) and using Lemme 1, we have

$$|a_2|^2 \leq \frac{(1-\gamma)^2(|k_1|^2 + |h_1|^2)}{2(1+\beta)^2[2]_{p,q}^2} \leq \frac{4(1-\gamma)^2}{(1+\beta)^2[2]_{p,q}^2}$$

and

$$|a_2|^2 \leq \frac{(1-\gamma)(|k_2| + |h_2|)}{2(1+2\beta)[3]_{p,q}} \leq \frac{2(1-\gamma)}{(1+2\beta)[3]_{p,q}}$$

respectively. Next, in order to find the bound on the coefficient $|a_3|$, we subtract (29) from (27), we thus get

$$a_3 = a_2^2 + \frac{(1-\gamma)(k_2 - h_2)}{2(1+2\beta)[3]_{p,q}}. \tag{33}$$

By substituting the value of a_2^2 from (31) into (33), it follows that

$$a_3 = \frac{(1-\gamma)^2(k_1^2 + h_1^2)}{2(1+\beta)^2[2]_{p,q}^2} + \frac{(1-\gamma)(k_2 - h_2)}{2(1+2\beta)[3]_{p,q}}.$$

By using lemma 2.3, we readily get

$$|a_3| \leq \frac{4(1-\gamma)^2}{(1+\beta)^2[2]_{p,q}^2} + \frac{2(1-\gamma)}{(1+2\beta)[3]_{p,q}}.$$

By substituting the value of a_2^2 from (32) into (33), it follows that

$$a_3 = \frac{(1-\gamma)(k_2+h_2)}{2(1+2\beta)[3]_{p,q}} + \frac{(1-\gamma)(k_2-h_2)}{2(1+2\beta)[3]_{p,q}} = \frac{(1-\gamma)k_2}{(1+2\beta)[3]_{p,q}}.$$

By using lemma 2.3 once again, we readily get

$$|a_3| \leq \frac{2(1-\gamma)}{(1+2\beta)[3]_{p,q}}.$$

This evidently completes the proof of Theorem 3.3. □

4. Corollaries and Consequences

Taking $q \to 1^-$ and $p = 1$ in Theorem 2.4, we obtain the following result.

Corollary 4.1. *Let f given by (1) be in the class $\mathcal{H}(\Sigma, \alpha)$. Then*

$$|a_2| \leq \ min\left\{\frac{\alpha}{1+\beta}, \frac{2\alpha}{\sqrt{2(\alpha+2)+4\beta(\alpha+\beta-\alpha\beta+2)}}\right\}$$

and

$$|a_3| \leq \frac{\alpha^2}{(1+\beta)^2} + \frac{2\alpha}{3(1+2\beta)}.$$

Remark 4.2. *Corollary 4.1 provides the estimate of $|a_2|$ and $|a_3|$, which was obtained previously by Frasin* [5, Theorem 2.2].

Taking $q \to 1^-$ and $p = 1$ in Theorem 3.3, we obtain the following result.

Corollary 4.3. *Let f given by (1) be in the class $\mathcal{H}(\Sigma, \beta)$. Then*

$$|a_2| \leq \min\left\{\frac{1-\gamma}{1+\beta}, \sqrt{\frac{2(1-\gamma)}{3(1+2\beta)}}\right\}$$

and

$$|a_3| \leq \min\left\{\frac{(1-\gamma)^2}{(1+\beta)^2} + \frac{2(1-\gamma)}{3(1+2\beta)}, \frac{2(1-\gamma)}{3(1+2\beta)}\right\} = \frac{2(1-\gamma)}{3(1+2\beta)}.$$

Remark 4.4. *Corollary 4.3 provides the estimate of $|a_2|$ and improves the estimate of $|a_3|$, which was obtained previously by Frasin* [5, Theorem 3.2].

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