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KILLING MAGNETIC FLUX SURFACES IN EUCLIDEAN 3-SPACE

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Abstract. In this paper, we give a geometric approach to Killing magnetic flux surfaces in Euclidean 3-space and solve the differential equations which expressed the mentioned surfaces. Furthermore we give some examples and draw their pictures by using the programme Mathematica.

1. Introduction

A divergence free vector field called as magnetic field. A smooth surface with normal vector is a *flux surface* of a smooth vector field if $g(V,N)$ is zero everywhere on M where g is inner product on M. The best example we can give to the flux surfaces (magnetic surface) is plasma. A plasma is a hot ionized gas consisting of approximately equal numbers of positively charged ions and negatively charged electrons. Considering the outermost bounding surfaces of magnetically confined plasma, the implication for plasma confinement become clear. If plasma is assumed magnetized everywhere, the magnetic field can not vanish on this surfaces. Hazeltine *et al.* have shown that the outermost, bounding surface must be a flux surface, it is natural to suppose the confinement region to be filled by a sequence of flux surfaces, each enclosing the next. In fact, flux surfaces provide a barrier to collisionless charged particles in the magnetic field [10]. Most of the universe is in the form of a plasma with a magnetic field perforated. The most common example of flux surfaces is flux tubes. As used in astrophysics, a flux tube generally has a

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larger magnetic field and other properties that differ from the surrounding space. These flux tubes are commonly found around many stars and around the sun. Some planets also have flux tubes. A well-known example is the flux tube between Jupiter and its moon "io" [13, 5, 10, 14]. It has been necessary to confine or limit the plasma to examine the magnetic field lines and the charged particles motions in the magnetic field. For this, Penning has developed a model called Penning trap. This model has been one of the most successful models and has contributed greatly to the work done in this area. However, this model can be use for confinement of single species plasmas. Later, the importance of flux surfaces has long been recognized in magnetic fusion research. For the first time, Pedersen et al. Showed that non-neutral plasmas can be confined to flux surfaces. These surface configurations, such as stellarators and tokamaks, are highly developed and studied in the context of thermonuclear fusion, and have recently become of interest for the confinement of non-neutral plasmas. They have certain advantages over open and closed field line systems, such as the Penning trap. These configurations confine both positive and negative species simultaneously, at any level of charge imbalance from pure electron to quasi-neutral. They may provide stabilization of diocotron modes, and confine energetic electrons and positrons at modest magnetic field strengths. Hence, such configurations have unique advantages for confine non-neutral electron-ion plasmas and antiproton positron plasmas in the positron electron laboratory. Moreover, can cause it to produce anti-hydrogen in abundant quantities [14].

In [5], a local non-orthogonal coordinate system, zero-framed with respect to the knot, is introduced, and the field is decomposed into toroidal and poloidal ingredients with respect to this system. The helicity of the field is then determined; this vanishes for a field that is either purely toroidal or purely poloidal. The magnetic energy functional is calculated under the simplifying assumptions that the tube is axially uniform and of circular cross-section. The case of a tube with helical axis is first considered, and new results concerning kink mode instability and associated bifurcations are obtained by Boozer.

On the other hand, a charged particle in a magnetic field experiences a force which called as *Lorentz force*. Due to this force it traces out a path called as *magnetic curves*. Magnetic curve plays an important role between physics and differential geometry. In terms of physics, when a charged particle enters the magnetic field besides the velocity vector (tangent vector) is expressed to the magnetic field and experience a force. So, charged particle follows a trajectory called magnetic curve with the influence of the force. For example, if T tangent vector field field enters (pass through) in the V static magnetic field with constant angle due to the Lorentz force particle traces helical path. Specifically, if the tangent vector field of the trajectory of the charged particle is perpendicular to magnetic field it traces a circular path. On the other hand, if the charged particle's tangent vector moves parallel to magnetic field, the Lorentz force acts zero. In terms of differential geometry, a magnetic field is defined by the property that its divergence is zero in $3D$ manifolds. Because of the fact that Killing vector field is divergence free, it is created magnetic field called [2, 3, 4].

The magnetic trajectories of magnetic field are curves satisfying the Lorentz force equation

$$\nabla_{\gamma'}\gamma' = \phi(\gamma') = V \times \gamma'$$

which generalizes the equation of geodesics under the condition $\nabla_{\gamma'}\gamma' = 0$. In contrast to geodesics, magnetic curves are not reversible and they cannot be rescaled, that is, the trajectories depend on the energy$\nu(t) = \|\gamma'\| = \nu_0$. In a sense, magnetic curves is one of the generalizations of the notion of geodesics. If the curve is an arc-length curve it is called a *normal magnetic curve*.

Through this equation magnetic curves can found many useful applications in analytical chemistry, biochemistry, atmospheric science, geochemistry, cyclotron, proton, cancer therapy, and velocity selector. Moreover, the solutions of the Lorentz force equation are Kirchhoff elastic rods. This provides an amazing connection between two apparently unrelated physical models and, in particular, it ties the classical elastic theory with the Hall effect (see for details [3, 8, 9]).

A given smooth surface M with normal vector N is a flux surface of a magnetic vector field V if $g(V, N)$ is zero everywhere on M. In other words, the magnetic field does not cross the surface M anywhere, i.e., the magnetic flux traversing M is zero. It is then possible to define a scalar flux function f such that its value is constant on the surface M, and

$$g(V, \nabla f) = 0.$$

In three dimensions, the only closed flux surface corresponding to a non-vanishing vector field is a topological toroid. This fact lies at the basis of the design of magnetic confinement devices.

Assuming the flux surfaces have this toroidal topology, the function$f$ defines a set of nested surfaces, so it makes sense to use this function to label the flux surfaces, i.e., f may be used as a "radial" coordinate. Each toroidal surface f encloses a volume $\mathbf{V}(f)$. The surface corresponding to an infinitesimal volume $\mathbf{V}$ is essentially a line that corresponds to the toroidal axis (called magnetic axis when V is a magnetic field). The flux F through an arbitrary surface M is given by

$$F = \int_S g(V, N) dS$$

When V is a magnetic field with toroidal nested flux surfaces, two magnetic fluxes can be defined from two corresponding surfaces [10]. The poloidal flux is defined by

$$\Psi = \int_{S_p} g(V, N) dS$$

where S_p is a ring-shaped ribbon stretched between the magnetic axis and the flux surface f. (Complementarily, Sp can be taken to be a surface spanning the central hole of the torus [5]) Likewise, the toroidal flux is defined by

$$\Phi \int_{S_t} g(V, N) dS$$

where S_t is a poloidal section of the flux surface. In the present paper, we have noted that the equation of a flux surface may be written in the form of differential equations. Then these equations can be solved as in partial differential equation. Also, it is known that the only closed flux surface corresponding to a non-vanishing vector field is a topological toroid, in three dimensions. We obtain the torus as the solution of the partial differential equation. The development of the physics of magnetically confined plasmas has important value for the study of space and astrophysical plasmas. Positive and negative charged ions and negatively charged electrons in a fusion plasma are at very high temperatures and accordingly have large velocities. To continue the fusion process, the particles from the hot plasma should be confined to a zone or the plasma should be allowed to cool rapidly. Magnetic confinement fusion devices take advantage of the fact that charged particles in a magnetic field experience a Lorentz force and track helical trajectories along the field lines.A number of topics in physics have been developed primarily through research on magnetically confined plasmas. Therefore, we give a geometrical approach these studies and the results in this study have some useful applications for the design of magnetic confinement devices.

2. Preliminaries

Let (M, g) be a $3-$dimensional Riemannian manifold with the standard flat metric g defined by

$$g(X, Y) = x_1 y_1 + x_2 y_2 + x_3 y_3 \tag{1}$$

for all $X = (x_1, x_2, x_3)$, $Y = (y_1, y_2, y_3) \in \chi(M)$.

The Lorentz force of a magnetic field F on M is defined to be a skew symmetric operator given by

$$g(\phi(X), Y) = F(X, Y) \tag{2}$$

for all X, $Y \in \chi(M)$.

The cross product of two vector fields X, $Y \in \chi(M)$ is given by

$$X \times Y = (x_2 y_3 - x_3 y_2, x_3 y_1 - x_1 y_3, x_2 y_1 - x_1 y_2). \tag{3}$$

Then, the mixed product of the vector fields X, Y, $Z \in \chi(M)$ is defined by

$$g(X \times Y, Z) = dv_g(X, Y, Z) \tag{4}$$

where dv_g denotes a volume on M.

A unit vector field V on M is Killing if and only if it satisfies the Killing equation

$$g(\nabla_Y V, X) + g(\nabla_X V, Y) = 0 \tag{5}$$

where X, $Y \in \chi(M)$ and ∇ is the Levi-Civita connection on M.

Let V be a Killing vector field and $F_V = \imath_V dv_g$ be the corresponding Killing magnetic force on M where $\imath$ denotes the inner product. Then the Lorentz force of the F_V given as

$$\phi(X) = V \times X. \tag{6}$$

for all $X \in \chi(M)$ [4]. Consequently, the magnetic trajectories γ determined by V are solutions of the Lorentz force equation written as

$$\phi(\gamma') = \nabla_{\gamma'} \gamma' = V \times \gamma'. \tag{7}$$

A unit speed curve γ is a magnetic trajectory of the magnetic field V if and only if V can be written along γ as

$$V(s) = \varpi(s) T(s) + \kappa(s) B(s) \tag{8}$$

where T and B are the tangent and binormal vectors of the curve γ, respectively (see ref. [4]).

The fundamental solutions of Eq.(5) $\left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}, -z\frac{\partial}{\partial y} + y\frac{\partial}{\partial z}, z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z} \right\}$ give a basis of

Killing vector fields on 3−dimensional Euclidian space $\mathbb{E}^3$. Where x, y and z denote the global coordinates on $\mathbb{E}^3$ and $\mathbb{R}^3 = span\left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\}$ is considered as a vector space [4, 6].

In this paper, we determine Flux surfaces of the Killing vector fields $V = \frac{\partial}{\partial z}$ and $V = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$. The other vector fields $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, -z\frac{\partial}{\partial y} + y\frac{\partial}{\partial z}$ and $z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}$ determine the same classifications for corresponding magnetic Flux surface. In this paper, our aim is to investigate all flux surfaces corresponding to Killing magnetic fields on 3−dimensional Euclidian space $\mathbb{E}^3$.

3. Killing Flux Surfaces in Euclidean 3-Space

Definition 3.1. *A given smooth surface M with its normal vector field N is a flux surface of a smooth vector field V when $g(V, N) = 0$ everywhere on M. If V is a Killing magnetic field then the flux surface M called as Killing magnetic surface of V.*

Theorem 3.2. *Let M be a surface in Euclidean 3-space and*

$$X(u, v) = (x_1(u, v), x_2(u, v), x_3(u, v))$$

be parametrization of M. Then M is a flux surface of magnetic vector field $V = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial x}$ if and only if

$$\frac{\partial x_3}{\partial u}\left(x_1\frac{\partial x_1}{\partial v} + x_2\frac{\partial x_2}{\partial v}\right) - \frac{\partial x_3}{\partial v}\left(x_1\frac{\partial x_1}{\partial u} + x_2\frac{\partial x_2}{\partial u}\right) = 0. \tag{9}$$

Proof. Let $X(u, v) = (x_1(u, v), x_2(u, v), x_3(u, v))$ be a parametrization of M in Euclidean 3-space. Then its unit normal vector given by

$$N = \frac{1}{\|X_u \times X_v\|}\begin{pmatrix} \frac{\partial x_2}{\partial u}\frac{\partial x_3}{\partial v} - \frac{\partial x_2}{\partial v}\frac{\partial x_3}{\partial u}, \\ \frac{\partial x_1}{\partial v}\frac{\partial x_3}{\partial u} - \frac{\partial x_1}{\partial u}\frac{\partial x_3}{\partial v}, \\ \frac{\partial x_1}{\partial u}\frac{\partial x_2}{\partial v} - \frac{\partial x_1}{\partial v}\frac{\partial x_2}{\partial u} \end{pmatrix}$$

Since $X(u, v)$ is a flux surface of magnetic vector field $V = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$ we have

$$g(N, V) = 0.$$

This implies that

$$\frac{\partial x_3}{\partial u}\left(x_1\frac{\partial x_1}{\partial v} + x_2\frac{\partial x_2}{\partial v}\right) - \frac{\partial x_3}{\partial v}\left(x_1\frac{\partial x_1}{\partial u} + x_2\frac{\partial x_2}{\partial u}\right) = 0.$$

Conversely if we assume that Eq.(9) holds, then it is easily seen that $g(N, V) = 0$. Thus $X(u, v)$ is a flux surface of magnetic vector field

$V = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$. Similar discussion can make for $-z\frac{\partial}{\partial y} + y\frac{\partial}{\partial z}$ and $-z\frac{\partial}{\partial x} + x\frac{\partial}{\partial z}$. □

Now we give a solution for differential equation Eq.(9) and draw its picture using the Mathematica programme.

Solution 3.3. *Let M be a flux surface of magnetic vector field $V = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial x}$ in Euclidean 3-space. Then M satisfy Eq.(9). Loss of the generality we may take $A(u,v) = x_1\frac{\partial x_1}{\partial v} + x_2\frac{\partial x_2}{\partial v}$, $B(u,v) = x_1\frac{\partial x_1}{\partial u} + x_2\frac{\partial x_2}{\partial u}$ in Eq.(9) we have following differential equation*

$$A(u,v)\frac{\partial x_3}{\partial u} - B(u,v)\frac{\partial x_3}{\partial v} = 0. \tag{10}$$

In order to give an example we solve Eq.(10), then we get $x_3(u,v) = c$, c is a constant. Then we can write

$$dx_3 = x_{3u}du + x_{3v}dv = 0.$$

This gives us

$$\frac{x_{3u}}{x_{3v}} = -\frac{dv}{du}. \tag{11}$$

Using Eq.(10) and Eq.(11), we obtain following differential equation

$$\frac{dv}{du} = \frac{B(u,v)}{A(u,v)}.$$

From the last equation we get characteristic coordinates $\xi = u$ and $\eta = \psi(u,v)$ in Eq.(10) where $\{\xi,\eta\}$ is linear independent set and $\frac{\partial(\xi,\eta)}{\partial(u,v)} \neq 0$. Consequently, we obtain $x_3(u,v) = f(\eta)$. Then, for certain $x_1(u,v)$ and $x_2(u,v)$ we have the flux surface as

$$X(u,v) = (x_1(u,v), x_2(u,v), f(\eta)).$$

Example 3.4. *Let M be a flux surface of magnetic vector field $V = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial x}$ in Euclidean 3-space. Then M satisfy Eq.(9). If $x_1(u,v) = \cos(u)$, $x_2(u,v) = \sin(v)$ then using the Eq.(9) we have*

$$\sin v\cos v\frac{\partial x_3}{\partial u} + \cos u\sin u\frac{\partial x_3}{\partial v} = 0$$

and this gives as

$$\frac{dv}{du} = \frac{\cos u\sin u}{\sin v\cos v} \Rightarrow \cos 2v - \cos 2u = c. \tag{12}$$

From Eq.(12) we have

$$\xi = u,$$
$$\eta = \cos 2v - \cos 2u.$$

Consequently, flux surface of magnetic vector field $V = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial x}$ *given by*

$$X(u, v) = (\cos u, \sin v, f(\cos 2v - \cos 2u)).$$

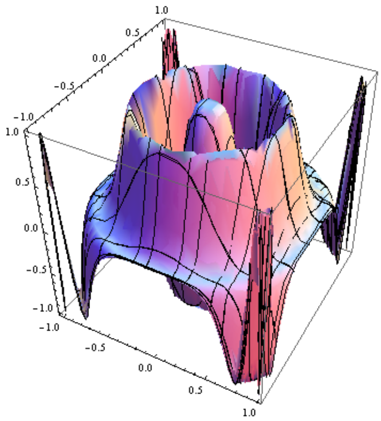

FIGURE 1. Flux surface X(u,v)=$(\cos u, \sin v, \sin(\cos 2v - \cos 2u)^3$.

Example 3.5. *Let* M *be a flux surface of magnetic vector field* $V = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$ *in Euclidean 3-space. Then* M *satisfy Eq.(9). If* $x_1(u, v) = (c + a\cos v)\cos u$, $x_2(u, v) = (c + a\cos v)\sin u$ *then using the Eq.(9) we have the solutions of the equation as*

$$X(u, v) = ((c + a\cos v)\cos u, (c + a\cos v)\sin u, f(v)).$$

We will give some examples below as a result of the equation.

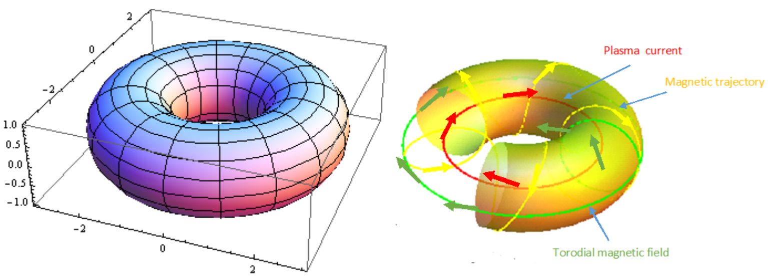

FIGURE 2. X(u,v)=$((2 + \cos v)\cos u, (2 + \cos v)\sin u, 2\sin v\cos v$.

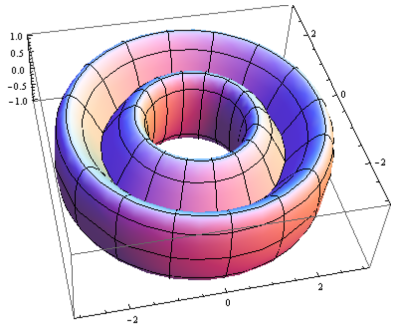

FIGURE 3. X(u,v)=$((2+\cos v)\cos u, (2+\cos v)\sin u, v^3-6v)$.

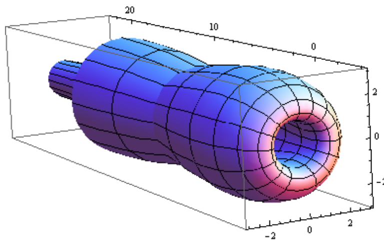

FIGURE 4. X(u,v)=$((2+\cos v)\cos u, (2+\cos v)\sin u, v^2+4v)$.

Corollary 3.6. *The topological torus is a flux surface of magnetic field* $V=-y\frac{\partial}{\partial x}+x\frac{\partial}{\partial x}$.

Proof. It is easily seen that the surface $X(u,v)$ satisfy Eq.(9), so we can say that torus is a flux surface of magnetic field $V=-y\frac{\partial}{\partial x}+x\frac{\partial}{\partial x}$. □

Theorem 3.7. *Let M be a surface in Euclidean 3-space and*

$$X(u,v)=(x_1(u,v),x_2(u,v),x_3(u,v)) \tag{13}$$

be parametrization of M. Then M is a flux surface of magnetic vector field $V=\frac{\partial}{\partial z}$ if and only if

$$\frac{\partial x_1}{\partial u}\frac{\partial x_2}{\partial v}-\frac{\partial x_1}{\partial v}\frac{\partial x_2}{\partial u}=0. \tag{14}$$

Proof. Let $X(u,v)=(x_1(u,v),x_2(u,v),x_3(u,v))$ be a parametrization of M in Euclidean 3-space. Then its unit normal vector N given by Eq.(3). Since $X(u,v)$ is a flux surface of magnetic vector field $V=\frac{\partial}{\partial z}$

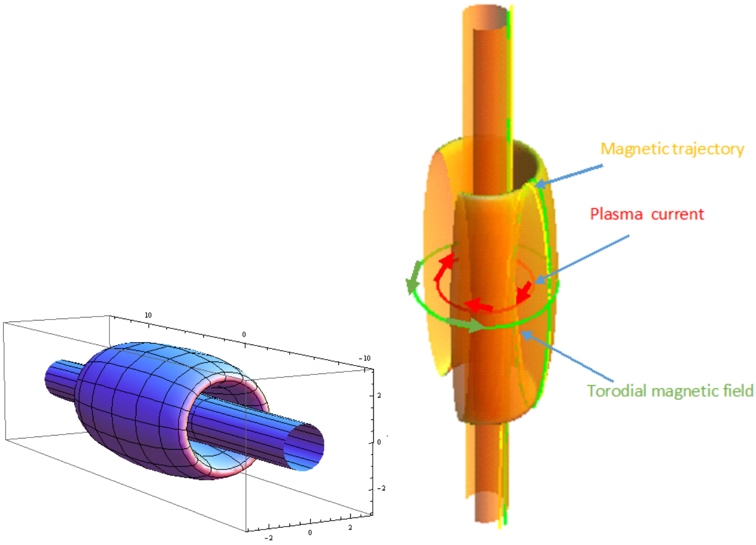

FIGURE 5. X(u,v)=$((2+\cos v)\cos u, (2+\cos v)\sin u, v^3+6v)$.

we have

$$g(N,V)=0$$

it implies that

$$\frac{\partial x_1}{\partial u}\frac{\partial x_2}{\partial v}-\frac{\partial x_1}{\partial v}\frac{\partial x_2}{\partial u}=0.$$

Conversely if we assume that $X(u,v)$ satisfy Eq.(14) then it is easily seen that $g(N,V)=0$. Thus $X(u,v)$ is a flux surface of magnetic vector field $V=\frac{\partial}{\partial z}$. Similar discussion can make for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$. □

Now we give a solution for differential equation Eq.(14) and draw its picture using the Mathematica programme.

Solution 3.8. *Let $X(u,v)=(x_1(u,v),x_2(u,v),x_3(u,v))$ be a flux surface of magnetic vector field $V=\frac{\partial}{\partial z}$ in Euclidean 3-space. Then M satisfy Eq.(14). Loss of the generality we may take $\frac{\partial x_1}{\partial u}=A(u,v)$, $\frac{\partial x_1}{\partial v}=B(u,v)$ in Eq.(14), then we have following differential equation*

$$A(u,v)\frac{\partial x_2}{\partial u}-B(u,v)\frac{\partial x_2}{\partial v}=0. \tag{15}$$

In order to solve Eq.(15) we get $\eta=x_2(u,v)=c$, c is a constant. Then we can write

$$dx_2=x_{2u}du+x_{2v}dv=0.$$

This gives

$$\frac{x_{2u}}{x_{2v}} = -\frac{dv}{du}. \tag{16}$$

Using Eq.(15) and Eq.(16) we obtain following differential equation

$$\frac{dv}{du} = \frac{B(u,v)}{A(u,v)}.$$

From the last equation we get characteristic coordinates $\xi = u$ *and* $\eta = \psi(u,v)$ *in Eq.(15) where* $\{\xi, \eta\}$ *is linear independent set and* $\frac{\partial(\xi,\eta)}{\partial(u,v)} \neq 0$. *Consequently, we obtain* $x_2(u,v) = g(\eta)$. *Then for certain* $x_1(u,v)$ *we have the flux surface as*

$$X(u,v) = (x_1(u,v), g(\eta), x_3(u,v)).$$

Example 3.9. *Let* $X(u,v) = (x_1(u,v), x_2(u,v), x_3(u,v))$ *be a flux surface of magnetic vector field* $V = \frac{\partial}{\partial z}$ *in Euclidean 3-space. Then* $X(u,v)$ *satisfy Eq.(14). If* $x_1(u,v) = \sqrt{uv}$ *then using the Eq.(14) we have*

$$\frac{v}{2\sqrt{uv}}\frac{\partial x_2}{\partial v} - \frac{u}{2\sqrt{uv}}\frac{\partial x_2}{\partial u} = 0$$

and this gives us

$$\frac{dv}{du} = -\frac{v}{u} \Rightarrow uv = c. \tag{17}$$

From Eq.(17) we have

$$\begin{aligned} \xi &= u, \\ \eta &= uv. \end{aligned} \tag{18}$$

Consequently flux surface of magnetic vector field $V = \frac{\partial}{\partial z}$ *given by*

$$X(u,v) = (\sqrt{uv}, f(uv), x_3(u,v)).$$

Theorem 3.10. *Let* γ *be a curve on the Killing Flux surface* M. *Then* γ *is a magnetic curve if and only if* γ *is a geodesic curve on* M.

Proof. Let γ be a magnetic curve on the Flux surface M. Then γ satisfy Eq.(8). Using Eq.(8) and Definition 1 we can easily see that γ is a geodesic curve.
Conversely, we assume that γ is a geodesic curve on M then γ satisfy Eq.(8). This gives us γ is a magnetic curve. □

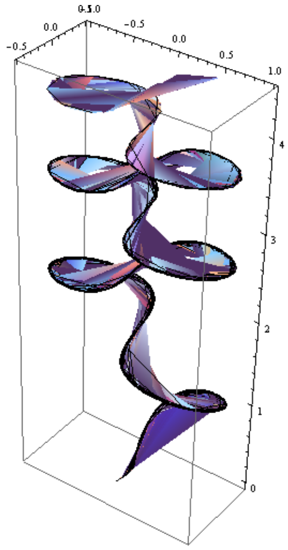

FIGURE 6. Flux surface X(u,v)=($\sqrt{uv}, \sin(uv)^3, 1/2\sin(2uv)$).

Definition 3.11. *$X(u,v) = (x_1(u,v), x_2(u,v), x_3(u,v))$ be a parametrization at a point $p \in S$ of a surface S in $3-$dimensional almost contact metric manifold $\mathbb{R}^3(-3)$. We call S is a distribution surface if the unit normal vector field of S belongs to contact distribution (see for details in* [7]*).*

Corollary 3.12. *Let M be a surface in Sasakian 3 manifold $\mathbb{R}^3(-3)$ and*
$X(u,v) = (x_1(u,v), x_2(u,v), x_3(u,v))$ be parametrization of M. Then M is a flux surface of magnetic vector field ξ if and only if M is a distribution surface.

Proof. It is obvious from Definition (3.1) and Definition (3.11). □

Example 3.13. *Let $X(u,v) = (\cos u, \sin v, \sin(\cos 2v - \cos 2u)^3)$ be a flux surface of magnetic field $V = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial x}$ and γ be magnetic trajectories of V. From last theorem we can easily say that $\gamma_1(s) = (0, \sin s, \sin(\cos 2s + 1)^3)$, $\gamma_2(s) = (\cos s, \sin s, 0)$ and $\gamma_3(s) = (\cos s, 1, -\sin(1 + \cos 2s)^3)$ are magnetic curves on the flux surface $X(u,v)$.*

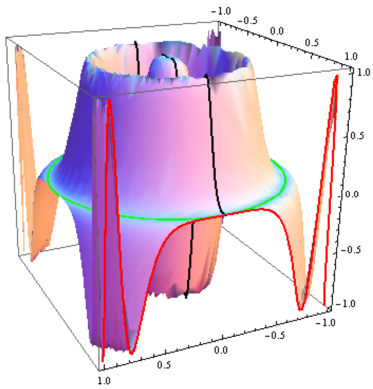

FIGURE 7. Flux surface and magnetic curves γ_1(Black), γ_2(Green) and γ_3(Red) on this surface.

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