# Estimation of Allowable Path-deviation Time in Free-space Optical Communication Links Using Various Aircraft Trajectories 

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#### Abstract

The allowable path-deviation time of aircraft in a free-space optical communication system has been estimated from various trajectories, using different values of aircraft speeds and turn rates. We assumed the existence of a link between the aircraft and a ground base station. First, the transmitter beam's divergence angle was calculated through two different approaches, one based on a simple optical-link equation, and the other based on an attenuation coefficient. From the calculations, the discrepancy between the two approaches was negligible when the link distance was approximately 110 km , and was under $5 \%$ when the link distance ranged from 80 to 140 km . Subsequently, the allowable path-deviation time of the aircraft within the tracking-error tolerance of the system was estimated, using different aircraft speeds, turn rates, and link distances. The results indicated that the allowable path-deviation time was primarily determined by the aircraft's speed and turn rate. For example, the allowable path-deviation time was estimated to be $\sim 3.5 \mathrm{~s}$ for an aircraft speed of $166.68 \mathrm{~km} / \mathrm{h}$, a turn rate of $90^{\circ} / \mathrm{min}$, and a link distance of 100 km . Furthermore, for a constant aircraft speed and turn rate, the path-deviation time was observed to be almost unchanged when the link distance ranged from 80 to 140 km .


Keywords : Free-space optical communication, Beam-divergence angle, Tracking error, Path-deviation time OCIS codes : (060.2605) Free-space optical communication; (060.4510) Optical communications

## I. INTRODUCTION

A free-space optical (FSO) communication system can easily provide a high-capacity data link without an installed optical-fiber infrastructure [1-3]. Various research efforts towards FSO communication links are underway to provide cost-effective broadband wireless Internet services in rural areas or underdeveloped countries that lack a well-established wired Internet infrastructure. In particular, high-capacity FSO communication links have been implemented between unmanned aerial vehicles (UAVs) in Facebook's Aquila project [2], and between balloons in Google's Loon project [3]. However, as compared to a conventional microwave-based wireless communication system, the FSO communication system must make additional efforts to maintain a line of sight (LOS), for reliable communication links. This is mainly owing to the narrow beam-divergence angle in FSO
communication systems. Thus, the efficient implementation of a pointing, acquisition, and tracking (PAT) subsystem would be a major contributor to the establishment of reliable FSO communication systems [4, 5].

A simple method to reduce the performance requirements for a PAT subsystem is to use a wide beam-divergence angle for FSO communication links. However, as the beamdivergence angle for the communication link increases, the received power decreases for a given transmitter power level. Thus the beam-divergence angle cannot be increased indefinitely while still meeting the power budget for an FSO communication system. There exists a tradeoff between the received power and the PAT performance requirements. In this study, we have calculated the optimum transmitter beam-divergence angle by using a simple equation that was first derived from a classical optical-link equation [6, 7]. Subsequently, the calculated results were compared to those from the previous approach in [8], which considered the

[^0]effect of the attenuation coefficient, instead of only utilizing a free-space range loss in the optical-link equation. The comparison of these two approaches for the calculation of the transmitter beam-divergence angle indicated that the discrepancy between the two approaches was negligible when the link distance was approximately 110 km . Using the calculated optimum beam-divergence angle at the transmitter side, we then estimated the tracking-error tolerance in the PAT subsystem.

Previously, the effect of tracking (or pointing) errors on the outage probability in general FSO communication links has been evaluated by including the standard deviation of pointing-error-induced jitter [9, 10]. For example, in [9] the jitter's standard deviation normalized to the receiver diameter was used for the calculation of the outage probability and link availability. In [11], the maximum tolerable tracking error was given as a design specification for the FSO communication link with a fast airborne platform. However, we think that, instead of the jitter's standard deviation and the maximum tracking-error tolerance, the allowable pathdeviation time for seamless communication with various aircraft trajectories would be straightforward for providing the tracking-time requirement for efficient implementation of PAT subsystems in the FSO communication link using a moving aircraft. Thus, in this study the allowable pathdeviation time of the aircraft in the FSO communication link was estimated using aircraft trajectories that varied based on their speeds and turn rates. The results imply that the allowable path-deviation time from the expected aircraft trajectories was mainly determined by the speed and turn rate of the aircraft. For example, the path-deviation time was estimated to be $\sim 3.5 \mathrm{~s}$ for an aircraft speed of 166.68 $\mathrm{km} / \mathrm{h}$ and a turn rate of $90^{\circ} / \mathrm{min}$ when the link distance ranged from 80 to 140 km . We believe that the PAT subsystems should be designed to finish their tracking procedure within the estimated path-deviation time in the FSO communication link using a moving aircraft platform.

## II. RESULTS AND DISCUSSION

The performance of the FSO communication link can be evaluated by using the received power, which can be determined by considering all causes of loss and gain in the communication link. In the FSO communication link, the received power can be expressed as follows [6, 7]:

$$
\begin{equation*}
P_{r x}=P_{t x} \cdot G_{t x} \cdot L_{r} \cdot G_{r x} \cdot A_{s y s t e m} \tag{1}
\end{equation*}
$$

where $P_{t x}$ is the transmitted signal power, and $G_{t x}$ and $G_{r x}$ are the transmit and receive antenna gains respectively. Additionally, $L_{r}$ is the free-space range loss, and $A_{\text {system }}$ is the system-dependent loss, which includes pointing loss and atmospheric loss. In particular, the transmit antenna gain is directly related to the beam-divergence angle $\Theta$ (which is
the full-angle $\mathrm{e}^{-2}$ divergence of the transmit beam [6]) as follows:

$$
\begin{equation*}
G_{t x}=\frac{32}{\Theta^{2}} \tag{2}
\end{equation*}
$$

Eq. (2) helps to approximate the gain of the transmit beam, which is only a fraction of the limiting aperture [7]. Using the established expressions for the receive antenna gain and the free-space range loss $[6,7]$, a simple expression for the calculation of the transmitter beam-divergence angle can be derived as follows:

$$
\begin{equation*}
\Theta=\frac{D}{L} \sqrt{\frac{2 P_{t x} A_{\text {system }}}{P_{r x}}} \tag{3}
\end{equation*}
$$

where $D$ and $L$ are the aperture diameter of the receive antenna and the link distance respectively. By using this simple equation derived from the link equation, the transmitter beam-divergence angle can be easily calculated for the FSO communication link. Previously, it was reported that the transmitter beam-divergence angle could be calculated by considering the attenuation-induced loss [8], which was given as

$$
\begin{equation*}
\Theta=\frac{1.72 D}{L} \sqrt{\frac{P_{t x} A_{\text {system }} h_{a}}{P_{r x}}} \tag{4}
\end{equation*}
$$

where the attenuation-induced loss $h_{a}=e^{-\alpha L}$ and $\alpha$ is the attenuation coefficient. Here the optimum transmitter beam-divergence angle was calculated using Eq. (3) first; subsequently, the calculated results were compared to the results based on Eq. (4). Table 1 summarizes the system parameters for our FSO communication link. Receiver sensitivity, which is the required power at the receiver to achieve a certain level of system performance and is denoted by $P_{r x^{\prime} \operatorname{sen}}$, was added to a system margin to calculate the received power. For the system-dependent loss $A_{\text {system }}$, a $3-\mathrm{dB}$ pointing loss plus a $2-\mathrm{dB}$ atmospheric loss were assumed in our calculation. For the attenuation-induced loss, the attenuation coefficient was assumed to be $3.5 \times$ $10^{-6} \mathrm{~m}^{-1}$, as given in [8].

TABLE 1. FSO communication system parameters

| Parameter | Value |
| :--- | :---: |
| Signal wavelength | 1550 nm |
| Transmitted signal power $\mathrm{P}_{\mathrm{tx}}$ | 20 dBm |
| Receiver sensitivity $\mathrm{P}_{\mathrm{rx}}$ sen | -38.9 dBm |
| Receiver antenna diameter D | 37 mm |
| Pointing loss | 3 dB |
| Atmospheric loss | 2 dB |
| System margin | 4.76 dB |
| Attenuation coefficient | $3.5 \times 10^{-6} \mathrm{~m}^{-1}$ |

Figure 1 shows the calculated transmitter beam-divergence angles as a function of link distance using two approaches, based on the simple link equation (Eq. (3), solid line) and the attenuation-induced loss (Eq. (4), dotted line). From the results, the discrepancy between the two approaches was negligible when the link distance was approximately 110 km . At a link distance of 112 km , the transmitter beam-divergence angle was calculated to be 133.81 and $133.78 \mu \mathrm{rad}$, using Eqs. (3) and (4), respectively. For link distances shorter than 110 km , the results calculated using Eq. (4) were slightly higher than those calculated using Eq. (3). On the other hand, for link distances greater than 110 km , the results calculated using Eq. (3) were higher than those calculated using Eq. (4). This was because in Eq. (4) the beam-divergence angle is proportional to a factor of $1.72 \sqrt{h_{a}}$, compared to a factor of $\sqrt{2}$ in Eq. (3). Thus, owing to the effect of $h_{a}$, the transmitter beam-divergence angle calculated using Eq. (4) decreases slightly faster than that from Eq. (3), as the link distance increases.

To estimate the tracking-error tolerance in the PAT subsystem, we assumed that the transmitter and receiver of the system were perfectly aligned, and that the diameter of the received beam was much larger than the aperture diameter of the receive antenna. In other words, the receive


FIG. 1. Transmitter beam-divergence angle calculated as a function of link distance. Solid and dotted lines represent the calculated results based on Eqs. (3) and (4) respectively.
antenna was located at the center of a large received beam. In this case, the maximum offset of the transmitter beam would be half of the transmitter beam-divergence angle on each side in any radial direction (i.e. $\pm \frac{D}{L} \sqrt{\frac{P_{t x} A_{s y s t e m}}{2 P_{r x}}}$ from Eq. (3), and $\pm \frac{1.72 D}{2 L} \sqrt{\frac{P_{t x} A_{\text {system }} h_{a}}{P_{r x}}}$ from Eq. (4)). For example, using Eq. (3) the optimum transmitter beamdivergence angle was calculated to be $150 \mu \mathrm{rad}$ at a link distance of 100 km , as shown in Fig. 1. Using this value, the maximum tracking-error tolerance was estimated as $\pm 75 \mu \mathrm{rad}$.

In FSO communication between a flying aircraft and a grounded base station, the established link would be maintained in a point-ahead manner by using the projected trajectory of the aircraft. However, the aircraft can change its trajectory (which is a path deviation from the projected trajectory in our estimation) with various speeds and turning rates, which may induce a tracking error in the PAT subsystem. Figure 2 shows the schematic diagram for estimating the maximum offset between the projected and actual trajectories of the aircraft during the turn [5]. $r$ and $\theta$ represent the turn radius and turn angle, respectively. Subsequently, the turn rate $\theta$ is given by $\theta / t$ at time $t$. In addition, $d$ is the expected location without an aircraft turn, which is given by $v t$ for an aircraft speed $v$ at time $t$. Then, the maximum offset $\Delta$ can be calculated as a function of $r, \theta$, and $d$ [5]. Finally, the tracking error $\alpha$ relative to the expected position can be calculated by the equation

$$
\begin{equation*}
\alpha=2 \tan ^{-1} \frac{\Delta}{2 L} . \tag{5}
\end{equation*}
$$

Figure 3 shows the tracking error $\alpha$ calculated with Eq. (5) as a function of path-deviation time for different turn rates and aircraft speeds. In this calculation, the link distance was assumed to be 100 km , for which the maximum tracking-error tolerance was previously estimated as $\pm 75$ $\mu \mathrm{rad}$. In Fig. 3(a), six different turn rates were considered for an aircraft speed of $166.68 \mathrm{~km} / \mathrm{h}$ ( 90 knots). For an allowable tracking error for a link distance of 100 km , the path-deviation times were estimated to be $\sim 3.5$ and $\sim 5 \mathrm{~s}$ for turn rates of 90 and $45 \% \mathrm{~min}$, respectively (the estimation


FIG. 2. Schematic diagram for the estimation of maximum offset between the projected and actual trajectories of the remote aircraft.


FIG. 3 Tracking error calculated as a function of path-deviation time, for (a) different turn rates and (b) different aircraft speeds. In this calculation, the link distance was assumed to be 100 km .
examples were indicated by dotted lines in Fig. 3). In Fig. 3(b), four different aircraft speeds were used to calculate the tracking error as a function of path-deviation time for a turn rate of $45^{\circ} / \mathrm{min}$. The allowable path-deviation times for aircraft speeds of $333.36 \mathrm{~km} / \mathrm{h}$ and $166.68 \mathrm{~km} / \mathrm{h}$ were estimated to be $\sim 3.5$ and $\sim 5 \mathrm{~s}$, respectively.

Figure 4 shows the tracking errors calculated as a function of path-deviation time for different link distances. In this calculation, the aircraft speed and the turn rate were assumed to be $166.68 \mathrm{~km} / \mathrm{h}$ and $90^{\circ} / \mathrm{min}$, respectively. We used four different link distances from 80 to 140 km , where the differences between the transmitter beam-divergence angles calculated using Eqs. (3) and (4) were less than 5\%. For a link distance of 100 km , the allowable path-deviation time was estimated to be $\sim 3.5 \mathrm{~s}$, which is identical to that in Fig. 3. In this estimation, the tracking-error tolerance


FIG. 4 Tracking error calculated as a function of path-deviation time, for different link distances. In this calculation, the aircraft speed and turn rate were assumed to be $166.68 \mathrm{~km} / \mathrm{h}$ and $90^{\circ} / \mathrm{min}$, respectively.
was estimated from a transmitter beam-divergence angle of $\sim 150 \mu \mathrm{rad}$, which was calculated using Eq. (3). The transmitter beam-divergence angle calculated using Eq. (4) was $\sim 153 \mu \mathrm{rad}$ for a link distance of 100 km ; thus, the tracking-error tolerance was $\pm 76.5 \mu \mathrm{rad}$. The allowable pathdeviation time was also estimated to be $\sim 3.5 \mathrm{~s}$, as shown in Fig. 4. For a link distance of 140 km , the transmitter beam-divergence angles were estimated to be $\sim 107 \mu \mathrm{rad}$ and $\sim 102 \mu \mathrm{rad}$ using Eqs. (3) and (4) respectively. Then, the tracking-error tolerances were estimated to be $\pm 53.5 \mu \mathrm{rad}$ from Eq. (3) and $\pm 51 \mu \mathrm{rad}$ from Eq. (4) respectively. As shown in Fig. 4, the path-deviation times were estimated to be $\sim 3.5$ and $\sim 3.4 \mathrm{~s}$ using the abovementioned trackingerror tolerances, respectively. For a link distance of 80 km , the allowable path-deviation times were estimated to be $\sim 3.5$ and $\sim 3.6 \mathrm{~s}$ from Eqs. (3) and (4) respectively. From these results, we have found that the allowable path-deviation time is mainly dependent on aircraft speed and turn rate, rather than link distance. The longer the link distance, the smaller the transmitter beam divergence, as shown in Fig. 1, which implies that the tracking-error tolerance is reduced. On the other hand, as shown in Fig. 2, the maximum offset in Eq. (5) can increase as the link distance increases. Thus, we conclude that the effect of link distance on the allowable path-deviation time is negligible.

## III. SUMMARY

The allowable path-deviation time from an expected aircraft trajectory has been estimated using various values of aircraft speeds, turn rates, and link distances. For the estimation, we assumed that a communication link was established between the aircraft and grounded base station in the FSO system. First, the transmitter beam-divergence angle was calculated through two different approaches, one
based on a simple link equation and the other based on an attenuation coefficient. From the results, we found that the differences between the calculated transmitter beam-divergence angles based on the two approaches were less than $5 \%$ when the link distance ranged from 80 to 140 km . Subsequently, the maximum tracking-error tolerance in the PAT subsystem was estimated using the calculated transmitter beamdivergence angle. Finally, the allowable path-deviation time was estimated, and the obtained values were within the maximum tracking-error tolerance. From the results, we found that the allowable path-deviation time was primarily dependent on the aircraft speed and turn rate. For example, the allowable path-deviation time was estimated to be $\sim 3.5 \mathrm{~s}$ for an aircraft speed of $166.68 \mathrm{~km} / \mathrm{h}$ and a turn rate of $90^{\circ} / \mathrm{min}$ when the link distance ranged from 80 and 140 km . Thus, we have also found that the effect of link distance on the allowable path-deviation time is negligible in the specified link distance range. We believe that these results can be utilized to design efficient PAT subsystems.

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