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In-Plane Extensional Vibration Analysis of Asymmetric Curved Beams with Linearly Varying Cross-Section Using DQM

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미분구적법(DQM)을 이용한 단면적이 선형적으로 변하는 비대칭 곡선보의 내평면 신장 진동해석

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Abstract The increasing use of curved beams in buildings, vehicles, ships, and aircraft has results in considerable effort being directed toward developing an accurate method for analyzing the dynamic behavior of such structures. The stability behavior of elastic curved beams has been the subject of a large number of investigations. Solutions of the relevant differential equations have traditionally been obtained by the standard finite difference. These techniques require a great deal of computer time as the number of discrete nodes becomes relatively large under conditions of complex geometry and loading. One of the efficient procedures for the solution of partial differential equations is the method of differential quadrature. The differential quadrature method(DQM) has been applied to a large number of cases to overcome the difficulties of the complex algorithms of programming for the computer, as well as excessive use of storage due to conditions of complex geometry and loading. In this study, the in-plane extensional vibration for asymmetric curved beams with linearly varying cross-section is analyzed using the DQM. Fundamental frequency parameters are calculated for the member with various parameter ratios, boundary conditions, and opening angles. The results are compared with the result by other methods for cases in which they are available. According to the analysis of the solutions, the DQM, used only a limited number of grid points, gives results which agree very well with the exact ones.

요 약 빌딩, 자동차, 선박, 항공기 등에서의 곡선보 사용 증가로 인해 이러한 구조물의 동적거동해석에 있어 괄목할 만한 성과가 있어 왔다. 탄성곡선보의 안정성 거동 해석분야는 많은 연구자들의 관심분야였다. 전통적으로 미분방정식의 해법은 유한차분법으로 해결해왔다. 이러한 방법들은 복잡한 기하학적 구조 및 하중에 따른 격자점의 증가로 많은 계산시간을 요구한다. 편미분방정식의 해를 구하기 위한 효율적인 방법 중의 하나는 미분구적법이다. 복잡한 기하학적 구조 및 하중으로 인한 과도한 컴퓨터 용량의 사용과 복합알고리즘 프로그램의 어려움을 극복하기 위하여 미분구적법(DQM)이 많은 분야에 적용되어왔다. 본 연구에서는 선형적으로 단면적이 변하는 비대칭 곡선보에 대하여 DQM을 적용하여 아크축 신장을 고려한 내 평면 진동해석을 수행하였다. 다양한 매개변수 비, 경계조건, 그리고 열림 각에 따른 기본진동수를 계산하였다. DQM 결과는 활용 가능한 다른 엄밀해와 비교하였다. 다양한 매개변수 비, 경계조건, 그리고 열림 각에 따른 기본진동수를 계산하였으며 DQM 결과를 활용 가능한 다른 엄밀해와 비교하였다. 해석결과에 따르면 DQM은, 적은 격자점을 사용하고도, 엄밀해 결과와 일치함을 보여주었다.

Keywords : Asymmetric Curved Beam, DQM, Extensional Vibration, Fundamental Frequency, New Result

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1. Introduction

The increasing usage of the curved beams in buildings, cars, and aircraft has resulted in developing an accurate method for analyzing the dynamic behavior of such structures. Accurate research of the vibration response of curved beams is of great importance in many engineering fields such as the design of the structures.

Hoppe[1] and Love[2] previously studied the in-plane vibration of beams. Love[2] extended the research on Hoppe's theory for stretching of the ring. Lamb[3] investigated the statics of incomplete rings with small curvature. Den Hartog[4] found the lowest natural frequency of circular beams with simply supported or clamped ends using the Rayleigh-Ritz method, and the study was extended by Volterra and Morell[5]. Archer[6] showed the basic equations of motion as given in Love[2] for the in-plane inextensional vibrations of an incomplete circular beam. Nelson[7] carried out for the vibration of a circular ring segment having simply supported ends using Lagrangian multipliers. Auciello and De Rosa[8] reviewed the vibrations of circular beams and briefly showed a number of other studies. Ojalvo[9] obtained the behavior of three-dimensional motions of elastic beams using classical beam-theory assumptions. Rodgers and Warner[10] also studied the frequencies of curved elastic beams with simply supported ends.

The differential quadrature method introduced by Bellman and Casti[11] is more effective method for the solution of differential equations. This simple technique can be applied to a large number of fields to solve the difficulties of complex program algorithms, as well as usage of excessive storage of the computer memories. In the present research, the in-plane extensional vibration of the asymmetric curved beams with linearly varying cross-section is analyzed using the DQM. Fundamental frequency parameters are calculated for the member with various parameter ratios of heights.

of slenderness, boundary conditions, and opening angles. The results are compared with the results by other methods for cases in which they are available. New results are also suggested.

2. Governing Differential Equations

In Fig. 1, the coordinate systems for the curved beam is shown. The beam axis is defined by the angle θ. Here, w is the tangential displacements of the beam axis, u is the radial displacements, r is the radius, h_0 is the height of the cross-section at the middle, and θ_0 is the opening angle. All displacements are positive directions as shown.

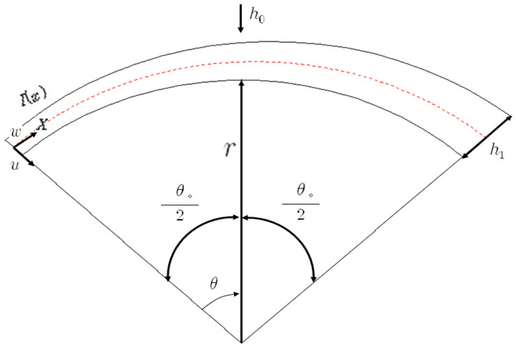

Fig. 1. Coordinates for a curved beam

The equilibrium conditions of a circular curved beam neglecting rotatory inertia and shear deformation, as shown in Fig. 2, give

$$\frac{\partial T}{\partial \theta} + N^* = mr\frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\frac{\partial N^*}{\partial \theta} - T = mr\frac{\partial^2 w}{\partial t^2} \tag{2}$$

$$\frac{\partial M}{\partial \theta} + Tr = 0 \tag{3}$$

where N^*, T, and M are the normal force, the

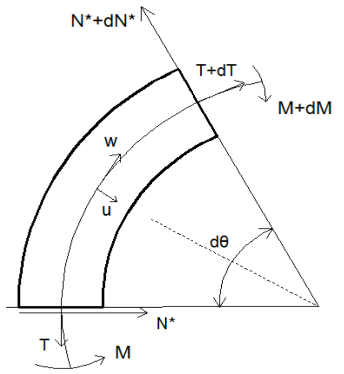

Fig. 2. Forces on a curved beam

shear force, and the bending moment, respectively. Here t is the time, and m is the mass per unit length. From the theory of curved beams, the normal force and the bending moment are given

$$N^{*}=(\frac{EA(\theta)}{r})(\frac{\partial w}{\partial \theta}-u) \tag{4}$$

$$M=(\frac{EI(\theta)}{r^2})(\frac{\partial w}{\partial \theta}+\frac{\partial^2 u}{\partial \theta^2}) \tag{5}$$

where E is the Young's modulus, A is the cross-section area, and I is the area moment of inertia.

The substitution of equations (4) and (5) into equations (1) and (2) using equation (3) shows the following differential equations:

$$-\frac{1}{r^3}(EI''(w'+u'')+2EI'(w''+u''')+EI(w'''+u^{iv})) + \frac{EA}{r}(w'-u)=mr\ddot{u} \tag{6}$$

$$\frac{1}{r^3}(EI'(w'+u'')+EI(w''+u'''))+\frac{1}{r}EA'(w'-u)) + EA(w''-u')=mr\ddot{w} \tag{7}$$

in which each prime and dot denote differentiation with respect to θ and t, respectively. Assume that the beam is under the vibration with a frequency ω and let

$$u(\theta,t)=U(\theta)T(t), \quad w(\theta,t)=W(\theta)T(t) \tag{8}$$

where $U(\theta)$ and $W(\theta)$ are the normal functions of $u(\theta)$ and $w(\theta)$, respectively, and $T(\theta)$ is $e^{i\omega t}$.

Introducing dimensionless distance coordinate X (see Fig. 1) defined as

$$X=\frac{\theta}{\theta_0} \tag{9}$$

Consider the beam with a rectangular cross sectional area shown in Figure 1. Here, $f(X)$ and $A(X)$ are the function of the cross-section variation law and the area of the varying cross section associated with the height of the cross-section h_0 at the middle of the beam. The simple case in which the cross-section varies linearly is studied, because the only law has been studied by Auciello and De Rosa[8]. The variation law is

$$I(X)=I_0F(X)=I_0 f(X)^3, \quad A(X)=A_0 f(X)$$
$$f(X)=[1+(2\eta(X-0.5))] \tag{10}$$

where η $(=\frac{h_1}{h_0}-1)$ is the ratio of the heights.

Using equations (8), (9), and (10), the differential equations (6) and (7) can be rewritten

$$-\frac{F''}{\theta_0^2}(\frac{W'}{\theta_0}+\frac{U''}{\theta_0^2})+2\frac{F'}{\theta_0}(\frac{W''}{\theta_0^2}+\frac{U'''}{\theta_0^3})+F(\frac{W'''}{\theta_0^3}+\frac{U^{(iv)}}{\theta_0^4}) + f(\frac{S}{R\theta_0})^2(\frac{W'}{\theta_0}-U)=\frac{mr^4\omega^2}{EI_0}U \tag{11}$$

$$\frac{F'}{\theta_0}(\frac{W'}{\theta_0}+\frac{U''}{\theta_0^2})+F(\frac{W''}{\theta_0^2}+\frac{U'''}{\theta_0^3})+\frac{f'}{\theta_0}(\frac{S}{R\theta_0})^2(\frac{W'}{\theta_0}-U) + f(\frac{S}{R\theta_0})^2(\frac{W''}{\theta_0^2}-\frac{U'}{\theta_0})=\frac{mr^4\omega^2}{EI_0}W \tag{12}$$

where S is the length of the beam axis, $r\theta_0$, and R is the radius of gyration of the cross sectional area, $(\frac{I_0}{A_0})^{\frac{1}{2}}$. Each prime denotes differentiation with respect to the dimensionless distance X.

The equations (11) and (12) are the governing equation of the in-plane extensional vibration of the asymmetric curved beams with varying cross-section.

The in-plane inextensional condition is starting with the basic equations where there is no extension of the center line. This condition requires that w and u be

$$u = \frac{\partial w}{\partial \Theta} \tag{13}$$

Using the equation (13) and eliminating u in equations (6) and (7), one can rewrite the equation

$$\frac{W^{(iv)}}{\theta_0^6}(F) + \frac{W^{(v)}}{\theta_0^5}(3\frac{F'}{\theta_0}) + \frac{W^{(vi)}}{\theta_0^4}(3\frac{F''}{\theta_0^2}+2F) + \frac{W'''}{\theta_0^3}(\frac{F'''}{\theta_0^3}+4\frac{F'}{\theta_0}) + \frac{W''}{\theta_0^2}(3\frac{F''}{\theta_0^2}+F) + \frac{W'}{\theta_0}(\frac{F'''}{\theta_0^3}+\frac{F'}{\theta_0}) = \frac{mr^4\omega^2}{EI_0}(-W+\frac{W''}{\theta_0^2}) \tag{14}$$

The equation (14) is the governing equation of the in-plane inextensional vibration of the asymmetric curved beams.

The boundary conditions of the beam for both ends clamped, both ends simply supported, and clamped-simply supported ends are, respectively,

$$W = U = U' = 0 \quad \text{at X=0 and 1} \tag{15}$$

$$W = U = M = 0 \quad \text{at X=0 and 1} \tag{16}$$

$$W = U = U' = 0 \quad \text{at X=0,}$$
$$W = U = M = 0 \quad \text{at X=1} \tag{17}$$

3. Differential Quadrature Method

Bellman and Casti[11] introduced the differential quadrature method (DQM) by formulating the quadrature rule in their introductory paper. They suggested the DQM as a new method for the numerical solution of ordinary and partial differential equations. Jang et al.[12] applied for the first time to structural components of the beams. The versatility of the DQM to engineering solutions in general and to structural solutions in particular is increasingly evident. Recently, Kang and Kim[13], and Kang and Park[14] studied the vibration and the buckling analysis of the asymmetric curved beams using the DQM, respectively. More recently, Kang and Park[15] also analyzed the extensional vibration of the curved beams using the DQM. The application of the DQM to a partial differential equation can be written

$$L\{g(x)\}_i = \sum_{j=1}^{N} W_{ij}\, g(x_j) \quad \text{for } i, j = 1, 2, \sim N \tag{18}$$

where L is the differential operator, x_j is the discrete point, i is the row vector of the N value, $g(x_j)$ is the function value, W_{ij} is the weighting coefficient of the function value, and N is the number of discrete point. This equation can be written as the derivatives of the functions in terms of the function values at all discrete points.

The form of the function $g(x)$ is

$$g_k(X) = X^{k-1} \quad \text{for } k = 1, 2 \sim N \tag{19}$$

If the differential operator represents an n^{th} derivative, the equation is

$$\sum_{j=1}^{N} W_{ij} x_j^{k-1} = (k-1)(k-2)\cdots(k-n)x_i^{k-n-1} \quad \text{for } i, k = 1, 2, \sim N \tag{20}$$

This expression consists of N sets of N linear algebraic equations, giving a unique solution for the weighting coefficients since the matrix is a Vandermonde matrix.

4. Numerical Application

The DQM is applied to the in-plane extensional vibration of the asymmetric curved beam with linearly varying cross-section.

Applying the DQM to equations (11) and (12), gives

$$-\frac{F''}{\theta_0^2}\left(\frac{1}{\theta_0}\sum_{j=1}^{N}A_{ij}W_j+\frac{1}{\theta_0^2}\sum_{j=1}^{N}B_{ij}U_j\right)$$
$$+2\frac{F'}{\theta_0}\left(\frac{1}{\theta_0^2}\sum_{j=1}^{N}B_{ij}W_j+\frac{1}{\theta_0^3}\sum_{j=1}^{N}C_{ij}U_j\right)$$
$$+F\left(\frac{1}{\theta_0^3}\sum_{j=1}^{N}C_{ij}W_j+\frac{1}{\theta_0^4}\sum_{j=1}^{N}D_{ij}U_j\right)$$
$$+f\left(\frac{S}{R\theta_0}\right)^2\left(\frac{1}{\theta_0}\sum_{j=1}^{N}A_{ij}W_j-U_i\right)=\frac{mr^4\omega^2}{EI_0}U_i \quad (21)$$

$$\frac{F'}{\theta_0}\left(\frac{1}{\theta_0}\sum_{j=1}^{N}A_{ij}W_j+\frac{1}{\theta_0^2}\sum_{j=1}^{N}B_{ij}U_j\right)$$
$$+F\left(\frac{1}{\theta_0^2}\sum_{j=1}^{N}B_{ij}W_j+\frac{1}{\theta_0^3}\sum_{j=1}^{N}C_{ij}U_j\right)$$
$$+\frac{f'}{\theta_0}\left(\frac{S}{R\theta_0}\right)^2\left(\frac{1}{\theta_0}\sum_{j=1}^{N}A_{ij}W_j-U_i\right)$$
$$+f\left(\frac{S}{R\theta_0}\right)^2\left(\frac{1}{\theta_0^2}\sum_{j=1}^{N}B_{ij}W_j-\frac{1}{\theta_0}\sum_{j=1}^{N}A_{ij}U_j\right)=\frac{mr^4\omega^2}{EI_0}W_i \quad (22)$$

where A_{ij}, B_{ij}, C_{ij}, and D_{ij} are the weighting coefficients for the first-order, second-order, third-order, and fourth-order derivatives, respectively.

The boundary conditions for both ends clamped, given by equation (15), can be expressed in differential quadrature as

$$\begin{aligned}
&W_1=0 && \text{at } X=0\\
&W_N=0 && \text{at } X=1\\
&U_1=0 && \text{at } X=0\\
&U_N=0 && \text{at } X=1\\
&\sum_{j=1}^{N}A_{2j}U_j=0 && \text{at } X=0+\delta\\
&\sum_{j=1}^{N}A_{(N-1)j}U_j=0 && \text{at } X=1-\delta \quad (23)
\end{aligned}$$

Here, δ is a very small distance from the boundary ends. In their paper on the application of DQM to the static analysis of the beams, Jang et al.[12] suggested the δ-technique to the boundary points of discrete points at a very small distance.

The boundary conditions for both ends simply supported, given by equation (16), can be expressed in differential quadrature as

$$\begin{aligned}
&W_1=0 && \text{at } X=0\\
&W_N=0 && \text{at } X=1\\
&U_1=0 && \text{at } X=0\\
&U_N=0 && \text{at } X=1\\
&\frac{1}{\Theta_0}\sum_{j=1}^{N}A_{2j}W_j+\frac{1}{\Theta_0^2}\sum_{j=1}^{N}B_{2j}U_j=0 && \text{at } X=0+\delta\\
&\frac{1}{\theta_0}\sum_{j=1}^{N}A_{(N-1)j}W_j+\frac{1}{\theta_0^2}\sum_{j=1}^{N}B_{(N-1)j}U_j=0 && \text{at } X=1-\delta \quad (24)
\end{aligned}$$

Similarly, the boundary conditions for one clamped and one simply supported ends, given by equation (17), can be expressed in differential quadrature as

$$\begin{aligned}
&W_1=0 && \text{at } X=0\\
&W_N=0 && \text{at } X=1\\
&U_1=0 && \text{at } X=0\\
&U_N=0 && \text{at } X=1\\
&\sum_{j=1}^{N}A_{2j}U_j=0 && \text{at } X=0+\delta\\
&\frac{1}{\theta_0}\sum_{j=1}^{N}A_{(N-1)j}W_j+\frac{1}{\theta_0^2}\sum_{j=1}^{N}B_{(N-1)j}U_j=0 && \text{at } X=1-\delta \quad (25)
\end{aligned}$$

This set of equations with the boundary conditions can be solved for the in-plane extensional vibration of the asymmetric curved beams with linearly varying cross-section.

5. Numerical Results and Comparisons

The fundamental frequency parameter, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$ for the extensional vibration of the asymmetric curved beam is evaluated for the rectangular cross sections under the various conditions. In the following, the simple cases in which the cross-section varies linearly are examined, because the only law has been studied by Auciello and De Rosa[8]. Kang and Kim[13] showed the convergence studies relative to the number of grid point N and the very small distance parameter δ, respectively. The optimal values for N are found to be 11 to 13 points, and the optimal values for δ are found to be 1×10^{-5} to 1×10^{-6} by trial-and-error calculations. Therefore, all results are calculated using 13 grid points and $\delta=1\times10^{-6}$ along the dimensionless axis.

The ratio of heights, $\eta(=\frac{h_1}{h_0}-1)$ is taken to be from 0.0 to 0.4, and the ratio of slenderness, S/r is 30, 100, and 500, respectively. The results by the DQM are summarized in Tables 1～8 without comparisons because no data are available. Tables 1 and 2 show the fundamental frequency parameters, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$ for the cases of simply-simply supported ends with $\eta=$ 0.1 and $\eta=$ 0.4. Tables 3～8 also show the frequency parameters for the case of fixed-fixed ends, fixed-simply supported ends, and simply supported-fixed ends, respectively. Table 9 also shows the comparisons of the frequency parameters for the uniform cross sectional area with non-uniform cross sectional area of the beams for the cases of fixed-fixed ends with S/R=30 and S/R=500.

As the values of the ratio of heights, $\eta(=\frac{h_1}{h_0}-1)$ beam become larger, the values of frequency parameters become higher for the cases of simply-simply supported and simply supported-fixed ends. On the other hand, as the values of $\eta(=\frac{h_1}{h_0}-1)$ become larger, the values of frequency parameters become lower for the cases of fixed-fixed and fixed-simply supported ends.

From Tables 1~8, it is seen that the values of frequency parameters of the bean with fixed ends are much higher than those of the beam with simply supported ends. The values of frequency parameters of the beam with simply supported-fixed ends are also higher than those of the beam with fixed-simply supported ends. The values of frequency parameters can be increased by decreasing the opening angle, Θ_0 and the slenderness ratio, S/R. However, when the value of the slenderness ratio, S/R is greater than 500, the difference between the values of frequency parameters is less than 2.0 percent. The variations of the slenderness ratio, S/R and the ratio of heights, $\eta(=\frac{h_1}{h_0}-1)$ affect the vibration behavior of fixed boundary conditions more significantly than the vibration behavior of simply supported boundary conditions. Table 9 shows that the values of frequency parameters of uniform cross-sectional beam are also slightly higher than those of non-uniform cross-sectional beam. The difference of the values of the parameters in the uniform and the non-uniform beams can be also reduced by increasing the values of S/R. The beam behaviors are affected more importantly by fixed-fixed end conditions, smaller opening angles, larger ratios of heights, and smaller slenderness ratios. Auciello and De Rosa[8] also calculated the fundamental frequencies of the inextensional vibration of the beams using the SAP IV finite element methods employing 60 elements. In Table 10, the results are summarized, and the solutions by the DQM are in good agreement with those by other numerical methods. For a thick beam, the shear deformable theory accounting the rotary inertia and shear effects gives a better approximation to the actual beam behavior. Therefore, the shear deformable theory for linearly varying cross sectional curved beams should be considered the next research.

Table 1. Fundamental frequency parameter, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$, for in-plane extensional vibration of asymmetric curved beams with simply-simply supported ends and $\eta=0.1$

θ_0 (degree)	$\lambda=(mr^4\omega^2/EI_0)^{1/2}$		
	S/R		
	30	100	500
30	143.0	142.8	142.8
60	34.80	34.65	34.63
90	14.81	14.69	14.66
120	7.861 θ_0	7.966	7.757
150	4.680	4.611	4.604
180	2.982	2.933	2.929

Table 2. Fundamental frequency parameter, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$, for in-plane extensional vibration of asymmetric curved beams with simply-simply supported ends and $\eta=0.4$

θ_0 (degree)	$\lambda=(mr^4\omega^2/EI_0)^{1/2}$		
	S/R		
	30	100	500
30	147.0	146.8	146.8
60	35.76	35.58	35.57
90	15.20	15.05	14.87
120	8.051	7.940	7.918
150	4.781	4.700	4.686
180	3.037	2.980	2.970

Table 3. Fundamental frequency parameter, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$, for in-plane extensional vibration of asymmetric curved beams with fixed-fixed ends and $\eta=0.1$

θ_0 (degree)	$\lambda=(mr^4\omega^2/EI_0)^{1/2}$		
	S/R		
	30	100	500
30	223.2	222.8	222.7
60	54.80	54.42	54.37
90	23.63	23.30	23.24
120	12.75	12.47	12.44
150	7.744	7.524	7.499
180	5.052	4.883	4.865

Table 4. Fundamental frequency parameter, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$, for in-plane extensional vibration of asymmetric curved beams with fixed-fixed ends and $\eta=0.4$

θ_0 (degree)	$\lambda=(mr^4\omega^2/EI_0)^{1/2}$		
	S/R		
	30	100	500
30	218.8	218.2	217.9
60	53.66	53.23	53.17
90	23.11	22.76	22.56
120	12.45	12.17	12.13
150	7.550	7.333	7.305
180	4.915	4.751	4.731

Table 5. Fundamental frequency parameter, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$, for in-plane extensional vibration of asymmetric curved beams with fixed-simply supported ends and $\eta=0.1$

θ_0 (degree)	$\lambda=(mr^4\omega^2/EI_0)^{1/2}$		
	S/R		
	30	100	500
30	179.7	179.2	176.8
60	43.94	43.60	43.17
90	18.82	18.56	18.45
120	10.07	9.879	9.846
150	6.055	5.913	5.897
180	3.905	3.802	3.784

Table 6. Fundamental frequency parameter, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$, for in-plane extensional vibration of asymmetric curved beams with fixed-simply supported ends and η= 0.4

θ_0 (degree)	$\lambda=(mr^4\omega^2/EI_0)^{1/2}$		
	S/R		
	30	100	500
30	176.8	176.5	176.5
60	43.17	42.89	43.12
90	18.45	18.22	16.01
120	9.846	9.660	9.497
150	5.897	5.757	5.681
180	3.784	3.683	3.637

Table 7. Fundamental frequency parameter, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$, for in-plane extensional vibration of asymmetric curved beams with simply supported-fixed ends and η= 0.1

θ_0 (degree)	$\lambda=(mr^4\omega^2/EI_0)^{1/2}$		
	S/R		
	30	100	500
30	181.9	181.2	180.7
60	44.47	44.08	44.01
90	19.05	18.78	18.89
120	10.20	10.00	9.975
150	6.142	5.998	5.980
180	3.967	3.864	3.852

Table 8. Fundamental frequency parameter, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$, for in-plane extensional vibration of asymmetric curved beams with simply supported-fixed ends and η= 0.4

θ_0 (degree)	$\lambda=(mr^4\omega^2/EI_0)^{1/2}$		
	S/R		
	30	100	500
30	184.9	184.0	183.2
60	45.19	44.72	44.60
90	19.36	19.06	19.60
120	10.36	10.15	10.14
150	6.237	6.087	6.073
180	4.029	3.922	3.913

Table 9. Fundamental frequency parameter, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$, for in-plane extensional vibration of uniform and non-uniform curved beams with fixed-fixed ends

θ_0 (degree)	$\lambda=(mr^4\omega^2/EI_0)^{1/2}$			
	η= 0.0 (uniform)		η= 0.4 (non-uniform)	
	S/R=30	S/R=500	S/R=30	S/R=500
30	223.6	223.1	218.8	217.9
60	54.88	54.45	53.66	53.17
90	23.67	23.29	23.11	22.56
120	12.77	12.46	12.45	12.13
150	7.758	7.512	7.550	7.305
180	5.061	4.874	4.915	4.713

Table 10. Fundamental frequency parameter, $\lambda=(mr^4\omega^2/EI_0)^{1/2}$, for in-plane vibrations of curved beams with fixed-fixed ends

θ_0 (degree)	$\lambda=(mr^4\omega^2/EI_0)^{1/2}$			
	η= 0.4			
	Galerkin	C.D.M.	SAP IV	DQM
10	1624.0	1624.6		1629.5
20	403.37	404.41	404.52	405.54
30	178.04	178.46		178.90
40	99.161	99.391	99.469	99.586
50	62.807	62.811		62.896
60	42.841	42.958	42.950	42.985

6. Conclusions

The in-plane extensional vibration of the asymmetric curved beams with linearly varying cross-section is analyzed by the differential quadrature method (DQM) neglecting the transverse shearing deformation. The frequency parameters are calculated for the beams with diverse parameter ratio, opening angles, and boundary conditions. The results are compared with other method for cases in which one is available. The present approach gives excellent results requiring only a limited number of grid points: only thirteen points were used for the solutions. New results are given for four sets of boundary conditions not considered by previous researchers for the in-plane extensional vibration of the asymmetric curved beams: fixed-fixed ends, simply-simply supported ends, simply supported-fixed ends, and fixed-simply supported ends.

The present method shows the followings:

1) The results by the DQM give the good precision compared with the other method in which one is available.
2) Only thirteen discrete points are used for the analysis.
3) It takes less than 1.0 second to compile the FORTRAN program with IMSL subroutine
4) The differential equations for the in-plane extensional vibration of the asymmetric curved beams with linearly varying cross-section are presented. The new results with various opening angles, boundary conditions, cross section ratios, and slenderness ratios are also shown. Those results can be also used in the comparisons with other numerical solutions or with other experimental test data by others.
5) For a thick beam, the shear deformable theory gives a better approximation to the actual beam behavior. Therefore, the shear deformable theory for linearly varying cross sectional curved beams should be considered the next research.

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〈Areas studied〉

Structural and Numerical Analysis, Buckling, Vibration