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Sliding Window Filtering for Ground Moving Targets with Cross-Correlated Sensor Noises

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Abstract

This paper reports a sliding window filtering approach for ground moving targets with cross-correlated sensor noise and uncertainty. In addition, the effect of uncertain parameters during a tracking error on the model performance is considered. A distributed fusion sliding window filter is also proposed. The distributed fusion filtering algorithm represents the optimal linear combination of local filters under the minimum mean-square error criterion. The derivation of the error cross-covariances between the local sliding window filters is the key to the proposed method. Simulation results of the motion of the ground moving target a demonstrate high accuracy and computational efficiency of the distributed fusion sliding window filter.

Keywords: Distributed fusion, Fusion formula, Kalman filter, Multi-sensor, Sliding window

1. INTRODUCTION

An increasing number of tracking systems for ground moving targets are being developed and deployed in real-world applications. Tracking and motion generation have been the predominant subjects in this regard [1-5]. Recent attention has been given to active sensing, which incorporates tracking and motion planning solutions in the presence of uncertainties. Ground moving targets often operate in unknown and inhospitable environments. To survive, such targets must be able to constantly monitor and appropriately react to variations and uncertainties in their environment. Model uncertainty appears owing to unmodeled dynamics and inaccurate parameters of the robot system. However, there are no mathematical models available that can perfectly represent an actual physical system. Thus, any discrepancies between the physical system and a mathematical model generates model uncertainty. The errors between the system parameters used in the mathematical model and those presented in the physical system also contribute to model uncertainties because it is almost impossible to obtain all system parameters correctly.

In practical applications, cross-correlations can occur among sensor noises. This is particularly true in practical situations in which a dynamic process is observed in a common noisy environment, such as when a target is taking an electronic countermeasure, e.g., noise jamming, or when the sensor noises are coupled owing to their dependence on the target state [6].

To obtain a more accurate and robust estimate of the state of a system under potential uncertainty, various types of techniques have been introduced and discussed. Among them, a sliding window estimation is one popular and successful strategy that is robust against temporal uncertainty and has been rigorously investigated [7-10]. Local sliding window Kalman filters (LSWKFs), which are fused, utilize finite measurements over the most recent time interval [7-9], [11]. It has been a general rule that an LSWKF is often more robust against dynamic model uncertainties and numerical errors than a standard local Kalman filter, which utilizes all measurements. Based on LSWKFs [7], [9] and the optimal fusion formula with matrix weights [10-13], we propose a decentralized sliding window fusion filtering for ground moving targets with cross-correlated sensor noise and uncertainties, which achieves a better accuracy than all LSWKFs and has a reduced computational burden compared to a centralized fusion sliding window filter.

The remainder of this paper is organized as follows. The problem is set up in Section II. The centralized fusion sliding window filter is described in Section III. In Section IV, we present the main results regarding the decentralized fusion sliding window

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filter for a multisensory environment. Herein, the key equations for cross-covariances between LSWKFs are derived. In Section V, an example application of the tracking error model using three sensors is described to illustrate the proposed decentralized filter. Finally, in Section VI, some concluding remarks are given.

2. PROBLEM STATEMENT

Considering a continuous-time linear dynamic system with additive white Gaussian noise,

$$\dot{x}_t = F_t x_t + G_t \xi_t, \quad t \ge t_0, \quad x_0 = x_{t_0}, \tag{1}$$

$$y_t^{(i)} = H_t^{(i)} x_t + w_t^{(i)}, \quad i = 1, \ldots, N, \tag{2}$$

where $x_t \in {}^{n}$ is the state; $y_t^{(i)} \in {}^{m_i}$ is the local sensor measurement; $\xi_t \in {}^{r}$ and $w_t^{(i)} \in {}^{m_i}$ are white Gaussian noises with zero mean and intensity matrices Q_t and $R_t^{(i)}$, respectively; and F_t, G_t, $H_t^{(i)}$ are matrices with compatible dimensions. Superscript (i) denotes the ith sensor, and N is the number of sensors.

Assuming the following for the initial state, $x_0 \quad (m_0; P_0)$, $m_0 = E(x_0)$, and $P_0 = \text{cov}\{x_0, x_0\}$, the system noise ξ_t is uncorrelated with the sensor noises $w_t^{(i)}, \ldots, w_t^{(N)}$, whereas all sensor noises $w_t^{(i)} \in {}^{m_i}$, $i = 1, \ldots, N$ are mutually correlated with the intensity matrices $R_t^{(ij)}$, $i, j = 1, \ldots, N$, $i \ne j$.

Our aim is to find the decentralized weighted fusion estimate of the state x_t based on the overall horizon cross-correlated sensor measurements.

$$y_{t-\Delta}^{t} = \left\{ y_s^{(1)}, \ldots, y_s^{(N)}, \ t - \Delta \le s \le t \right\}. \tag{3}$$

3. CENTRALIZED FUSION SLIDING WINDOW FILTER

Let us consider the original dynamic system (1) and rewrite the measurement model (2) in an equivalent form. We therefore obtain the following:

$$\begin{aligned} &\dot{x}_t = F_t x_t + G_t \xi_t, \ t \ge t_0, \ x_0 = x_{t_0}, \\ &Y_t = H_t x_t + w_t, \end{aligned} \tag{4}$$

where

$$Y_t = \begin{bmatrix} y_t^{(1)} \\ \vdots \\ y_t^{(N)} \end{bmatrix}, \ H_t = \begin{bmatrix} H_t^{(1)} \\ \vdots \\ H_t^{(N)} \end{bmatrix}, \ w_t = \begin{bmatrix} w_t^{(1)} \\ \vdots \\ w_t^{(N)} \end{bmatrix}. \tag{5}$$

The optimal *centralized fusion sliding window filter* (CFSWF) $\hat{x}_t^{opt}$ of state x_t based on the overall sliding window measurements (3) is described through the following differential equations [7]-[9]:

$$\begin{aligned} &\dot{\hat{x}}_s^{opt} = F_s x_s^{opt} + K_s^{opt}\left[Y_s - H_s \hat{x}_s^{opt} \right], \\ &\dot{P}_s^{opt} = F_s P_s^{opt} + P_s^{opt} F_s^T - P_s^{opt} H_s^T R_s^{-1} H_s P_s^{opt} + \tilde{Q}_s, \\ &K_s^{opt} = P_s^{opt} H_s^T R_s^{-1}, \ \ \tilde{Q}_s = G_s Q_s G_s^T, \\ &R_s = \begin{bmatrix} R_s^{(11)} & \cdots & R_s^{(1N)} \\ \vdots & \ddots & \vdots \\ R_s^{(N1)} & \cdots & R_s^{(NN)} \end{bmatrix}, \ R_s^{(ii)} \equiv R_s^{(i)}, \ i = 1, \ldots, N; \ t - \Delta \le s \le t, \end{aligned} \tag{6}$$

where the horizon initial conditions at time instant $s = t - \Delta$ represent the unconditional mean

$$\hat{x}_{t-\Delta}^{opt} \overset{def}{=} m_{t-\Delta} = E\left(x_{t-\Delta}\right)$$

and covariance

$$P_{t-\Delta}^{opt} \overset{def}{=} P_{t-\Delta} = E\left[\left(x_{t-\Delta} - m_{t-\Delta}\right)\left(x_{t-\Delta} - m_{t-\Delta}\right)^T \right]$$

of the horizon state $x_{t-\Delta}$ satisfying the Lyapunov equations

$$\begin{aligned} &\dot{m}_\tau = F_\tau m_\tau, \quad t_0 \le \tau \le t - \Delta, \quad m_{t_0} = m_0 = E\left(x_0\right), \\ &\dot{P}_\tau = F_\tau P_\tau + P_\tau F_\tau^T + \tilde{Q}_\tau, \quad P_{t_0} = P_0 = \text{cov}\left\{x_0, x_0\right\}, \\ &t_0 \le \tau \le t - \Delta. \end{aligned} \tag{7}$$

Let us now summarize the algorithmic procedure for the CFSWF. First, the horizon initial conditions $\hat{x}_{t-\Delta}^{opt}$ and $P_{t-\Delta}^{opt}$ are determined using the Lyapunov equations in (7). Then, the CFSWF equations in (6) are solved based on the horizon interval $s \in \left[t - \Delta, t\right]$ using the current horizon measurements (3).

The CFSWF represents a joint estimator. To compute the state estimate $\hat{x}_t^{opt}$, the implementation of the CFSWF requires all horizon sensor measurements (3) jointly at each time instant t. Therefore, under several limitations, such as the high computational cost and low communication resources, the CFSWF cannot produce well-timed results, particularly for a large number of sensors. Thus, a decentralized sliding window filter is preferable because there is no need to estimate the state using the *overall* sensor measurements in (3) simultaneously.

4. DISTRIBUTED FUSION SLIDING WINDOW FILTER

Here, we show that the fusion formula [10], [12], [13] can serve as the basis for the design of a decentralized fusion filter. A new suboptimal *decentralized fusion sliding window filter* (DFSWF) is described as follows: First, the local sensor measurements $y_t^{(1)},\dots,y_t^{(N)}$ are processed separately using the optimal LSWKF [7]-[9], and second, the obtained local estimates (filters) are fused in an optimal linear combination.

Let us denote the local sliding window of a Kalman estimate of state x_t based on the individual sensor $y_t^{(i)}$ using $\hat{x}_t^{(i)}$. To find $\hat{x}_t^{(i)}$, we can apply the LSWKF to system (1) with sensor $y_t^{(i)}$ [7-9]. We thus obtain the following differential equations:

$$\begin{aligned}
&\dot{\hat{x}}_s^{(i)} = F_s\hat{x}_s^{(i)} + K_s^{(i)}\left[y_s^{(i)} - H_s^{(i)}\hat{x}_s^{(i)}\right],\\
&\dot{P}_s^{(ii)} = F_sP_s^{(ii)} + P_s^{(ii)}F_s^T - P_s^{(ii)}H_s^{(i)^T}R_s^{(i)^{-1}}H_s^{(i)}P_s^{(ii)} + \tilde{Q}_s,\\
&K_s^{(i)} = P_s^{(ii)}H_s^{(i)^T}R_s^{(i)^{-1}},\\
&P_s^{(ii)} = \operatorname{cov}\left\{e_s^{(i)}, e_s^{(i)}\right\},\quad e_s^{(i)} = x_s - \hat{x}_s^{(i)},\ t-\Delta \le s \le t,
\end{aligned} \tag{8}$$

with the horizon initial conditions $\hat{x}_{t-\Delta}^{(i)} = m_{t-\Delta}$, $P_{t-\Delta}^{(ii)} = P_{t-\Delta}$ determined using (7).

Thus, the decentralized fusion sliding window of the suboptimal estimate $\hat{x}_t^{sub}$ based on the overall sensor measurements in (3) is constructed using the fusion formula, i.e.,

$$\hat{x}_t^{sub} = \sum_{i=1}^{N} c_t^{(i)}\hat{x}_t^{(i)},\qquad \sum_{i=1}^{N} c_t^{(i)} = I_n, \tag{9}$$

where I_n is the identity matrix, and $c_t^{(1)},\dots,c_t^{(N)}$ are the time-varying weighted matrices determined based on the mean square criterion.

Theorem 1 [10], [12] *(a) The optimal weights* $c_t^{(1)},\dots,c_t^{(N)}$ atisfy the linear algebraic equations

$$\sum_{i=1}^{N} c_t^{(i)}\left[P_t^{(ij)} - P_t^{(iN)}\right] = 0,\ \sum_{i=1}^{N} c_t^{(i)} = I_n, \tag{10}$$

and can be explicitly written in the following form:

$$c_t^{(i)} = \sum_{j=1}^{N} W_t^{(ij)}\left(\sum_{l,h=1}^{N} W_t^{(lh)}\right)^{-1},\quad i = 1,\dots N, \tag{11}$$

where $W_t^{(ij)}$ *is the* $(ij)th$ $(n\times n)$ *submatrix of the* $(nN\times nN)$ *block matrix* P_t^{-1}, $P_t = \left[P_t^{(ij)}\right]_{i,j=1}^{N}$.

(b) The fusion error covariance $P_t^{sub} \overset{def}{=} \operatorname{cov}\left\{e_t^{sub}, e_t^{sub}\right\}$, $e_t^{sub} = x_t - \hat{x}_t^{sub}$, *is given by*

$$P_t^{sub} = \sum_{i,j=1}^{N} c_t^{(i)}P_t^{(ij)}c_t^{(j)^T}. \tag{12}$$

Equations (10)–(12) defining the unknown weights $c_t^{(i)}$ and fusion error covariance P_t^{sub} depend on the local covariances $P_t^{(ii)}$, which are determined by (8) and the local cross-covariances,

$$P_t^{(ij)} = \operatorname{cov}\left\{e_t^{(i)}, e_t^{(j)}\right\},\quad i,j = 1,\dots,N,\quad i\ne j, \tag{13}$$

given in Theorem 2.

Theorem 2. *The local cross-covariances (13) satisfy the following differential equations:*

$$\begin{aligned}
&\dot{P}_s^{(ij)} = \tilde{F}_s^{(i)}P_s^{(ij)} + P_s^{(ij)}\tilde{F}_s^{(j)^T} + \tilde{Q}_s,\quad t-\Delta\le s\le t,\\
&\tilde{F}_s^{(i)} = F_s - K_s^{(i)}H_s^{(i)},\quad i,j = 1,\dots,N,\quad i\ne j
\end{aligned} \tag{14}$$

with the horizon initial conditions $P_{t-\Delta}^{(ij)} = P_{t-\Delta}$ *and gains* $K_s^{(i)}$ *determined using (7) and (8), respectively.*

The derivation of (14) is given in Appendix. Thus, (8)–(14) completely define the DFSWF.

Remark 1. The LSWKFs $\hat{x}_t^{(i)}$, $i = 1,\dots N$ are separated for different types of sensors, i.e., each local estimate $\hat{x}_t^{(i)}$ is found independently of the other estimates. Therefore, the LSWKFs can be implemented in parallel for different sensors using (2).

Remark 2. We should note that the local error covariances $P_t^{(ij)}$, $i,j = 1,\dots N$ and weights $c_t^{(i)}$ may be pre-computed because they do not depend on the sensor measurements $y_{t-\Delta}^{t}$, but only on the noise statistics Q_t and $R_t^{(i)}$, and the system matrices F_t, G_t, $H_t^{(i)}$, which are parts of the system model described in (1) and (2). Thus, once the measurement schedule has been settled, the real-time implementation of the DFSWF requires only the computation of the LSWKFs $\hat{x}_t^{(i)}$, $i = 1,\dots N$ and the final suboptimal fusion estimate $\hat{x}_t^{sub}$.

5. EXAMPLE

5.1 Tracking Error Model

There are two basic control approaches to solving the motion task of the ground moving target: stabilization to a fixed posture and tracking of the reference trajectory. For nonholonomic ground moving targets, the trajectory-tracking problem is easier to solve and more natural than posture stabilization. The state of the tracking error can be expressed in the frame of the real ground moving target [14].

The reference target [14] is an imaginary ground moving target that ideally follows the reference path. In contrast, the real ground moving target (when compared to the reference target) has a certain error when following the reference path. This linearized tracking error model is described as follows:

$$\dot{x}_t = \begin{bmatrix} 0 & \omega_r & 0 \\ -\omega_r + 0.5\delta_t & 0 & v_r + \delta_t \\ 0 & 0 & 0 \end{bmatrix} x_t + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \xi_t, \quad t \geq 0, \tag{15}$$

where $x_t = \begin{bmatrix} x_{1,t} & x_{2,t} & x_{3,t} \end{bmatrix}^T$, which $x_{1,t}$ is the x-position error, $x_{2,t}$ is the y-position error, $x_{3,t}$ is the ground moving target heading angle error, v_r is the tangential velocity, and ω_r is the angular velocity. The linearization obtained was shown to be controllable [15], [16] as long as the trajectory does not come to a stop. Thus, the reference inputs v_r and ω_r are constant (linear and circular paths), δ_t is an uncertain model parameter, and ξ_t is a white Gaussian noise. Here, $v_r = 0.1643\ m/s$, $\omega_r = 0.3788\ rad/s$, the system noise intensity Q_t is 0.02^2, and $\delta_t = 1$ for the interval $2 \leq t \leq 6$.

The horizon length of the LSWKFs is taken as $\Delta_1 = 0.4$, $\Delta_2 = 0.5$, and $\Delta_3 = 0.6$.

The second coordinate related to the y-position error between the real and reference ground moving targets is observable through the measurement model containing three identical local sensors with a different level of accuracy given by

$$\begin{aligned} y_t^{(1)} &= H^{(1)} x_t + w_t^{(1)}, \quad H^{(1)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \\ y_t^{(2)} &= H^{(2)} x_t + w_t^{(2)}, \quad H^{(2)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \\ y_t^{(3)} &= H^{(3)} x_t + w_t^{(3)}, \quad H^{(3)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \end{aligned} \tag{16}$$

where $w_t^{(1)}$, $w_t^{(2)}$, and $w_t^{(3)}$ are cross-correlated white Gaussian noises with zero-mean and stationary intensities $R_t^{(ij)}$, $i, j = 1, 2, 3$.

The parameters are subjected to

$$\begin{aligned} &R_t^{(i)} = 0.01^2, \quad i = 1,2,3 \quad R_t^{(12)} = R_t^{(21)} = 0.02^2, \\ &R_t^{(13)} = R_t^{(31)} = 0.018^2, \quad R_t^{(23)} = R_t^{(32)} = 0.015^2. \end{aligned}$$

The two fusion sliding window filters, namely, CFSWF $\left(\hat{x}_t^{opt}, P_t^{opt}\right)$ and DFSWF $\left(\hat{x}_t^{sub}, P_t^{sub}\right)$, and two fusion non-sliding window filters, i.e., a centralized Kalman filter (CKF) and a decentralized Kalman filter (DKF), for the system model with uncertainty δ_t are compared. Here, the model is considered to show robustness of the sliding window filters against uncertainty [11].

5.2 Results and Analysis

The behavior of the CFSWF and DFSWF estimates $\left(\hat{x}_t^{opt}, \hat{x}_t^{sub}\right)$ and their fusion error covariances $\left(P_t^{opt}, P_t^{sub}\right)$ were studied. We focused on the mean square errors (MSEs)

$$P_{22,t}^{opt} = E\left[\left(x_{2,t} - \hat{x}_{2,t}^{opt}\right)^2\right], \; P_{22,t}^{sub} = E\left[\left(x_{2,t} - \hat{x}_{2,t}^{sub}\right)^2\right] \tag{17}$$

for the second coordinate $x_{2,t}$, which is called a y-position error, because the uncertainty δ_t appears in (15) only at this coordinate. The simulation results of other coordinates $x_{1,t}$ and $x_{3,t}$ are similar. The point of interest is the comparison of the optimal and suboptimal sliding window estimates $\left(\hat{x}_{2,t}^{opt}, \hat{x}_{2,t}^{sub}\right)$ of the y-position error, and corresponding MSEs (17).

The MSEs are shown in Figures 1–3. Fig. 1 shows two pairs of estimates of the y-position error using the sliding window and non-sliding window fusion filters under the uncertainty $\delta_t = 1$. The first pair includes the CFSWF and DFSWF, and the second contains the CKF and DKF [10].

In Fig. 2 we can see that, at around the uncertainty interval $2 \leq t \leq 6$, the sliding window filters CFSWF and DFSWF demonstrate a good performance compared to the non-sliding window filters CKF and DKF. This is confirmed based on the general agreement with the robustness of the sliding window strategy. In addition, in each pair of filters, the centralized versions

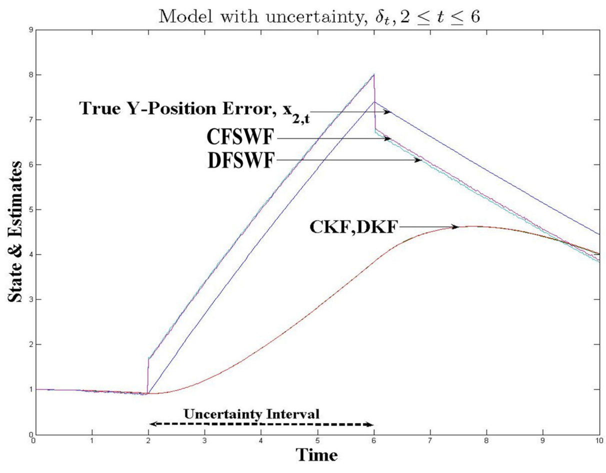

Fig. 1. The y-position error $x_{2,t}$ and its estimates using CFSWF, DFSWF, CKF, and DKF.

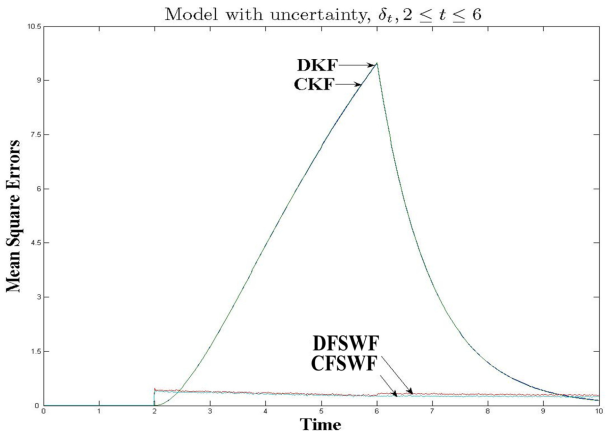

Fig. 2. MSE comparison for $x_{2,t}$ with uncertainty $\delta_t = 1$

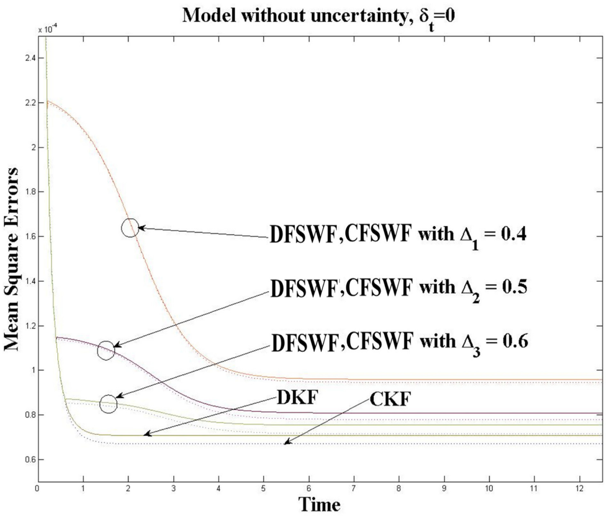

Fig. 3. MSE comparison for $x_{2,t}$ without uncertainty $\delta_t = 0$

provide more accurate estimates than the decentralized version. However, these differences are negligible.

The estimation accuracy of the filters can be more clearly compared through the MSEs in Fig. 2. Here, the actual MSEs are calculated using the Monte-Carlo method with 1,000 runs. In Fig. 2, we can see that within the uncertainty interval $2 \le t \le 6$, the MSEs of the non-sliding window filters CKF and DKF are remarkably large. However, the differences between all optimal and suboptimal filters are negligible outside of the uncertainty interval. This means that, for our example, the application of the DFSWF can produce good results under the real-time processing requirements.

Fig. 3 shows the filter performances of the CFSWF and DFSWF as a function of the horizon length Δ. Because uncertainty does not appear, the actual MSEs can be calculated directly through (6)–(8) and (12)–(14) without Monte-Carlo simulations, as shown in Fig. 3. The results in Fig. 4 demonstrate that the sliding window filters (CFSWF and DFSWF) reach the non-sliding window Kalman filters (CKF and DKF) with increasing horizon length Δ.

6. CONCLUSION

In this paper, a new decentralized sliding window filter for ground moving targets in a cross-correlated sensor environment was proposed. The proposed filter represents the weighted sum of the LSWKFs. Each LSWKF is fused based on the minimum MSE criterion. The matrix weights depend on the cross-covariances between the LSWKFs. The key differential equations for these filters are derived.

Furthermore, the DFSWF has a parallel structure and allows parallel processing of the observations making it reliable because the remaining faultless sensors can continue with the fusion estimation if some sensors incur a fault. Taking into account the tracking error model of the ground moving targets, the proposed DFSWF provides good tracking results under uncertainty. A simulation analysis and comparison with the optimal CFSWF verifies the effectiveness of the proposed DFSWF.

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APPENDIX

Derivation of equations (14)

According to (8), the local errors $e_s^{(i)}$ and $e_s^{(j)}$, $i \neq j$ satisfy the following differential equations on the sliding window interval $s \in \left[t-\Delta;\ t\right]$:

$$\dot{e}_s^{(i)} = \tilde{F}_s^{(i)} e_s^{(i)} + B_s^{(i)} \eta_s, \qquad B_s^{(i)} = \left[-K_s^{(i)} \quad 0 \quad G_s\right],$$

$$\dot{e}_s^{(j)} = \tilde{F}_s^{(j)} e_s^{(j)} + B_s^{(j)} \eta_s, \quad B_s^{(j)} = \left[0 \quad -K_s^{(j)} \quad G_s\right],$$

$$\eta_s = \left[w_s^{(i)^T} \quad w_s^{(j)^T} \quad \xi_s^T\right]^T \in {}^{m_i+m_j+r}, \quad t-\Delta \le s \le t,$$

where η_s is the composite white noise with intensity matrix

$$Q_s^{(\eta)} = \begin{bmatrix} R_s^{(ii)} & R_s^{(ij)} & 0 \\ R_s^{(ji)} & R_s^{(jj)} & 0 \\ 0 & 0 & Q_s \end{bmatrix}, \quad R_s^{(ii)} \equiv R_s^{(i)}, \quad R_s^{(ij)} = R_s^{(ji)^T}.$$

The cross-covariance $P_s^{(ij)}$ then represents the expectation of the product $E\left[e_s^{(i)} e_s^{(j)^T}\right]$, satisfying the following differential equation [17]:

$$\dot{P}_s^{(ij)} = \frac{d}{ds} E\left[e_s^{(i)} e_s^{(j)^T}\right]$$

$$= \tilde{F}_s^{(i)} P_s^{(ij)} + P_s^{(ij)} \tilde{F}_s^{(j)^T} + B_s^{(i)} Q_s^{(\eta)} B_s^{(j)^T}.$$

In addition, after simple manipulations with the item

$$B_s^{(i)} Q_s^{(\eta)} B_s^{(j)^T} = K_s^{(i)} R_s^{(ij)} K_s^{(j)^T} + G_s Q_s G_s^T,$$

(14) is obtained.

This completes the proof of Theorem 2.