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# On Multipliers of Lattice Implication Algebras for Hierarchical Convergence Models

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# 계층적 융합모델을 위한 격자함의 대수의 멀티플라이어

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**Abstract** Role-based access or attribute-based access control in cloud environment or big data environment need requires a suitable mathematical structure to represent a hierarchical model. This paper define the notion of multipliers and simple multipliers of lattice implication algebras that can implement a hierarchical model of role-based or attribute-based access control, and prove every multiplier is simple multiplier. Also we research the relationship between multipliers and homomorphisms of a lattice implication algebra $L$, and prove that the lattice $[0,u]$ is isomorphic to a lattice $[u',1]$ for each $u \in L$ and that $L$ is isomorphic to $[u,1] \times [u',1]$ as lattice implication algebras for each $u \in L$ satisfying $u \vee u' = 1$.

**Key Words :** Lattice implication algebras, Multipliers, Simple multipliers, Role-based access control, Attribute-based access control

**요 약** 클라우드 환경이나 빅데이터 환경에서의 역할기반 또는 속성기반의 접근제어에는 계층적 모델을 표현하는 적당한 수학적 구조가 필요하다. 본 논문에서는 역할기반 또는 속성기반의 접근제어의 계층적 모델을 구현할 수 있는 격자함의 대수에서 멀티플라이어와 단순 멀티플라이어의 개념을 정의하고, 모든 멀티플라이어는 단순 멀티플라이어임을 증명한다. 또한 격자함의대수 $L$의 멀티플라이어와 준동형사상의 관계를 조사하고, 각각의 $u \in L$에 대하여 격자 $[0,u]$와 격자 $[u',1]$이 동치임과 $u \vee u' = 1$인 $u \in L$에 대하여 $L$과 $[u,1] \times [u',1]$이 격자함의대수로써 동치임을 보인다.

**주제어 :** 격자함의대수, 멀티플라이어, 단순멀티플라이어, 역할기반접근제어, 속성기반접근제어

## 1. Introduction

Lattice implication algebra was introduced in [1] as a bounded lattice equipped with a logical implication "→" and an involution "′". This algebra is one of many-valued logical systems with a conjunction and a disjunction and a logical implication, which has many interesting properties as algebraic structure and has been studied in many literatures on the algebraic viewpoint[2-6]. The many-valued lattice logic is closely related to computer science dealing with decision making, inference system and artificial intelligence, etc. Lattice implication algebra is a generalization of fuzzy sets with Łukasiewicz fuzzy implication[7]. So it can be used to simplify the logical operations of fuzzy sets, and for the

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antitone-involution on lattice implication algebra has the similar characteristics with polar maps of formal concept which is applied to Machine Learning[8], this algebra can be studied to analysis the formal concept or the fuzzy formal concept. Also, as a lattice implication algebra is a partially ordered set, this algebra has a good mathematical structure to display hierarchical model such as role-based access control and attribute-based access control in cloud environment or big data environment[9, 10].

The notion of lattice implication algebras is equivalence with that of quasi lattice implication algebras[11] which is an algebra of type (2,1,0) with a binary operation, implication, and a unary operation, involution and the greatest element.

After a partial multiplier on a commutative semigroup had been introduced in [12], the notion of multipliers was studied and applied to many other algebraic structures[13-15]. The definitions and properties of derivations, which are similar operators to multipliers, of lattice implication algebras have been researched in [16-19], and the derivation defined in [17] becomes a multiplier.

In this paper we define the notion of multipliers and simple multipliers of lattice implication algebras, and prove every multiplier is simple multiplier. Also we research the relationship between multipliers and homomorphisms, and prove that for each element $u$ in a lattice implication algebra $L$, the interval $[0,u]$ is isomorphic to $[u',1]$ as lattices, and that for each $u \in L$ satisfying $u \vee u' = 1$, $L$ is isomorphic to $[u,1] \times [u',1]$ as lattice implication algebras.

## 2. Lattice Implication ALgebras

A lattice implication algebra is an algebraic system $(L, \cdot, ', 1)$ with a binary operation "$\cdot$", an involution "$'$" and an element 1 satisfying the following axioms: for all $x, y, z \in L$,

(L1) $x(yz) = y(xz)$,

(L2] $xx = 1$,

(L3) $(xy)y = (yx)x$,

(L4) $xy = 1$ and $yx = 1$ imply $x = y$,

(L5) $xy = y'x'$.

**Lemma 2.1.** ([4,11]) Let $L$ be a lattice implication algebra. Then $L$ satisfies the following: for any $x$, $y$, $z \in L$,

(1) $1x = x$,

(2) $x \le y$ if and only if $xy = 1$,

(3) $1' = 0$ and $0' = 1$,

(4) $x' = x0$,

(5) $y \le xy$,

(6) $x \le y$ implies $zx \le zy$ and $yz \le xz$,

(7) $x \le y$ implies $y' \le x'$,

(8) $((xy)y)y = xy$,

(9) $(xy)y = (yx)x = x \vee y$ and $x \wedge y = (x' \vee y')'$,

(10) $(x \vee y)' = x' \wedge y'$,

(11) $(x \wedge y)' = x' \vee y'$,

(12) $(x \vee y)z = (xz) \wedge (yz)$,

(13) $(x \wedge y)z = (xz) \vee (yz)$,

(14) $z(x \wedge y) = (zx) \wedge (zy)$.

Table 1. Cayley table of binary operation $\cdot$ on $L$

| $\cdot$ | 0 | $a$ | $b$ | $c$ | $d$ | 1 |
|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a$ | $d$ | 1 | $d$ | 1 | 1 | 1 |
| $b$ | $c$ | $c$ | 1 | $c$ | 1 | 1 |
| $c$ | $b$ | $d$ | $b$ | 1 | $d$ | 1 |
| $d$ | $a$ | $c$ | $d$ | $c$ | 1 | 1 |
| 1 | 0 | $a$ | $b$ | $c$ | $d$ | 1 |

**Example 2.2.** Let $L = \{0, a, b, c, d, 1\}$ be a set. If we define a binary operation $\cdot$ on $L$ by the Cayley table of Table 1 and define $x' = x0$ for every $x \in L$, then $(L, \cdot, ', 1)$ is a lattice implication algebra with the Hasse diagram of Fig. 1.

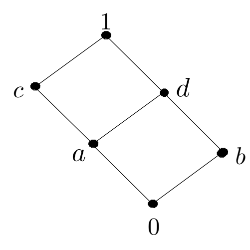


Fig. 1. Hasse diagram of $(A, \cdot)$

**Lemma 2.3.** Let $L$ be a lattice implication algebra.

Then for every $x, y, z \in L$,

$$x \wedge y \leq z \Rightarrow x \leq yz.$$

Proof. Let $x \wedge y \leq z$. Then we have

$$\begin{aligned} 1 &= (x \wedge y)z = (xz) \vee (yz) \\ &= ((xz)(yz))(yz) = (y((xz)z))(yz) \\ &= (y(x \vee z))(yz). \end{aligned}$$

That is $y(x \vee z) \leq yz$. Also, since $z \leq x \vee z$, $yz \leq y(x \vee z)$. This implies $yz = y(x \vee z)$. Hence we have

$$x(yz) = x(y(x \vee z)) = y(x(x \vee z)) = y1 = 1,$$

and $x \leq yz$. □

**Lemma 2.4.** Let $L$ be a lattice implication algebra. Then for every $x, y, z \in L$,

$$x(y \vee z) = (xy) \vee (xz).$$

Proof. Let $x, y, z \in L$. Then we have

$$\begin{aligned} x(y \vee z) &= (y \vee z)'x' = (y' \wedge z')x' \\ &= (y'x') \vee (z'x') = (xy) \vee (xz) \end{aligned}$$

by Lemma 2.1. □

**Example 2.5.** Let $L = [0,1]$ be the closed interval in real numbers $\mathbb{R}$ and the partial order $\leq$ is the usual order in $\mathbb{R}$. If we define a binary operation $\cdot$ and a unary operation $'$ on $L$ by

$$xy = 1 \wedge (1 - x + y) \text{ and } x' = 1 - x,$$

respectively, for every $x, y \in L$. Then $L$ is a lattice implication algebra.

## 3. Multipliers of Lattice Implication Algebras

A map $f : L \rightarrow M$ on lattice implication algebras $L$ and $M$ is called a lattice−homomorphism of $L$ to $M$ if

$f(x \vee y) = f(x) \vee f(y)$ and $f(x \wedge y) = f(x) \wedge f(y)$, and a homomorphism of $L$ to $M$ if

$$f(xy) = f(x)f(y)$$

for every $x, y \in L$.

**Lemma 3.1.** Let $L$ and $M$ be lattice implication algebras. If $f : L \rightarrow M$ is a homomorphism, then $f$ is a lattice−homomorphism.

Proof. Let $f$ be a homomorphism of $L$ to $M$ and $x$, $y$, $z$ $\in L$. Then

$$\begin{aligned} f(x \vee y) &= f((xy)y) = (f(x)f(y))f(y) \\ &= f(x) \vee f(y). \end{aligned}$$

This implies that

$$\begin{aligned} f(x \wedge y) &= f((x' \vee y')') = f((x' \vee y')0) \\ &= f(x' \vee y')f(0) = (f(x') \vee f(y'))f(0) \\ &= (f(x')f(0)) \wedge (f(y')f(0)) \\ &= f(x'0) \wedge f(y'0) = f(x'') \wedge f(y'') \\ &= f(x) \wedge f(y). \end{aligned}$$

Hence $f$ is a lattice−homomorphism. □

The converse of Lemma 3.1 is not true in general as the following example show.

**Example 3.2.** In Example 2.2, if we define a map $f : L \rightarrow L$ by $f(0) = f(a) = a$, $f(b) = f(d) = d$, $f(c) = c$, $f(1) = 1$, then $f$ is a lattice−homomorphism of $L$, but it is not homomorphism, because

$$f(ab) = f(d) = d \neq 1 = ad = f(a)f(b).$$

Let $Hom(L)$ and $LHom(L)$ denote the families of all homomorphisms and all lattice−homomorphisms of a lattice implication algebra $L$ respectively. Then we know that $Hom(L) \subsetneq LHom(L)$ from Lemma 3.1 and Example 3.2.

**Definition 3.3.** Let $L$ be a lattice implication algebra. A map $\rho : L \rightarrow L$ is called a multiplier of $L$ if

$$\rho(xy) = x\rho(y)$$

for every $x, y \in L$.

**Lemma 3.4.** Let $\rho$ be a multiplier of a lattice implication algebra $L$. Then the following properties are satisfied: for every $x, y, z \in L$,

(1) $x\rho(yz) = y\rho(xz)$,

(2) $x \leq \rho(x)$, in particular $\rho(1) = 1$,

(3) $x \leq \rho\rho(x)$,

(4) $x \le y$ implies $\rho(x) \le \rho(y)$,
(5) $x\rho(y) = y'\rho(x')$,
(6) $\rho(x) = x'\rho(0)$.

Proof. (1) Let $x,y,z \in L$. Then we have
$$x\rho(yz) = x(y\rho(z)) = y(x\rho(z)) = y\rho(xz).$$
(2) For any $x \in L$, we have
$$x \le \rho(0)'x = x'\rho(0) = \rho(x'0) = \rho(x'') = \rho(x).$$
(3) For any $x \in L$, by (2) of this lemma, we have
$$x\rho\rho(x) = \rho(x\rho(x)) = \rho(1) = 1.$$
(4) Let $x \le y$ in $S$. Then $xy = 1$. Since $\rho(x) \le (yx)\rho(x)$, we have
$$\rho(x)\rho(y) = \rho(x)\rho(1y) = \rho(x)\rho((xy)y) = \rho(x)\rho((yx)x) = \rho(x)((yx)\rho(x)) = 1.$$
(5) For any $x,y \in L$, we have
$$x\rho(y) = \rho(xy) = \rho(y'x') = y'\rho(x').$$
(6) For any $x \in L$, by (5) of this lemma, we have
$$\rho(x) = 1\rho(x) = x'\rho(1') = x'\rho(0).$$ □

**Lemma 3.5.** Every multiplier of a lattice implication algebra is a lattice-homomorphism.

Proof. Let $\rho$ be a multiplier of $L$ and $x,y \in L$. Then by Lemma 3.4(6), we have
$$\rho(x \vee y) = (x \vee y)'\rho(0) = (x' \wedge y')\rho(0) = x'\rho(0) \vee y'\rho(0) = \rho(x) \vee \rho(y),$$
and
$$\rho(x \wedge y) = (x \wedge y)'\rho(0) = (x' \vee y')\rho(0) = x'\rho(0) \wedge y'\rho(0) = \rho(x) \wedge \rho(y).$$ □

Every multiplier is a lattice-homomorphism, but the converse of Lemma 3.5 is not true in general. In fact the map $f$ in Example 3.2 is a lattice-homomorphism of $L$, but not multiplier because of $f(ab) = f(d) = d \ne 1 = ad = af(b)$.

**Example 3.6.** Let $L$ be a lattice implication algebra and $u \in L$. If we define a map $\rho_u : L \to L$ by
$$\rho_u(x) = ux$$
for every $x \in L$, then $\rho_u$ is a multiplier.

The multiplier $\rho_u$ defined in Example 3.6 is called a simple multiplier.

Let $M(L)$ and $SM(L)$ denote the families of all multipliers and all simple multipliers, respectively, of a lattice implication algebra $L$. Then it is clear that $SM(L) \subseteq M(L)$. And from Lemma 3.5 and Example 3.2 we know that $M(L) \subsetneq LHom(M)$.

**Theorem 3.7.** Every multiplier $\rho$ of a lattice implication algebra $L$ is simple with $\rho = \rho_{\rho(0)'}$.

Proof. Let $\rho : L \to L$ be a multiplier of $L$. Then we have
$$\rho(x) = \rho(x'') = \rho(x'0) = x'\rho(0) = \rho(0)'x'' = \rho(0)'x = \rho_{\rho(0)'}(x)$$
for every $x \in L$. Hence $\rho = \rho_{\rho(0)'}$. □

From Theorem 3.7, we know that $SM(L) = M(L)$ for any lattice implication algebra $L$.

**Lemma 3.8.** Let $L$ be a lattice implication algebra. Then for any $u \in L$, the interval
$$[u,1] := \{x \in L | u \le x \le 1\}$$
is a lattice implication algebra with an involution $\perp_u$ defined by $x^{\perp_u} = xu$ for every $x \in L$.

Proof. Let $\perp_u$ be a unary operation defined by $x^{\perp_u} = xu$ for every $x \in [u,1]$. Then for any $x,y \in [u,1]$, we have
$$x^{\perp_u \perp_u} = (xu)u = x \vee u = x$$
and
$$y^{\perp_u} x^{\perp_u} = (yu)(xu) = x((yu)u) = x(y \vee u) = xy.$$
This implies $\perp_u$ is an involution of $[u,1]$ satisfying the axiom (L5) in definition of lattice implication algebra. Other axioms (L1)-(L4) are satisfied trivially in $[u,1]$ since $[u,1] \subseteq L$. □

**Lemma 3.9.** Let $u$ be an element of a lattice implication algebra $L$. Then the restriction $\rho_u^*$ to $[0,u]$ of multiplier $\rho_u$ is a lattice-isomorphism from $[0,u]$ to $[u',1]$.

Proof. Suppose that $\rho_u^* : [0,u] \to L$ is the restriction to $[0,u]$ of a multiplier $\rho_u$. Then for any $x \in L$, $\rho_u(x) = ux = x'u' \in [u',1]$ and $\rho_u$ is a lattice-homomorphism by Lemma 3.5, hence $\rho_u^*$ is also a lattice-homomorphism.

Let $\rho_u^*(x) = \rho_u^*(y)$ for any $x,y \in [0,u]$. Then $ux = uy$ and $x'u' = y'u'$. Since $x \le u$ and $y \le u$, $u' \le x'$ and $u' \le y'$, and we have

$$x' = x' \vee u' = (x'u')u' = (y'u')u' = y' \vee u' = y'.$$

This implies $x = x'' = y'' = y$, and $\rho_u^*$ is injective.

Let $x \in [u',1]$. Since $u' \le xu'$ and $(xu')' \le u'' = u$, $(xu')' \in [0,u]$ and

$$\rho_u^*((xu')') = u(xu')' = (xu')u' = x \vee u' = x.$$

This implies $\rho_u^* : [0,u] \to [u',1]$ is surjective. Hence $\rho_u$ is a lattice-isomorphism from $[0,u]$ to $[u',1]$. □

In Example 2.2, the multiplier $\rho_a$ is not a homomorphism because of $\rho_a(db) = \rho_a(d) = ad = 1 \ne d = 1d = (ad)(ab) = \rho_a(d)\rho_a(b)$.

Also there is an example of homomorphism but not multipliers as the following example show. So we can know that multiplier and homomorphism are different notions to each other.

**Example 3.10.** In Example 2.2, if we define a map $f: L \to L$ by

$$f(0) = f(a) = f(c) = a,\ f(b) = f(d) = f(1) = 1,$$

then $f$ is a homomorphism of $L$ but not a multiplier because of $f(ba) = f(c) = a \ne c = ba = bf(a)$.

**Lemma 3.11.** Let $L$ be a lattice implication algebra and $x, y, z \in L$. Then the multipliers of $L$ satisfy the following properties:

(1) $Ker(\rho_x) := \{y \in L \mid \rho_x(y) = 1\} = [x,1]$,

(2) $\rho_x(yz) = y\rho_x(z)$,

(3) $\rho_x(y) = \rho_{y'}(x')$,

(4) $y \le \rho_x(y)$ and $x' \le \rho_x(y)$,

(5) $\rho_x(y)y = \rho_y(x)x$.

If $u \in L$ such that $u \vee u' = 1$, then the following are satisfied:

(6) for every $x \in [u,1]$ and $y \in [u',1]$, $\rho_x(y) = y$ and $\rho_y(x) = x$, in particular $\rho_u(y) = y$ and $\rho_{u'}(x) = x$,

(7) $\rho_u \rho_u = \rho_u$.

Proof. (1)-(5) are clear from the definition of simple multipliers.

(6) Let $x \in [u,1]$ and $y \in [u',1]$ with $u \vee u' = 1$. Since $1 = u \vee u' \le x \vee y$, $x \vee y = 1$. Hence we have

$$\rho_x(y) = xy = ((xy)y)y = (x \vee y)y = 1y = y.$$

Similarly, we can show $\rho_y(x) = x$.

(7) Let $x \in L$. Since $\rho_u(x) \in [u',1]$ by (4) of this lemma, $\rho_u(\rho_u(x)) = \rho_u(x)$ by (6) of this lemma. □

Let $L$ be a lattice implication algebra and $u \in L$ such that $u \vee u' = 1$. Then the multiplier $\rho_u$ is a closure operator of $L$ by (2) and (4) of Lemma 3.4 and (7) of Lemma 3.11.

**Theorem 3.12.** Let $u$ be an element of lattice implication algebra $L$ such that $u \vee u' = 1$. Then $\rho_u$ is a homomorphism of $L$ to the lattice implication algebra $[u',1]$.

Proof. Let $u \in L$ with $u \vee u' = 1$, and $x, y \in L$. Then $x \le \rho_u(x)$, and this implies

$$\rho_u(x)\rho_u(y) \le x\rho_u(y) = \rho_u(xy)$$

by Lemma 2.1(6). Conversely, we have

$$\rho_u(xy)\rho_u(y) = (x\rho_u(y))\rho_u(y) = x \vee \rho_u(y) \ge x \vee u'$$

by Lemma 2.1(9) and Lemma 3.11(4). This implies

$$\begin{aligned}\rho_u(xy)(\rho_u(x)\rho_u(y)) &= \rho_u(x)(\rho_u(xy)\rho_u(y)) \\ &\ge \rho_u(x)(x \vee u') = \rho_u(x)x \vee \rho_u(x)u' \\ &= \rho_x(u)u \vee \rho_u(x)u' \ge u \vee u' = 1\end{aligned}$$

by Lemma 2.4 and Lemma 2.1(5). This implies

$\rho_u(xy)(\rho_u(x)\rho_u(y))=1$ and $\rho_u(xy)\le\rho_u(x)\rho_u(y)$. Hence $\rho_u(xy)=\rho_u(x)\rho_u(y)$, and $\rho_u$ is a homomorphism from $L$ to $[u',1]$. □

Let $L_1$ and $L_2$ be lattice implication algebras. Then $L_1\times L_2$ is also a lattice implication algebra with a binary operation · and an involution ′ defined by $(x_1, y_1)(x_2, y_2)=(x_1x_2, y_1y_2)$ and $(x, y)'=(x', y')$ for every $(x, y), (x_1, y_1), (x_2, y_2)\in L_1\times L_2$.

**Theorem 3.13.** Let $u$ be an element of a lattice implication algebra $L$ such that $u\vee u'=1$. If $\phi_u : L\to[u,1]\times[u',1]$ is a map given by

$$\phi_u(x)=(\rho_{u'}(x), \rho_u(x))$$

for every $x\in L$, then $\phi_u$ is an isomorphism of $L$ to the lattice implication algebra $[u,1]\times[u',1]$.

Proof. Let $\phi_u : L\to[u,1]\times[u',1]$ be the map given by $\phi_u(x)=(\rho_{u'}(x), \rho_u(x))$ for every $x\in L$. Since $\rho_{u'} : L\to[u,1]$ and $\rho_u : L\to[u',1]$ are homomorphisms of lattice implication algebras by Theorem 3.12, $\phi_u$ is a homomorphism.

Let $\phi_u(x)=\phi_u(y)$ for any $x, y\in L$. Then $u'x=\rho_{u'}(x)=\rho_{u'}(y)=u'y$ and $ux=\rho_u(x)=\rho_u(y)=uy$. Hence we have

$$x=1x=(u'\vee u)x=(u'x)\wedge(ux)$$
$$=(u'y)\wedge(uy)=(u'\vee u)y=1y=y,$$

and $\phi_u$ is injective.

Let $(x,y)\in[u,1]\times[u',1]$. Then $\rho_u$ and $\rho_{u'}$ are lattice-homomorphisms by Lemma 3.5, and $\rho_{u'}(x)=x$, $\rho_u(y)=y$ and $\rho_{u'}(y)=\rho_u(x)=1$ by (6) and (1) of Lemma 3.11. This implies

$$\rho_{u'}(x\wedge y)=\rho_{u'}(x)\wedge\rho_{u'}(y)=x\wedge 1=x$$

and

$$\rho_u(x\wedge y)=\rho_u(x)\wedge\rho_u(y)=1\wedge y=y.$$

So for every $(x,y)\in[u,1]\times[u',1]$, there is an element $x\wedge y\in L$ such that

$$\phi_u(x\wedge y)=(\rho_{u'}(x\wedge y), \rho_u(x\wedge y))=(x, y),$$

i.e., $\phi_u$ is surjective. Hence $\phi_u$ is an isomorphism of $L$ to $[u,1]\times[u',1]$. □

## 4. Conclusions

The partial ordered sets have good structure for representing hierarchical objects and relationships of them. As lattice implication algebras, one of posets, is a generalization of Boolean algebras, it could be applied to more application problems than Boolean algebras. In this paper we define the multipliers of lattice implication algebras and research some properties of it, and using this properties, we showed that a lattice implication algebra have same structure with the Cartesian product of subalgebras. Finite totally ordered (chain) lattice is a lattice implication algebra. Theorem 3.13 shows a method to make a lattice implication algebra by using chain lattice implication algebras with hierarchical structure. This study can be used in a variety of future cloud and big data environments, and extended to the specific research applied to role-based or attribute-based access control.

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