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Original Article

Multivariate analysis of critical parameters influencing the reliability of thermal-hydraulic passive safety system

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ABSTRACT

Thermal-hydraulic passive safety systems (PSSs) are incorporated into many advanced reactor designs on the bases of simplicity, economics and inherent safety nature. Several factors among which are the critical parameters (CPs) that influence failure and reliability of thermal-hydraulic (t-h) passive systems are now being explored. For simplicity, it is assumed in most reliability analyses that the CPs are independent whereas in practice this assumption is not always valid. There is need to critically examine the dependency influence of the CPs on reliability of the t-h passive systems at design stage and in operation to guarantee safety/better performance. In this paper, two multivariate analysis methods (covariance and conditional subjective probability density function) were presented and applied to a simple PSS. The methods followed a generalized procedure for evaluating t-h reliability based on dependency consideration. A passively water-cooled steam generator was used to demonstrate the dependency of the identified key CPs using the methods. The results obtained from the methods are in agreement and justified the need to consider the dependency of CPs in t-h reliability. For dependable t-h reliability, it is advisable to adopt all possible CPs and apply suitable multivariate method in dependency consideration of CPs among other factors.

1. Introduction

Recent development in nuclear engineering tends towards ensuring safe, stable, reliable and economically competitive design and operations of advanced and innovative nuclear power reactors through adoption of more passive safety components and systems. In line with these, research efforts are focused on safety, performance and reliability analysis of nuclear passive safety systems (PSSs) which are the front-line systems that depend on physical principles to mitigate the progression of incidents in a plant or facility.

The evolutionary reactor designs (Generation III$^+$ reactors) are now fully incorporated with robust passive safety systems or with a combination of both passive systems and the conventional active ones. One of the key challenges that must be surmounted to ensure the desired target of safe, stable, dependable and effective operation of those reactors is the quantification of reliability of the incorporated passive systems [1] which are characterized by several critical parameters (CPs) and physical phenomena whose interactions and effects are not fully explored and thoroughly investigated.

Passive systems are known to be inherently safe but are not failure-proof while fulfilling their safety missions due to the deviation of the natural forces on which their operations are based. This can be caused by the onset of any of the physical phenomena capable of infringing on the condition(s) of operation or change their initial or boundary conditions [2,3].

The reliability of natural circulation (NC)-based passive systems may be influenced by phenomena and consequently CPs which are dependent in behaviour such as the *build-up of non-condensable gases which affect heat transfer, thermal stratification and undetected leakage which also affect heat transport rate* to mention a few. Besides the influencing phenomena, the components dependency is due to components of the same system which operate in the same environment and are therefore subjected to the same conditions (stress). As a result, malfunction of a component in the system (sub-system) affects the effective functioning of the others

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and consequently the overall system (sub-system).

Therefore, to ascertain and improve on safety which translate to reliability of the NC-based plants (systems) whether at design stage or in operation, consideration of the dependency influence of the CPs of such plants (systems) is critical.

1.1. Overview of the existing thermal-hydraulic reliability methods and their technical challenges

For quite some years now, intensive research have been on the development of reliability methodologies for assessing the reliability of thermal-hydraulic (t-h) PSSs. Some methodologies were already developed through collaborative efforts though with associated uncertainties. The common ones being Reliability Evaluation of Passive Systems (REPAS), Reliability Methods for Passive Systems (RMPS) and Assessment of Passive System ReliAbility (APSRA*)* methodology.

The REPAS methodology *evaluates the t-h reliability of a passive system* (the reliability connected with the occurrence of t-h phenomena). The overall reliability of the system is obtained by combining the t-h reliability of the different associated phenomena and the reliability of active sub-systems and components connected with the main system which influences its operation. The overall reliability is evaluated by conventional statistical methods.

RMPS is an improvement on REPAS and both involve a combination of both probabilistic and deterministic t-h approaches and t-h Best Estimate (BE) codes are used for uncertainty propagation. The uncertainties in key variables are modeled by probability density functions (pdfs) with expert judgment (EJ) commonly applied while statistical analyses are used for treating the results obtained from experiments [4]. Introduction of the *formal EJ approach* to estimate distributions for parameters whose values are either inadequate or unavailable and the use of *efficient sensitivity analysis techniques* to gauge the effects of changes in the input parameter distributions on the estimated reliability values greatly improved the RMPS.

Lastly, APSRA, another novel reliability method that is both probabilistic and deterministic is based on the use of experimental data and EJ for root diagnosis [4]. The method generates a failure surface by considering the deviation of all the CPs which influence the system performance through the t-h code. The causes of such deviations from nominal conditions are determined through *root diagnosis*.

Most of the existing reliability methods are based on independent failure modes approach, failure modes of passive system hardware components approach and functional failure approach to mention a few.

The challenges associated with these methods are briefly discussed as follows:

- Less effective performance is inherent in passive systems compared to the active ones which directly influence their reliability [5]. This performance observed with NC-based passive systems are caused by the driving force and resistance influenced by many yet to be explored phenomena (uncertainties). These uncertainties therefore account for most associated physical process failure [6,7] and eventually the failure of overall system. Thus, both component and phenomena failures in passive system reliability are important to be determined and *most reliability methodologies do not account for the relevant uncertainties especially with t-h reliability*. For instance, the associated uncertainties are out of scope of REPAS methodology and thus not captured [8]. Some limitations and inaccuracies were discovered by Ref. [8] when the REPAS was applied to a two-phase NC system which involve boiling and condensation phenomena.
- For the independent failure mode approach, *a high level of conservatism is associated many atimes*. This implies that the probability of failure of the system is relatively high as the combination of those failure modes are in a series system configuration. In this case, a single failure is enough to challenge the fulfillment of the mission of the system which may not necessarily be so in practice [9,10].
- To sum up, with the functional failure mode approach (FFA), the mission of the passive system defines which parameter values are considered a failure by comparing the corresponding pdfs according to defined safety criteria. The major deficiency of this methodology is *the selection and definition of the representative pdfs which is solely based on experience and subjective/engineering judgment.*

1.2. Approach to filling some research gaps on reliability methodologies of PSSs

This paper laid emphasis on multivariate analysis methods for dependency consideration of the CPs associated with the t-h reliability of PSSs with the aim of partly filling the first two research gaps listed above. Two of the conventional multivariate analysis (dependency) methods were presented. The methods - *the covariance matrix* and *the conditional subjective probability density function approaches*-were applied to a passively water-cooled steam generator with the purpose of demonstrating the dependency analysis of the CPs and the influence on the t-h reliability of the study system. The two methods were compared and contrasted with essential remarks on the methods and results enumerated.

2. Multivariate analysis of critical parameters influencing thermal-hydraulic reliability

The CPs (which can also be referred to as influencing factors/uncertainties) have been discovered to affect the performance and thus reliability of PSSs regardless of the fact that passive systems are expected to be inherently safe (all things being equal) [11].

For ease of analysis during evaluation of the overall t-h reliability, the essential components which are parts of the main system are usually considered. *The failure analysis of the individual components of passive systems in most cases does not consider environmental interactions and thus could not account for systems interactive effects on failure behaviour* [12]. This dependency behaviour can affect the failure rate positively or negatively. For the purpose of quantifying the effects of the CPs, the indicating (quantifying) factors of the CPs must be adopted such as the non-condensable fraction, valve closure coefficient etc. In this regard, previous studies have covered extensively five phenomena which are *build-up of non-condensable gases, undetected leakage, heat loss, partially opened valve (POV) in the discharge line* and *heat exchanger plugged pipes* with their corresponding CPs [13]. *Reliability analysis based on the influencing factors therefore involves defining the failure rate which is a function of the adopted indicator values-CPs.*

The CPs are therefore the physical parameters responsible for the failure modes of passive systems [14] while the t-h phenomena are the physical processes responsible for the behaviour of the t-h systems and are thus known to influencing their performance and consequently reliability.

The CPs in general can be considered and treated in two ways as given in previous studies [13] which are *independency* and *dependency considerations*. Evaluation of reliability on the basis of independency consideration of CPs may likely result in the overall failure probability that is over conservative (over-estimated), far from reality and therefore be of less practical application as the CPs interactive influence on failure mode or performance are not considered. Based on the parameters independency methodology adopted by Ref. [6], the conclusion of the work by Ref. [13] led to development of suitable models for CPs dependency consideration.

This model developed by Ref. [13] puts the dependency of the CPs into consideration which seems more reliable and realistic though there is inadequacy of knowledge of the behaviours of t-h passive systems concerned. In addition, evaluation of failure is more complicated as a result of introduction of dependency among the phenomena involved. This dependency consideration is very suitable as it factors-in the interactions among the phenomena where two or more factors can influence the overall system performance (in terms of the probability of occurrence or severity of the resulting events) [13].

If the dependencies existing among the CPs of a system are known (at least to some degree), the conventional multivariate methods that can be applied to consider the effects of dependency are multivariate distributions (e.g. joint probability distribution or bivariate normal distributions), functional relations between the parameters, conditional subjective probability density functions and covariance matrices. The last two methods were presented and applied in this paper.

2.1. *Established models in dependency consideration of PSSs reliability*

With variables *R* and *S* representing the functional requirement of a safety plant parameter and system's state (operating plant parameter) accordingly, where such parameters can be flow rate, temperature etc. Both *R* and *S* can be defined by pdfs (adopting the probabilistic model) and depend on many variables *x* which are characterized by uncertainties. The pdfs adopted can be in form of simple normal distribution with its associated mean, standard deviation and variance. The performance equation of a t-h nuclear passive system is thus expressed by the limit state function,

$$G_I(x) = R(x) - S(x)\ . \tag{1}$$

P_I, the overall failure probability evaluated over the failure region $G_I\,(.) \leq 0$ is expressed as:

$$P_I = 1 - R_I = \int\limits_{G_I(x)\leq 0} \cdots \iint f_X(x_1, x_2, \ldots, x_n)dx_1 dx_2 \ldots dx_n\ , \tag{2}$$

where $f_x\ (.)$ is the joint pdf depicting the uncertainty in the parameters *x* [6].

The above is for the case of maximum failure threshold i.e. $G_I > 0$, for safe function; $G_I < 0$, for failure of the system mission and vice versa for the minimum failure threshold. $G_I = 0$ is of course the limit state for all cases.

Based on the above preliminary definitions, the essential models in dependency analysis of CPs with the simplest case of two dependent CPs (bivariate parameters) are obtained from the first principles as presented below.

With the quantitative measures of two time-variant CPs denoted as $X_1(t)$ and $X_2(t)$ at time *t*, for the purpose of simplification, time-invariant nature of the CPs is assumed which leads to the following relationships [15]:

$$\Pr(X_1 \leq L_1 | X_2 \leq L_2) = \frac{\Pr\{(X_1 \leq L_1)(X_2 \leq L_2)\}}{\Pr(X_2 \leq L_2)}\ ,$$

i. e. by applying the statistically dependent theorem on the two CP. Hence,

$$\Pr\{(X_1 \leq L_1)(X_2 \leq L_2)\} \equiv \Pr(X_1 \leq L_1 | X_2 \leq L_2) * \Pr(X_2 \leq L_2)\ , \tag{3}$$

where, L_1 and L_2 are the failure limits (thresholds).

The reliability *R*, of the study system can be estimated based on the predicted joint pdfs of the CPs influencing the performance of the system. Hence,

$$R = \Pr\{X_1 \leq L_1, X_2 \leq L_2\} = \int\limits_0^{L_1}\int\limits_0^{L_2} f(x_1, x_2)dx_1 dx_2\ , \tag{4}$$

where $f(x_1, x_2)$ is the joint pdf of CPs X_1 and X_2.

Introducing time into the reliability Equation; the reliability, $R(t)$ for *n* dependent CPs [16,17] can be generalized as;

$$R(t) = \Pr\left\{ X_1(t) \leq L_1, X_2(t) \leq L_2, \ldots, X_n(t) \leq L_n \right\}$$
$$= \int\limits_0^{L_1}\int\limits_0^{L_2} \cdots \int\limits_n^{L_n} f\{x_1(t), x_2(t), \ldots, x_n(t)\} dx_{1t} dx_{2t} \ldots dx_{nt}\ , \tag{5}$$

where $f\{x_1(t), x_2(t), \ldots, x_n(t)\}$ is the joint pdf of CPs $X_1(t), X_2(t), \ldots, X_n(t)$ at time *t*.

2.2. *Theory of the covariance matrix method (bivariate option)*

Application of multivariate analysis in treating the CPs (dependency consideration) is essential to account for interactive effects as dependent CPs can influence the likelihood of occurrence or even the extent of failures. The effects of the CPs are evident through their contributions to the failure modes (FMs). A suitable probability distribution function such as the truncated normal distribution (TND) or s-normal pdf, Weibull distribution etc can be adopted to characterize the identified failure modes (evident through the CPs) associated with a system under study. For commonly adopted t-h PSSs like the isolation condenser and passively cooled steam generator; the TND has been applied in previous studies [13,16] and established to be very suitable from the simulation and experimental point of view. In addition, from the mathematical perspective, the distribution is adopted based on the fact that over the estimated range, the standard deviation is very small with respect to the mean and besides, the TND is an engineering distribution that is commonly applied on the basis of its universality and simplicity. The normal distribution is of the form:

$$f(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) * e^{-\left[\frac{(x-\mu)^2}{2\sigma^2}\right]}\ , \tag{6}$$

where μ and σ^2 represent the mean and the variance respectively for parameters observed from a sample x_i with a number of observations *n* each for the samples.

For two CPs 1 and 2, the necessary condition for dependency is,

$$S_{x_1 x_2} = Cov(X_1(t), X_2(t)) \neq 0\ ; \tag{7}$$

where,

$$S_{x_i x_j} = Cov(x_i, x_j) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x}_i)(x_j - \overline{x}_j). \tag{7a}$$

To assess the correlation of the CPs in the dependency procedure, the Pearson's product moment correlation coefficient, *r* can be used and it is expressed [15] as:

$$r_i = \frac{S_{xy}}{S_x S_y} = \frac{\sum (x_i - \overline{x})\left(y_j - \overline{y}\right)}{\sqrt{\sum (x_i - \overline{x})^2 \left(y_j - \overline{y}\right)^2}} . \tag{8}$$

In addition, the generalized variance-covariance matrix [17,18] for *n* set of CPs which can also be used to ascertain dependency of the CPs in matrix form is given as:

$$\begin{pmatrix} Var(X_1(t)) & Cov(X_1(t), X_2(t)) & \cdots & Cov(X_1(t), X_n(t)) \\ Cov(X_2(t), X_1(t)) & Var(X_2(t)) & \cdots & Cov(X_2(t), X_n(t)) \\ \vdots & \vdots & \vdots & \vdots \\ Cov(X_n(t), X_1(t)) & Cov(X_n(t), X_2(t)) & \cdots & Var(X_n(t)) \end{pmatrix} . \tag{9}$$

Var (X$_n$*(t))* denotes the variance of a sample of quantified CP *n* and *Cov (X*$_n$*(t), X*$_i$*(t))*, covariance of quantified CPs *n* and *i*. The matrix gives the variances of each variable (CP) as the main diagonal entries while the covariances between each pair of variables (*i*,*j*) are the other entries of the matrix. Each of the entries must satisfy Equation (7) for any of the CPs combinations to be dependent.

The reliability is determined using generalized Equation (5) taking into consideration the permissible limits for each of the CPs. The integral is evaluated with a suitable computational tool like MATLAB, Ms Excel program etc.

The pdf (the adopted normal distribution) is thus integrated over the permissible limits (allowable range of the parameters) by applying a suitable numerical method of integration on Equation (5) [17], i.e.

$$\begin{aligned} R(t) &= \Pr\{X_1(t) \le L_1, X_2(t) \le L_2, \ldots, X_n(t) \le L_n\} \\ &= \int_{L_{0,1}}^{L_1} \int_{L_{0,2}}^{L_2} \ldots \int_{L_{0,n}}^{L_n} f\{x_1(t), x_2(t), \ldots, x_n(t)\} dx_{1t} dx_{2t} \ldots dx_{nt} . \end{aligned}$$

In order to obtain the required parameters for the RHS of Equation (5), the adopted pdf (Equation (6)) is rewritten in such a way that the variance is substituted with variance-covariance matrix (*V*) as,

$$f_x(x_1, \ldots, x_n) = \left(\frac{1}{\sqrt{|V|}\ \sqrt[n]{(2\pi)}}\right) {*} e^{-\left[\frac{(x-\mu)V^{-1}(x-\mu)^T}{2}\right]} , \tag{10}$$

where n = the number of dependent CPs which is 4, $|V|$ = the determinant of the variance-covariance matrix, V^{-1} is the inverse of the matrix V, $\boldsymbol{x}$ = vector of the x_i values, $\boldsymbol{\mu}$ = the vector of the means of the i distribution, and superscript T = the transpose of the matrix.

The overall t-h reliability for the system is therefore obtained by substituting the values of the variables above into Equation (5).

2.3. Theory of the conditional subjective probability density functions method (bivariate option)

The conditional subjective probability density functions method is based on the highlighted main assumptions for two given CPs *X* and *Y* (bivariate analysis) as follows:

a. a critical parameter *Y* follows a normal pdf,
b. the conditional mean of *Y* given *x* is linear in *x* is $E(Y|x)$ and
c. the conditional variance of *Y* given *x* is constant is $Var(Y|x)$,

where the CP *X* can also be taken as *x*..

In addition to the above conditions which are sufficient to obtain an expression for the conditional distribution of *Y* given that $X = x$, *X* also is assumed to follow a suitable pdf (normal distribution in this case). The four conditions make up necessary and sufficient conditions to obtain an expression for the joint probability density function of the two CPs.

With the limits for the normal distribution, $-\infty < x < +\infty$, the pdf of *X* using the adopted distribution is:

$$f_x(x) = \left(\frac{1}{\sigma_X \sqrt{2\pi}}\right) {*} e^{-\left[\frac{(x-\mu_X)^2}{2\sigma_X^2}\right]} . \tag{11}$$

The conditional mean ($E(Y|x)$ or $\mu_{Y|X}$) and the conditional variance ($\sigma^2_{Y|X}$) of *Y* can be expressed as:

$$\mu_{Y|X} = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X}(x - \mu_X) , \tag{12}$$

and

$$\sigma^2_{Y|X} = \sigma^2_Y \left(1 - \rho^2\right) , \tag{13}$$

respectively.

The conditional subjective distribution, *h* (*y*|*x*) of *Y* (with $X = x$ as earlier defined) can be derived based on the conditional mean and variance already obtained. Therefore,

$$\begin{aligned} h(y|x) &= \frac{1}{\sigma_{Y|X}\sqrt{2\pi}} \exp\left[-\frac{\left(Y - \mu_{Y|X}\right)^2}{2\sigma^2_{Y|X}}\right] \\ &= \frac{1}{\sigma_Y \sqrt{2\pi}\sqrt{(1-\rho^2)}} \exp\left[-\frac{\left[y - \mu_Y - \rho\frac{\sigma_Y}{\sigma_X}(X - \mu_X)^2\right]}{2\sigma^2_Y(1-\rho^2)}\right] . \end{aligned} \tag{14}$$

The joint pdf of the CPs *X* and *Y* can then be expressed as:

$$f(x,y) = f_X(x) {*} h(y|x) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{(1-\rho^2)}} \exp\left[-\frac{q(x,y)}{2}\right] , \tag{15}$$

where the variable *q(x, y)* actually represents

$$\left(\frac{1}{1-\rho^2}\right) {*} \left[\left(\frac{X-\mu_X}{\sigma_X}\right)^2 - 2\rho\left(\frac{X-\mu_X}{\sigma_X}\right)\left(\frac{Y-\mu_Y}{\sigma_Y}\right) + \left(\frac{Y-\mu_Y}{\sigma_Y}\right)^2\right] , \tag{16}$$

in the expression for the joint pdf (Equation (15)).

With a combination of just two CPs at a time, (bivariate option), the above expression is the simplest form of multivariate distribution (bivariate normal distribution) using conditional subjective pdf method.

2.4. Generalized procedure for evaluating thermal-hydraulic reliability based on dependency consideration

The procedure for evaluating the reliability of t-h PSS based on dependency consideration of the CPs involves:

a. selection of the case study t-h PSSs.

b. identification of the possible failure modes (FMs) associated with the study system.
c. selection of the relevant CPs.
d. quantification of the CPs by the influencing coefficients.
e. definition of the range of the CPs and failure criteria (FC)/thresholds.
f. application of statistical approach to test for dependency of the CPs.
g. development of the joint pdf for the CPs by using suitable multivariate analysis approach based on failure characteristics once the CPs are tested and confirmed dependent [5].
h. the t-h reliability is evaluated by the joint pdf developed (with the permissible range as integration limit).

The procedure is time dependent as each of the CPs varies with time and therefore needed to be repeated for the time under consideration to ensure precision as the behaviour of the t-h PSSs is stochastic and non-stationary. The calculation is simplified by assuming the behaviour of the selected CPs to be time-invariant which makes the distribution parameters (mean, variance etc) constant.

A concise form of the above procedure for the dependency analysis of parameters influencing t-h reliability was given by Ref. [18] as a flow chart.

3. Application of the multivariate methods in dependency analysis of a simple NCS

A generic simple form of the NCS in which the operation of most t-h PSSs is based is used for illustration. This NC-based system is adopted in most advanced nuclear power reactor purposely for removing residual heat from the reactor core after an emergency shutdown. The systems in this group of PSSs based on the classification of [19] are passively cooled steam generator based on NC, passive residual heat removal heat exchangers (PRHR-HX) and passively cooled core isolation condensers.

3.1. Description of the passively water-cooled steam generator

A passive system whose mission is to remove reactor core decay heat through the steam generator (SG) like this case study is usually incorporated in some of the advanced pressurized water reactor (PWR) designs. The pressurized water flowing to the steam generator (from the core) - at close to full system pressure and temperature - rises through the inlet of the PRHR (hot leg) and is condensed in a heat exchanger (HX) submerged in a pool of water-condenser (Fig. 1). The cold coolant returns to the SG through the outlet of PRHR system (cold leg) which is connected to the SG at a lower elevation compared to the hot leg [19]. System-integrated Modular Advanced ReacTor (SMART) and Chinese HPR1000 are examples of advanced reactors that operate based on the principle of the passively water-cooled steam generator earlier described [19].

3.2. Characterization of the passively water-cooled steam generator

The major CPs which influence the performance of the study NC loop together with their failure mechanisms and thresholds are highlighted (Table 1). These identified CPs are responsible for failure modes to a greater extent like pipe break, blockage of HX pipes etc known the NC-based system. A thorough examination of the expected safety mission(s) is required for proper identification of the key CPs [20].

Expert judgment, engineering assessment and practical experience are normally adopted to characterize t-h PSSs as can also be seen in the Table. This is due to inadequacy of experimental and operational data associated with the operations of such systems [9]. Nominal/average values in the Table characterize the status of the system at steady state while permissible range which falls within all possible range of the indicator gives the normal operational/allowable range of the CPs. In the case of this study system, the data and the adopted pdf have been used in literature for similar t-h PSSs like isolation condensers and have been confirmed through simulations, relevant experiments [13,17,20] and justified from mathematical perspective.

3.3. Application of the covariance matrix method

The four identified major CPs of the system are denoted as: $x_1 =$ *NCF (non-condensable fraction)*, $x_2 =$ *HXP (heat exchanger plugging)*, $x_3 =$ *VCC (valve closure coefficient)* and $x_4 =$ *UL (undetected leakage)*.

A set of four observations each for the CPs (similar values were adopted in literature based on expert judgment, engineering experience etc) [17] were considered. The vector $\boldsymbol{x}_i$ is thus a set of randomly sampled values of the CPs within the success/permissible range (Table 2) for the NC-based system.

This characterization is followed by adoption of a suitable probability density functions (pdfs) which are required in evaluating the reliability of t-h PSSs.

The mean vector for each of the selected CPs is the mean for the set of observations while the variance-covariance matrix is the variances of each variable along the main diagonal and the covariances between each pair of variables in the other positions of the matrix as earlier defined.

The mean vector obtained is, $\overline{\mu} = [0.216 \quad 0.033 \quad 0.150 \quad 0.500]$.

With the use of Ms Excel statistical function tools (COVAR and PEARSON), all the six possible combinations (${}^4C_2=3*2$) of the key CPs satisfied the dependency condition (Equation (7)) as in Table 3.

The combinations of the CPs are found to be correlated as the r values are all above 0.4 which implies that the combinations display a high positive correlation (Table 3).

Considering the results (Table 3), one can reasonably take the CPs to be dependent. The variance-covariance matrix, V which results from the earlier stated conditions (Equation (9)) is:

$$V = \begin{bmatrix} 0.007224 & 0.000210 & 0.005270 & 0.013400 \\ 0.000210 & 0.000053 & 0.000253 & 0.001150 \\ 0.005270 & 0.000253 & 0.007267 & 0.013000 \\ 0.013400 & 0.001150 & 0.013000 & 0.066667 \end{bmatrix}.$$

The t-h reliability can now be evaluated using Equation (5) by taking the permissible limits for each of the CPs i.e. ($x_1 < 0.5$, $x_2 < 0.10$, $x_3 < 0.3$ and $x_4 < 1.0$) as limits of integration using a suitable computational tool. This involves integrating the pdf (represented by the multivariate normal distribution) over the defined limits by applying an appropriate integration technique on Equation (5) [17], i.e.

$$R(t) = \Pr\{X_1(t) \le L_1, X_2(t) \le L_2, \ldots, X_n(t) \le L_n\}$$
$$= \int_{L_{0,1}}^{L_1} \int_{L_{0,2}}^{L_2} \cdots \int_{L_{0,n}}^{L_n} f\{x_1(t), x_2(t), \ldots, x_n(t)\} dx_{1t} dx_{2t} \ldots dx_{nt}\,.$$

The adopted normal pdf as defined in Equation (6) is then rewritten in a way to describe the CPs (as in Equation (10)); the new Equation is therefore applied as described in Section 2.2.

The results obtained through the variance-covariance matrix $\boldsymbol{V}$

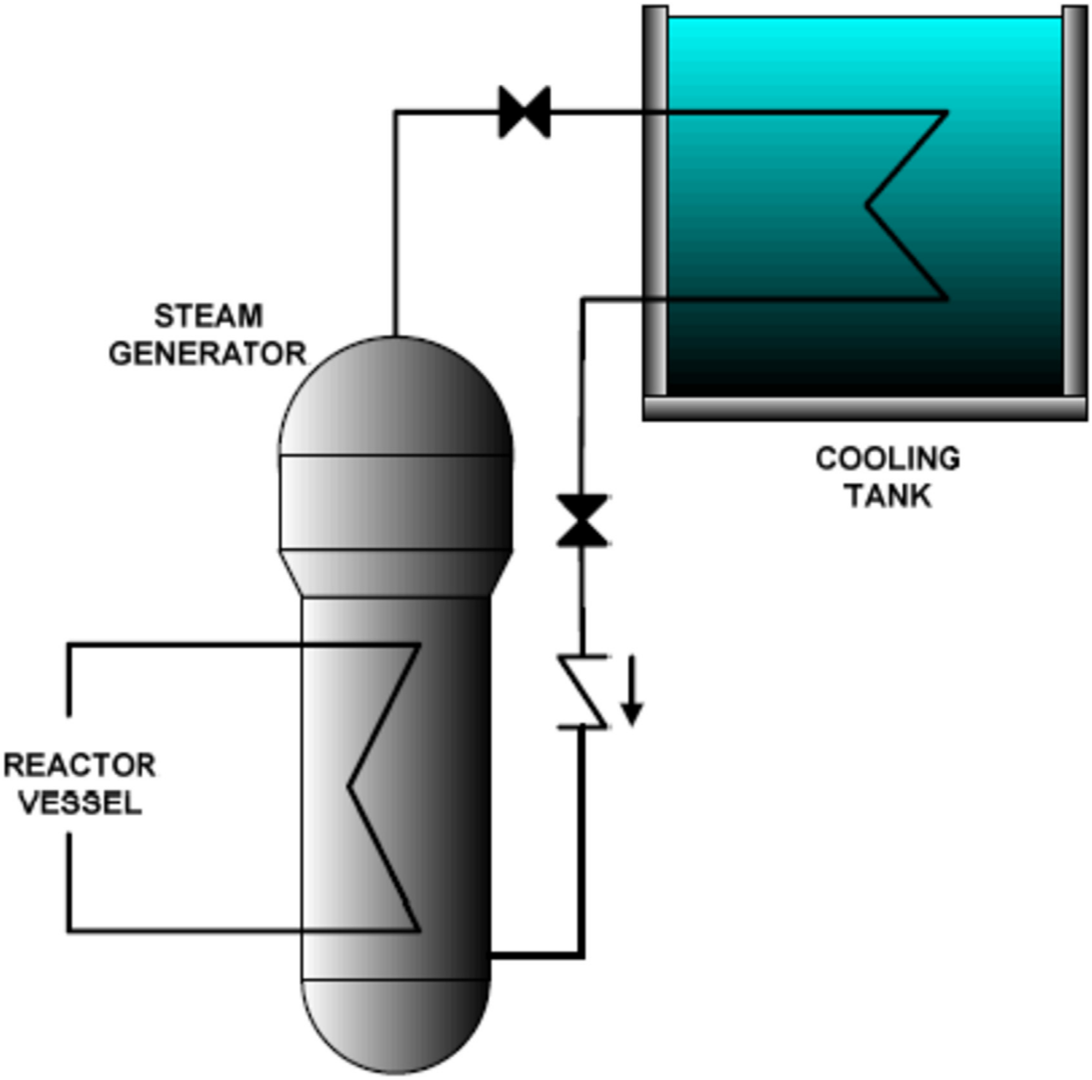

Fig. 1. Schematic representation of core decay heat removal by a passively water-cooled steam generator [19].

Table 1
Range of critical parameters and failure thresholds adopted [9,13,14].

S/N	Critical parameter	Range of indicator	Nominal/average operating value	Failure threshold	Permissible range	PDF
1	Non-condensable fraction	0 .01–0.8	0	0.5	0.01–0.5	D-TND
2	Heat exchanger plugging	0–0.15	0	0.10	0.0–0.10	D-TND
3	Valve closure coefficient	0–0.5	0	0.3	0.0–0.30	D-TND
4	Undetected leakage (cm^2)	0–10	0	1.0	0.0–1.0	D-TND

Non-condensable fraction: 0-absence of non-condensables, 0.8-presence of about 80% non-condensables … as the presence of 100% non-condensables is not realistic.
HX plugged pipes: 0-total absence of plugged pipes, 0.15-unacceptable condition requiring replacement/fixing.
Valve closure coefficient: 0-valve completely open, 1-valve completely closed.
Undetected leakage: 0 cm^2-no leakage, 10 cm^2 – maximum area of undetected leakage, failure threshold of 1.0 cm^2 (approximately 3/8 in.) is adopted as it is approximately the lower limit for Small Break Loss of Coolant Accident (LT-SBLOCA) in nuclear reactors.
D-TND: Doubly Truncated Normal Distribution.

and the pdfs are presented (Table 4).

The inverse of the variance-covariance matrix, $\boldsymbol{V}^{-1}$ obtained is,

$$10^{4*}\begin{bmatrix} 0.0334 & 0.0377 & -0.0190 & -0.0037 \\ 0.0377 & 3.0741 & -0.0399 & -0.0528 \\ -0.0190 & -0.0399 & 0.0320 & -0.0017 \\ -0.0037 & -0.0528 & -0.0017 & 0.0035 \end{bmatrix}.$$

The above values are substituted into Equation (10), and Equation (5) is further applied which gives the final t-h reliability for the system to be 0.21948.

3.4. Application of the conditional subjective probability density functions method

For this method, the same identified CPs (Table 1) with the same sets of four observations each for the CPs (Table 2) on the basis of

Table 2
Value sets for the identified critical parameters for the study system [14,17].

	Selected critical parameters			
	x_1	x_2	x_3	x_4 (cm^2)
Sample sets/observations	0.300	0.035	0.25	0.60
	0.250	0.042	0.18	0.80
	0.214	0.025	0.12	0.40
	0.100	0.030	0.05	0.20
Mean of the sets	0.216	0.033	0.150	0.500
Standard deviation	0.084994	0.007257	0.085245	0.258199

Table 3
The correlation coefficients and covariance values for the combinations of the key CPs.

S/N	Critical parameters combination	Correlation coefficient, r	Covariance value (Cov)
1	$x_1\ x_2$	0.453	0.00021
2	$x_1\ x_3$	0.970	0.00527
3	$x_1\ x_4$	0.814	0.00134
4	$x_2\ x_3$	0.544	0.00253
5	$x_2\ x_4$	0.818	0.00115
6	$x_3\ x_4$	0.788	0.01300

Table 4
Variance-covariance matrix and pdf related results.

Variables	Values
Determinant of V	$3.1184{*}10^{-11}$
f(x$_1$)	$1.4850{*}10^{-5}$
f(x$_2$)	$3.0559{*}10^{4}$
f(x$_3$)	$2.9088{*}10^{-4}$
f(x$_4$)	0.0

randomly selecting possible permissible values for normal operating conditions (within the allowable range) were used. Adoption of the sets of observations are on the basis of engineering and expert judgment and the values have already been adopted in literature for similar passive systems [14,17,20] which w-ere adjudged to be suitable as real plant and experimental data are scarce for such systems. In addition, in practical applications of such systems, these values are all realistic for the indicators adopted for the identified CPs.

In this method, just like the covariance matrix approach, of importance is the determination of the Pearson's product moment correlation coefficient, r mathematically defined in Equation (8) and the test for dependency too with covariance necessary condition (Equation (7)).

Applying Equations (7) and (8), using a suitable computation tool (MATLAB, Ms Excel etc) as in the case of the covariance method produced same results (Table 3) since the same set of observations were used for the CPs. All assumptions and necessary conditions for conditional subjective probability still hold for each of the six (6) combinations. The values of r for all the possible combinations revealed that the CPs are correlated as the r values ranges between 0 and 1, with the lowest value of 0.453 which connotes a high positive correlation (Table 3). The values of r as well as the covariance of the combinations presented (Table 3) justify their dependency consideration.

To obtain the expressions for the joint pdfs, the conditional mean and conditional variance were obtained using Equations (12) and (13). In addition, Equation (14) was used to obtain the conditional distribution, $h(y|x)$ for the combinations (Table 5) using the conditional mean and variance values (also in Table 5). Finally, with the conditional distributions obtained, the expressions for conditional subjective pdfs, $f(x, y)$ (Table 6) were derived using Equation (15) for the CPs combinations after obtaining the expression for $q(x,y)$ by Equation (16).

3.4.1. Determination of the thermal-hydraulic reliability based on the derived conditional subjective pdfs

The reliability is therefore determined using Equation (5) in conjunction with the derived pdfs (Table 6). The mission fulfillment regions (permissible limits) are used as limits of integration ($x_1 < 0.5, x_2 < 0.10, x_3 < 0.3$ and $x_4 < 1.0$) as in Table 1. The evaluation involves integrating each of the pdfs over the permissible limits using a suitable numerical integration approach. In this illustration, the *Simpson's rule extension to multiple integrals* was found suitable and adopted.

The Simpson's rule extension to multiple integrals takes the form,

$$S_x(y_i) = f\left(x_0, y_j\right) + f\left(x_n, y_j\right) + 4\sum_{i-1}^{\frac{n-2}{2}} f\left(x_{2i-1}, y_j\right) + 2 \times \sum_{i=1}^{\frac{n-2}{2}} f\left(x_{2i}, y_j\right), \tag{17}$$

with n being the number of steps in evaluating the integral in the x (first CP in the combination) domain and $S_x\,(y_n)$ depicts Simpson's rule applied along the x-axis as a function of y_n on the y-axis for the inner integral.

The Simpson's rule is re-applied in order to evaluate the outer integral. The original equation is therefore expressed again as,

$$S = \frac{h_x h_y}{9}\left[S_x(y_0) + S_x(y_n) + 4\sum_{j=1}^{\frac{n-2}{2}} S_x\left(y_j\right) + 2\sum_{j=1}^{\frac{n-2}{2}} S_x\left(y_j\right)\right], \tag{18}$$

where h is the increment between the calculated steps. The computation can be done by any suitable computational tool like MATLAB program, C language routine or Ms Excel program as convenient.

To illustrate the integration procedure, the pdf for the CP combination $x_1\,x_2$ is used (i.e. Equation (19), selected from Table 6). The numerical integration approach for multiple integrals is illustrated as follows:

Table 5
Variables for deriving the conditional subjective pdfs.

Combination of CP	Correlation coefficient, r	Conditional mean, $\mu_{(Y\|X)}$	Conditional variance, $\sigma^2_{Y\|X}$	Conditional distribution, $h(y\|x)$
$x_1\ x_2$	0.453	$0.0387x + 0.0217$	0.000042	$61.654 * \exp[-11942(Y - 0.0387x - 0.02465)^2]$
$x_1\ x_3$	0.970	$0.9727x - 0.06$	0.000432	$19.194 * \exp[-1157.4(Y - 0.9727x - 0.06)^2]$
$x_1\ x_4$	0.814	$2.4734x - 0.0342$	0.022479	$2.660 * \exp[-22.222(Y - 2.4734x + 0.0342)^2]$
$x_2\ x_3$	0.544	$6.3926x - 0.0610$	0.005115	$5.578 * \exp[-97.752(Y - 6.3926x + 0.0610)^2]$
$x_2\ x_4$	0.818	$29.115x - 0.461$	0.022037	$2.687 * \exp[-22.727(Y - 29.115x + 0.461)^2]$
$x_3\ x_4$	0.788	$2.385x + 0.1422$	0.025321	$2.508 * \exp[-19.763(Y - 2.385x - 0.1422)^2]$

Table 6
The conditional subjective probability distribution functions.

Combination of CPs	Probability distribution function, *f(x, y)*
$x_1\ x_2$	$289.3 * \exp[-87.05(X - 0.216)^2 + 923.5(X-0.216)(Y - 0.033) - 11933.8(Y-0.033)^2]$
$x_1\ x_3$	$90.07 * \exp[-1164.2(X - 0.216)^2 + 2250.1(X-0.216)(Y - 0.15) - 1156.1(Y-0.15)^2]$
$x_1\ x_4$	$12.49 * \exp[-205.2(X - 0.216)^2 + 110(X-0.216)(Y - 0.5) - 22.2448(Y-0.5)^2]$
$x_2\ x_3$	$306.64 * \exp[-13492.2(X - 0.033)^2 + 1250(X-0.033)(Y - 0.15) - 97.775(Y-0.15)^2]$
$x_2\ x_4$	$147.8 * \exp[-28738.8(X - 0.033)^2 + 1321.9(X-0.033)(Y - 0.5) - 22.71(Y-0.5)^2]$
$x_3\ x_4$	$11.37 * \exp[-181.15(X - 0.15)^2 + 94.21(X-0.15)(Y - 0.5) - 19.75(Y-0.5)^2]$

$$289.3 * \exp[-87.05(X - 0.216)^2 + 923.5(X-0.216)(Y - 0.033) - 11933.8(Y-0.033)^2] \tag{19}$$

The integration limits for the CPs in the selected combination (x_1x_2) are x_1: [0.01, 0.5] and x_2: [0, 0.10] and the observed values (Table 2) for the CPs are within the permissible limits as the failure thresholds are 0.5 and 0.10 respectively (Table 1).

Equation (4) which is the time-invariant form of generalized reliability Equation (5) can now be written for the CPs 1 and 2 as;

$$R = \Pr\{X_1 \le L_1, X_2 \le L_2\} = \int_{L_{0,1}}^{L_1} \int_{L_{0,2}}^{L_2} f\{x_1, x_2\} dx_1 dx_2\ , \tag{20}$$

which translates to,

$f(x_1, x_2) = f(X,Y) = 289.3 * \exp[-87.05(X - 0.216)^2 + 923.5(X-0.216)(Y - 0.033) - 11933.8(Y-0.033)^2]$, therefore,

$$R = \int_{0.0}^{0.1} \int_{0.01}^{0.50} f\{289.3 * \exp[-87.05(X - 0.216)^2 + 923.5(X - 0.216)(Y - 0.033) - 11933.8(Y - 0.033)^2]\} dx_1 dx_2\ . \tag{21}$$

The *Simpson's rule extension to multiple integrals* requires an even number of steps for the two integrals. On this basis, *eight (8) steps* were used for the computation which translates to the step-sizes of 0.06125 and 0.01250 for the first (X) and second (Y) CPs (i.e. the inner and outer integrals) respectively in the selected combination *(x_1x_2)*. The result obtained for the combination is 0.0948. The numerical procedure was repeated for the other five (5) combinations taking into consideration the permissible limits of the other CPs which are, x_3: [0.0, 0.3] and x_4: [0.0, 1.0]. The t-h reliability results for all the six combinations (Table 7) were used to determine the overall t-h reliability of the study system.

With the conditional subjective pdf dependency approach applied, the overall t-h reliability is obtained by using the reliability combination rule for redundant components in parallel (in which at least one component or sub-system must operate for the overall system to operate) which gave *0.398* as the overall t-h reliability for the study system.

Table 7
The t-h reliability for the combinations based on their conditional subjective pdfs.

Probability distribution function, *f(x, y)*	T-H reliability
$f(x_1, x_2)$	0.0948
$f(x_1, x_3)$	0.1106
$f(x_1, x_4)$	0.0934
$f(x_2, x_3)$	0.0526
$f(x_2, x_4)$	0.0475
$f(x_3, x_4)$	0.0859

3.5. Discussion/comparison of the results from the multivariate methods

For the covariance method, the overall t-h reliability obtained for the system is 0.219 and that obtained for the conditional subjective probability density approach is 0.398. The t-h reliability obtained for the study system compares relatively well for the two multivariate methods applied to the study system. The results are also comparable even to systems of similar configurations in literature (such as the isolation condenser) in which other dependency methods were applied [16]. The slight disparity observed in the results of the two multivaraiate methods may be attributed to the inherent errors/uncertainties associated with their different mathematical bases.

It is important to note that this t-h reliability result is actually a sub-part of the overall system reliability which comprises both the component and phenomenal (t-h) reliability of which the results obtained is the latter. Therefore to obtain the overall system reliability, the component reliability must be obtained through suitable methods like those for active systems and then appropriately combined with the t-h reliability.

4. Remarks on the presented methods and results

The multivariate analysis methods (covariance and conditional subjective pdf) in most cases rely on the use of engineering experience and expert judgment in identifying key CPs, defining their range of values, selecting the suitable pdfs and setting the failure limits for the CPs as a result of unavailability of adequate real plant/experimental data. This will have some implications on the true nature and value of the estimated t-h reliability for the system under consideration. For instance, the selected pdfs may not be able to suitably (fully) characterize the performance distributions (behaviours) of the selected CPs.

In practice, dependency of CPs is a factor among several other environmental factors that influence quantified reliability which also need to be taken into consideration in reliability analysis to ensure accuracy. These other factors that are unaccounted for make the quantified reliability characterized with some level of uncertainties.

In addition, the time-invariant characteristics of the CPs assumed for simplification will also cause some discrepancies between the estimated reliability and the real value as the CPs are time-variant and stochastic in practice.

Furthermore, assignment of suitable models/correlations and pdfs binding the selected key CPs cannot be said to be explicitly justified as a lot of factors are involved practically due to the complexity of the operating nature of the systems. The more some of these issues are put into consideration, the less the uncertainties become and the more realistic the estimated reliability obtained.

These two approaches as presented seem more acceptable and realistic to some extent compared to independent consideration of the CPs which completely neglect the dependency influence of the CPs on reliability.

5. Conclusions

Multivariate analysis of critical parameters (CPs) influencing the reliability and performance of thermal-hydraulic (t-h) passive safety systems was presented from two perspectives which are the covariance matrix and conditional subjective probability density function methods. The methods followed a generalized procedure for evaluating the reliability of t-h PSSs based on dependency consideration of identified key critical parameters. The two multivariate analysis methods were applied to a generic t-h PSSs (a passively water-cooled steam generator) in order to evaluate its thermal-hydraulic reliability. With the identified four key CPs, the results obtained agree relatively well and justified the essence of dependency consideration of CPs and also call for introduction of more influencing factors (not only CPs) in dependency consideration of reliability of t-h PSSs. The two methods gave reliability results that appear more realistic and thus can be of practical application compared to independent analysis.

In addition, the analysis justified the dependency consideration of the CPs as they were proved dependent by showing non-zero covariance values and highly positive correlation coefficients which depict strong relationships among the CPs. The influence of the CPs also reflected on the final value of the thermal-hydraulic reliability for the passive safety system studied.

In order to improve on the suitability/accuracy of the multivariate methods, it is recommended that more key CPs should be considered with less expert judgment in selecting the pdfs for characterizing the CP, setting of failure limits etc and other environmental factors of performance be taken into consideration when sufficient data and information are available . In addition, sourcing for scarce experimental and real plant data are encouraged as much as possible in the analysis for better quantification of reliability through reduction of uncertainties.

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