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# Elementary School Teachers' Beliefs of the Common Core State Standards for Mathematical Practice

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Pennsylvania is one of the states that adopted the Common Core State Standards for Mathematics (CCSSM) and crafted its own standards (The PA Core State Standards). Pennsylvania teachers are required to have a clear understanding of the PA Core Standards. It is timely and appropriate to study Pennsylvania teachers' beliefs, as the standards have been adopted and implemented for several years since the revision of the PA Core Standards (2014). This study examined how eight western Pennsylvania elementary school teachers' beliefs about teaching and learning mathematics related to the SMP.
To this end, I conducted an in-depth interview with each participating teacher. The in-depth interviews featured the teachers' overarching mathematical instructional goals and their productive beliefs. Furthermore, I linked these beliefs with the CCSSM Standards for Mathematical Practice (SMP).

*Keywords*: Common Core State Standards for Mathematical Practice (SMP), elementary school teachers, teacher beliefs, case study, in-depth interview
ZDM classification: A80, C70, D40
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## I. INTRODUCTION

The field of mathematics education in K-12 schools has focused increasingly on the development of standards for students' learning of mathematics for the past three decades. National Council of Teachers of Mathematics (NCTM) published several Standards documents to improve mathematics education during the 1990s and 2000s (NCTM, 1989, 1991, 1995, and 2000). To complement the NCTM Standards, state education standards in a majority of the states have been developed since the 1990s. Most recently, the National Governors Association

(NGA) and Center for Best Practices Council of Chief State School Officer (CCSSO) "recognized the value of consistent, real-world learning goals and launched this effort to ensure all students, regardless of where they live, are graduating high school prepared for college, career and life," leading the effort to develop common standards in literacy and mathematics standards (NGA & CCSSO, 2010a, Development Process Section, para. 1).

The Common Core State Standards for Mathematics (CCSSM) define a level of quality regarding what K-12 students should know and be able to do in mathematics. In particular, the CCSSM explicitly delineate the Standards for Mathematical Practice (SMP) that mathematics teachers should nurture in all students (NGA & CCSSO, 2010a). The SMP is considerably influenced by the two leading documents in U.S. mathematics education—NCTM Process Standards (NCTM, 2000) and National Research Council (NRC) report, *Adding It Up* (NRC, 2001). The CCSSM claims to be research-based, internationally benchmarked, and developed using the best existing state standards and the NCTM standards with involvement of teachers, content experts, state leaders, parents, and students (NGA & CCSSO, 2010a).

In the midst of widespread recognition of and focus on the CCSSM, however, there exist questions relating to teachers' beliefs about the CCSSM. Although there are a number of studies investigating teachers' beliefs about the previous standards and teachers' standards-based mathematics instruction, some of the studies with respect to the recent CCSSM and the SMP in particular, are only emerging now (Heck et al., 2011).

Over the past three decades, the NCTM's Standards documents have assumed a leading role in the reform movement in mathematics education for the purpose of improving mathematics education (NCTM, 1989, 1991, 1995, 2000). The ensuing research studied the influence of these Standards relating to teachers' beliefs (Battista, 1994; Borko et al., 1992; Prawat, 1992; Raymond, 1997). For example, Battista (1994) asserted that teachers would not be able to understand or achieve the goals of the mathematical activity that incorporated the reform movement, no matter how well-written it was, if a teacher's beliefs were incongruous with the intention of the activity. In other words, teachers who perceived a more traditional, procedural way of mathematics education tended to focus on following procedures rather than making sense out of mathematics. The studies relevant to teachers' beliefs and their practices have shown meaningful correlation between the two (Ball, 1990; Ball, 1991; Brendefur & Frykholm, 2000; Fennema et al., 1996; Lloyd & Wilson, 1998; Pajares, 1992; Staub & Stern, 2002).

The aforementioned studies have shown that many teachers' beliefs seemed inconsistent with those of the standards-based movement, even after a couple decades of endeavoring to change those beliefs. Succeeding NCTM standards, the CCSSM and its practice standards, the SMP have pursued the vision of a balance of conceptual and procedural understanding of mathematics, problem solving and reasoning, strategic use of mathematical tools, mathematical discussion, and making sense of mathematics. Moreover, the recently adopted CCSSM standards have required teacher's revised or extended understanding of the standards.

Since the release of the CCSSM in 2010, researchers, mathematics educators, and mathematicians have studied various areas related to the CCSSM. However, studies on teachers' beliefs about the CCSSM (SMP in particular) are limited. Moreover, studies discussed teachers' perceptions in the form of an online poll in which they surveyed how teachers think about their readiness to implement the CCSSM (e.g., Bostic & Matney, 2013; Burks et. al., 2015; Davis, Choppin, McDuffie, & Drake, 2013; Cogan, Schmidt, & Houang, 2013; Davis, Drake, Choppin, & McDuffie, 2014; Troia & Graham, 2016). Despite the advantage of survey type of research such as the usefulness when a researcher aims to explain features of a very large group (Blackstone, 2012) and its time- and cost-efficiency, it would be more appropriate to conduct a qualitative study when investigating teachers' beliefs.

It is called to investigate teachers' beliefs of the CCSSM more in-depth, as the teachers might have learned the new standards, have used the new curriculum, and have implemented them in classroom settings for some years. For example, Heck and his colleagues (2011) proposed a priority research agenda to understand the impact and implementation of the CCSSM. One of the suggestions for case study of the CCSSM is to examine teachers' beliefs of the CCSSM as follows:

> Since teachers'…beliefs…affect what transpires in classrooms, it is critical to understand how teachers respond to the CCSSM, and what kinds of classroom learning opportunities for their students [might] result. (Heck et al., 2011, p. 13)

For this study, I conducted in-depth interviews with eight elementary school teachers in western Pennsylvania. I probed into how their beliefs about teaching and learning mathematics related to the SMP. To guide this study and to fulfill its purpose, I used the research question: *How do the participating elementary school teachers' beliefs align with the SMP?*

## II. BACKGROUND

In this chapter, I define some key words for clarity of meaning first. Then I examine the CCSSM (focusing on SMP) and Pennsylvania Core Standards for Mathematics (PA Core Standards for Mathematics. Next, I review the literature on the CCSSA. Finally, I discuss the literature on teachers' beliefs.

## 1. DEFINITIONS OF TERMS

First, "*standard* is a statement describing what a person should know or be able to do [emphasis added]." (Weiss, 2002, p. 23). Standards influence teacher development such as teacher education and professional development. Weiss noted that standards also guide what kinds of assessment should be used as well as to guide the purposes of the assessments. Similarly, Hiebert (2003) defined standards as "the goals we set for our students. They are value judgments about what we would like our students to know and be able to do" (p. 6). I use the definition of beliefs as Philipp (2007) described as follows:

> psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action. (p. 259)

Furthermore, the authors of *Principles to Actions* (NCTM, 2014) outlined the *productive beliefs* about teaching and learning mathematics in contrast to *unproductive beliefs* (Figure 1). Both productive and unproductive beliefs are linked to effective practices. Productive beliefs enhance more effective instructional practice or allow student access to important mathematics content and practices. Unproductive beliefs, however, obstruct effective teaching and learning.

| **Unproductive Beliefs** | **Productive Beliefs** |
|---|---|
| Mathematics learning should focus on practicing procedures and memorizing basic number combinations. | Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse. |
| Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems. | All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures. |
| Students can learn to apply mathematics only after they have mastered the basic skills. | Students can learn mathematics through exploring and solving contextual and mathematical problems. |
| The role of the teacher is to tell students exactly what definitions, formulas, and rules they should | The role of the teacher is to engage students in tasks that promote reasoning and problem solving and |

| | |
|---|---|
| know and demonstrate how to use this information to solve mathematics problems. | facilitate discourse that moves students toward shared understanding of mathematics. |
| The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests. | The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others. |
| An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused. | An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics. |

**Figure 1**. Beliefs about teaching and learning mathematics (NCTM, 2014, p. 11)

Teachers' beliefs influence their decisions about lesson plans, tasks, instructional practices, and assessment. Students' beliefs influence their perception and attitudes about mathematics and what it means to learn mathematics. Teachers who possess productive beliefs plan lessons to trigger student discourses, implement the standards-based instructions, and reflect their lessons with the goal of helping students make sense of mathematical concepts and procedures, to ensure successful mathematics learning for all students. Likewise, students who hold productive beliefs actively face challenges in problem solving, construct viable argument, critique others' reasoning, justify their own solutions, make connections to their prior knowledge and to other subjects, and create and use multiple representations (NCTM, 2014).

## 2. STANDARDS FOR MATHEAMTICAL PRACTICE

Studies reported that students gained better or similar learning outcomes when their teachers used student-centered approaches and standards-based pedagogy in their mathematics instructions, compared to more traditional ways of teaching. On the basis of prior mathematics education research results, this study assumes that teachers' instructional practices are one of the most important school-based factors to account for students' learning. Consequently, the standards for practice itself and understanding of such standards is paramount to guide teacher's instructional practices.

The CCSSM was created in response to the calls for a coherent, focused, and rigorous standards for the school mathematics. The work team of the CCSSM described the standards as “what students should understand and be able to do in their study of mathematics” (NGA & CCSSO, 2010a, p. 4). The CCSSM was developed to raise mathematically proficient students who exhibit both conceptual and procedural understanding of mathematics in a way appropriate to their mathematical maturity. The standards are grade-specific from Kindergarten to grade 12, but do not suggest specific intervention methods or materials (NGA & CCSSO, 2010a).

The CCSSM begins with the SMP that describes what knowledge and skills mathematics educators at all levels should pursue in developing students for mathematical proficiency. Including classroom teachers, all mathematics educators who design and perform curricula, assessments, and professional development should attend to the need to connect the mathematical practices to the content in mathematics teaching. The eight SMPs advocate the NCTM’s Process Standards (2000) and *Adding It Up* (NRC, 2001), to complement the CCSSM content standards by describing the varieties of expertise that K-12 teachers need to develop in their students. These mathematical practices are the refinements from the NCTM’s Process Standards: problem solving, reasoning and proof, communication, connections, and representation (NCTM, 2000) and the Strands of Mathematical Proficiency from the National Research Council (NRC)’s report, *Adding It Up*: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (NRC, 2001).

The SMP briefly explains how each of the SMPs can be implemented in the mathematics classroom in a variety of grade levels and mathematical topics. The descriptions of the SMP from the Common Core State Standards Initiative (CCSSI; NGA & CCSSO, 2010a) will guide the processes of data collection and analysis for this study. I list the eight Standards in Figure 2, only the title of each Standard for brevity. (See Appendix A for more in-detailed descriptions of each standard).

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

**Figure 2**. Eight standards for mathematical practice (NGA & CCSSO, 2010a)

## 3. THE PENNSYLVANIA CORE STANDARDS FOR MATHEMATICS

The CCSSM is the most recent mathematical standards that are adopted nationwide. Some states have crafted the states' own standards based upon the CCSSM, but to meet state-specific needs. For example, Pennsylvania created PA Core Standards for Mathematics that mirrored the CCSSM. The newly revised 2014 PA Core Standards for Mathematics emulated the content and rigor of the CCSSM. The standards were also drawn from the best of the PA Academic Standards and represent the input of Pennsylvania educators. The PA Core Standards for Mathematics were distributed to schools in the 2013-2014 academic year and professional development for teacher training followed in the 2014-2015 academic year (Commonwealth of Pennsylvania, 2018a). The Pennsylvania Department of Education (PDE) states that teachers in Pennsylvania are responsible for "a clear understanding of the instructional shifts and rigorous demands of PA Core Standards" (Commonwealth of Pennsylvania, 2018b, p. 1).

The PDE provides The Standards Aligned System (SAS), offering resources for teachers to improve student achievement. The SAS classifies and describes Standards, Assessment, Curriculum Framework, Instruction, Materials & Resources, and Safe and Supportive Schools. Above all, the Standards are accessible to the public and are downloadable. The 2014 PA Core Standards for Mathematics describes what students in Pennsylvania should understand and be able to do (PDE, 2014). It consists of Standards for Mathematical Content and Standards for Mathematical Practice. The introductory page of the PA Core Standards for Mathematics depicts the importance of both standards.

## 4. LITERATURE ON THE CCSSM

Since the release of the CCSS in 2010, researchers, mathematics educators, and mathematicians have studied various areas related to the CCSS. Although my focus is on the CCSSM, often I refer to the literature on CCSS as one body, combining the mathematics standards and the ELA/Literacy standards together. The key areas of the studies around the CCSS have been content, curriculum, and alignment; professional development and implementation; the effects of the CCSS on instructional practices and student outcomes; and teacher conceptions (beliefs, knowledge, interpretation, understanding, etc.) about the CCSS.

First, researchers have studied and analyzed the CCSS content, curriculum, and alignment (e.g., Cobb & Jackson, 2011; Dingman, Teuscher, Newton, & Kasmer, 2013; Gamson, Lu, & Eckert, 2013; Nagle & Moore-Russo, 2014; Polikoff, 2015; Porter, McMaken, Hwang, & Yang, 2011; Porter, Polikoff, Barghaus, & Yang, 2013; Williamson, Fitzgerald, & Stenner, 2013).

They assessed the quality of the CCSS, compared the CCSS with other standards (e.g., state standards, NCTM Standards, etc.), examined the degree of textbook alignment to the CCSS, or compared the CCSS-aligned textbooks with former textbooks.

Some of the researchers studied whether, in their view, the CCSS were appropriate. Those who advocated the CCSS explained why integrating the CCSS in classroom instruction was necessary and how the CCSS should be implemented (e.g., Burns, 2013; Bostic & Matney, 2013; Kendall, 2011; Schoenfeld, 2015; Sztajn, Marrongelle, Smith, & Melton, 2012; Wu, 2011; Wu, 2014). These researchers suggested ways to assist teachers in developing an understanding of the content and practice standards of the CCSS for desirable implementation. There have been, however, voices of concern about the CCSS (e.g., Cobb & Jackson, 2011; Mathis, 2010; Tienken, 2011), critiquing the CCSS for the lack of research on the impact of the standards, little provision of necessary educational resources, and plausible implementation obstacles.

Second, there have been studies about professional development for and implementation of the CCSS (e.g., Elias, 2014; Herman, Epstein, & Leon, 2016; Holliday & Smith, 2012; Jenkins & Agamba, 2013; Kane, Owens, Marinell, Thal, & Staiger, 2016; Liebtag, 2013; Opfer, Kaufman, & Thompson, 2016; Simpson & Linder, 2014; Sztajn et al., 2012). Most of the studies in the early 2010s suggested desirable implementation and professional development rather than delving into teachers' instructional practice using the CCSSM. Since 2014, more researchers probed into teachers' within-classroom implementation of the CCSS.

Third, researchers have investigated the relationship of the CCSS to student achievement (e.g., Hiebert & Mesmer, 2013; Schmidt & Houang, 2012). These studies discussed the potential impact of the CCSS on students' learning.

Lastly, researchers have been interested in teacher perspectives on the CCSS (e.g., Burks et. al., 2015; Davis et al., 2013; Davis et al., 2014; Troia & Graham, 2016). These studies discussed conceptions of teachers in the form of an online poll in which they surveyed how teachers think about their readiness to implement the CCSS. Moreover, there has been research examining teachers' perception, knowledge, or beliefs about the CCSSM (e.g., Bostic & Matney, 2013; Cogan et al., 2013; Davis, Choppin, McDuffie, & Drake, 2013; Davis, Drake, Choppin, & McDuffie, 2014; McDuffie et. al., 2017; Olson, Olson, & Capen, 2014). For example, Bostic and Matney (2013) examined elementary and middle school teachers' perceived mathematical content needs and pedagogical needs, both related to the CCSSM, and the connection between these perceptions. Olson and his colleagues (2014) analyzed teachers' responses of the surveys associating their reading of the SMP. These studies investigated teachers' conceptions (perception, knowledge, and beliefs) that would affect their instructional practices. However, these studies were limited either to a survey method (e.g., Bostic & Matney, 2013; Burks et. al., 2015; Choppin et al., 2013; Cogan et al., 2013; Davis et al., 2013; Davis et al., 2014; Troia & Graham, 2016) or to the setting of professional development (e.g., Carney, Brendefur, Thiede, Hughes, & Sutton, 2016; Olson et al., 2014).

## 5. TEACHERS' BELIEFS

In education field, it is widely accepted that teachers' beliefs influence their behaviors in the classrooms and that it is necessary to understand teachers' beliefs structures to improve their teaching practices (Bauch, 1984; Beswick, 2012; Brookhart & Freeman, 1992; Brosnan, 1994; Ernest, 1989; Feiman-Nemser & Floden, 1986; Ferrini-Mundy, 1986; McLeod, 1989; Pajares, 1992; Philipp, 2007; Prawat, 1992; Richardson, 1996; Weinstein, 1988, 1989). As Fenstermacher (1978) predicted several decades ago, the study of teachers' beliefs for effective teaching has received great attention. In the early 1980s, researchers began to view more the importance of teachers' active and cognitive factors that would affect student learning in classrooms (Peterson, Fennema, Carpenter, & Loef, 1989).

Since the 1980s, researchers have actively studied mathematics teachers' beliefs. Studies have reported that teachers tend to view mathematics as a static body of knowledge that is skill-based, complete, and rigid (Agudelo-Valderrama, 2008; Collier, 1972; Gregg, 1995; Philipp, 2007; Steffe, 1990; Wilson, 1994). Ernest (1989) noted that "teachers' mental contents or schemas, particularly the system of beliefs concerning math and its teaching and learning" (p. 249) as key elements that influence teachers' instructional practice.

The body of research examined an impact of in- or pre-service teachers' beliefs on their instructional practices or a relationship between what teachers believe and what they do in classrooms. These studies showed some variability in degrees of the relationships. Some of them discovered that the teachers' beliefs and their mathematics teaching practice were not consistent (e.g., Raymond, 1997; Skott, 2001; Sztajn, 2003), while others found that teachers' beliefs were closely related with their teaching of mathematics (e.g., Borko, Mayfield, Marion, Flexer, & Cumbo, 1997; Guskey, 1986).

Researchers examined not only the relationships between teachers' beliefs and their instructional practices, but they also probed into other areas such as teachers' beliefs within curriculum material or curriculum reform. These studies took teachers' beliefs about reform-based teaching of mathematics into account, based on the beliefs that "teachers are key to the success of the…reform movement in U.S.A. mathematics education" (Battista, 1994, p. 462) and "teaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change" (Ernest, 1989, p. 249). Among these studies, some investigated teachers' beliefs and mathematics teaching practices in relation to the NCTM standards. These studies focused on measuring teachers' belief systems about the Standards. Some investigated in-service teachers' beliefs of the NCTM vision (e.g., Alba, 2001; Carter & Norwood, 1997; Futch & Stephens, 1997; Zollman & Mason, 1992).

Researchers have continued to examine teachers' beliefs since the CCSSM was adopted (e.g., Buehl & Beck, 2015; Carney et al., 2016; Chen et al., 2014; Davis et al., 2013; Davis et al., 2014; Felbrich, Kaiser, & Schmotz, 2014; Lui, & Bonner, 2016; Polly et al., 2013; Porter, Fusarelli, & Fusarelli, 2015; Skott, 2015). The results of these studies consistently showed the importance of teachers' beliefs about teaching and learning mathematics as well as nature of mathematics. For example, Lui and Bonner (2016) asserted that both in-service and preservice teachers had shown more constructivists than traditional beliefs. However, the participating teachers' beliefs and their conceptual or procedural knowledge didn't present significant relationships.

## III. METHODOLOGY

In order to learn a more in-depth and detailed beliefs of teachers, it is best to conduct a qualitative research. My methodological orientation is rooted in case study, focusing on the elementary school mathematics teachers in western Pennsylvania. This qualitative case study analysis was conducted through in-depth interviews to identify teachers' beliefs that might be aligned with the SMP.

A number of studies that investigated teacher' beliefs about standards employed qualitative methods such as case study. These studies employed interviews and classroom observations including vignettes and conversations of teachers and students, teacher talk, etc., to provide detailed descriptions of teacher beliefs (e.g., Alba, 2001; Battista, 1994; Brosnan et al., 1996; Collier, 1972; Porter et al., 2013; Raymond, 1997).

### 1. PARTICIPANTS

The case for this qualitative research is defined as in-service elementary school teachers who have taught mathematics in four school districts of western Pennsylvania.

**Table 1.** Profiles of the interviewed elementary school teachers

| Teachers | Years of teaching elementary school mathematics | Levels and areas of educational attainment | Teaching assignments |
|---|---|---|---|
| **Alicia** | 5 | B.S.Ed. in Early Childhood-Special Education, Pursuing M.Ed. in Mathematics Education (Elementary-Middle School Track) | 3rd and 4th grades reading and mathematics in the special education setting, co-teaching 6th grade mathematics intervention, English Language Arts (ELA), social studies, autistic and emotional support |
| **Betsy** | 5 | M.Ed. in Mathematics Education (Elementary-Middle School Track) | 2nd grade (previously, kindergarten) |
| **Cecilia** | 6 | M.Ed. in Special Education, B.S.Ed. in Early Childhood-Special Education | 4th grade mathematics |
| **Jane** | 7 | M.Ed. in Special Education, ABD in D.Ed. in Curriculum and Instruction | 3rd grade general mathematics (previously, K to 3 math coach, K to 6 mathematics and reading learning support) |
| **Daisy** | 13 | M.Ed. in Literacy Education (Elementary-Middle School Track) | Kindergarten (previously, 4th grade) |
| **Grace** | 6 (previously, middle school mathematics for 3 years) | M.Ed. in Special Education, B.S. Ed in Elementary Education | K to 5 math specialist, co-teaching K to 5 (previously, middle school mathematics) |
| **Olivia** | 20 | M.Ed. in Mathematics Education (Elementary-Middle School Track) | 5th grade mathematics (previously, 4th and 6th grades), math coach |
| **Rachel** | 34 | M.Ed. in Literacy and Early Childhood Development | Recently retired (previously, 1st grade for 10 years, mathematics instructional support for 4 years, 2nd grade for 20 years) |

## 2. DATA SOURCES AND COLLECTION

Data for this study is from an in-depth interview of each participant. Qualitative interviews "are particularly well-suited for studying people's understanding of the meanings in their lived world, describing their experiences and self-understanding, and clarifying and elaborating their own perspective on their lived world" (Kvale & Brinkmann, 2009, p. 116). The purpose of the interviews for this study was to uncover and analyze the elementary school teachers' beliefs of the SMP.

In this study, I often use the terms "in-depth interview" and "intensive interview" interchangeably. According to Seidman (2006), "the purpose of in-depth interviewing is not to get answers to questions, nor to test hypotheses, and not to 'evaluate' as the term is normally used" (p. 9). He suggested three characteristics for in-depth questions. First, in-depth interviews use open-ended questions to give the participants capacity to reconstruct their experience. Second, in-depth interviews comprise a three-interview series, because a one-time interview may not provide the researcher a full insight of the participants' experience. Third, the length of in-depth interviews is between 1 and 2 hours. Boyce and Neale (2006) defined in-depth interviewing as "a qualitative research technique that involves conducting *intensive individual interviews* with a small number of respondents to explore their perspectives on a particular idea, program, or situation [emphasis added]" (p. 3). My interviews are intensive interviews in which only eight participants were involved and questions were asked to gain detailed information about teachers' thoughts.

## 3. DATA ANALYSIS

The interviews each took 1.5 - 2 hours and all interviews were audio-recorded with participants' consent. I transcribed all eight audio-recorded interview data verbatim. Later, I categorized the statements from the verbatim transcription around the themes in the context of the research question. To minimize different, if not wrong, interpretation from the intention of the speakers, the verbatim transcription was checked with the interviewees later.

| Turn | Speakers | Talk (Activity) |
|---|---|---|
| 11 | Researcher | Okay, that's cool. In terms of the practice, what do you most value? |
| 12 | Alicia | That's a tough one. Um, I think, as far as the practice goes, again it's all problem-based learning, teaching students through experience, and I think that the best practice is teaching students through experience, rather than… |

**Figure 3.** Verbatim transcription example

In transcription, I used the "turn numbers" to easily keep track of the exchanges on the dialogues between the interviewee and the researcher (Figure 3). The odd turn numbers are the researcher's questions or comments while the even turn numbers are the interviewees' responses. Henceforth, I write an interviewee's name (in pseudonym) and turn number in the parenthesis whenever I refer the quotes from the transcripts. For example, I put (Alicia, Turn 12) for Alicia's statement cited from turn number 12.

After completing the first draft of all transcriptions, I reviewed them for two reasons: (1) to check for errors and (2) to highlight the key words. First, I made any change, if needed, to ensure the words were transferred correctly by both carefully listening to the recordings and reading the verbatim transcripts at the same time. While proofreading the transcripts, I paid close attention to the content and found the key words or key statements. I highlighted the texts that disclosed the characteristics of the participants such as to identify their mathematical instructional goals and their focused practices that aligned with the SMP.

At completion of the transcribing, I began analysis. It required re-reading the transcripts and listening the audio recordings several times. In doing so, I highlighted the key statements on the transcripts and made notes. I categorized the meaningful themes to answer the research questions for this study. I narrowed down the themes to subcategories for additional classification. Quotes from the transcripts, email exchanges, and personal communication were added to support the themes. Lastly, I reviewed the categorization multiple times before writing the results section.

## IV. RESULTS

This case study examined the eight teachers' teaching and learning beliefs that corresponded with the vision of the CCSSM. To recognize the teachers' beliefs about teaching and learning mathematics and how their beliefs align with the SMP, I paid close attention to their stated goals in teaching mathematics and their responses to the productive beliefs questions (Figure 1). In this section, I present the eight interviewed elementary school teachers' goals in teaching mathematics and their productive beliefs.

### 1. ELEMENTARY SCHOOL TEACHERS' OVERARCHING GOALS IN TEACHING MATHEMATICS

First, I speculated which SMPs were closely associated to the teachers' overarching goals in teaching mathematics. As teaching is a complex activity and teacher behavior cannot be defined with only one SMP, it is common that I relate several SMPs to describe a goal of teaching mathematics or an instructional activity that a teacher exemplified. The reason is that teaching (or learning) is not a clean-cut task that can be explained simplistically, and the SMP is situated in an intertwined teaching and learning setting. However, I pointed out the most directly connected SMPs to each participant's comment, rather than analyzing each word and any latent meaning of it to link with the SMP.

To gauge each teacher's overarching goals in teaching mathematics, I asked the following question at the beginning of the interviews: *"Throughout your experience, what is your goal in teaching mathematics?"* In this section, I discuss not only what each participant described explicitly as their goals in teaching mathematics, but also what they seemed to value implicitly through the interview conversations.

Five out of eight teachers listed "love of learning" or "love of mathematics" as a goal (Betsy, Jane, Daisy, Grace, and Olivia). For example, Betsy mentioned, "I love learning, so I guess my goal is teach kids to love learning as well" (Turn 16). Other non-mathematics-specific, non-pedagogy-related goals were to help students overcome their unconstructive view of their mathematical ability (Daisy), to have a good attitude towards learning (Grace), and to feel confidence in themselves (Olivia).

Six teachers specifically emphasized content goals: basic mathematics (Cecilia), number sense (Daisy and Rachel), be prepared for the next grade (Jane), rigor of the selected tasks or cognitively demanding tasks (Betsy and Grace), and understanding mathematics conceptually (Olivia and Rachel). The rest of the goals are pedagogical in nature: problem-based learning (Alicia), seeing mathematics as applicable or having real-life connections (Alicia Daisy, and Rachel), experiencing hands-on activities (Alicia, Rachel, and Daisy), cultivating students' problem-solving ability (Betsy and Daisy), be willing to learn and make mistakes (Grace), encountering productive struggles (Betsy and Jane), sustaining a mathematically productive discourse environment (Olivia), being challenged (Jane), being engaged (Grace), persevering (Alicia), valuing differentiated instruction (Grace), explaining students' strategies (Betsy and Jane), understanding other students' strategies (Jane and Olivia), and emulating the pedagogical model, "I do, We do, You do" (Cecilia). In the following sections, I present each teacher's overarching goals that arose during the interviews and the SMPs that align with such goals. When aligning the teachers' overarching goals of teaching mathematics with the SMP, I carefully examined the descriptions of the SMP (NGA & CCSSO, 2010a). In this report, I present some examples of the teachers' statements that aligned the specific SMPs. The in-detailed descriptions of the SMP is found in Appendix A.

Alicia emphasizes problem-based learning, hands-on learning, real-life connections, and stamina (perseverance). Her goals in teaching mathematics are closely related with pedagogical perspectives and proficiencies. She mentioned that "I really like teaching through problem-based lessons because I feel like students learn more from experiences than they do from me lecturing. Plus, teaching through problem-based learning is more valuable for the mathematical practices because then the students are able to engage with the mathematics" (Turn 8). When asked what Alicia thought her students needed to develop in their learning of mathematics, she immediately answered, "Stamina" (Turn 16). She elaborated what she meant by stamina, stating, "Something I've noticed with students is they kind of expect you to give them answers. They don't wanna struggle with the math; they want to have a quick and easy solution to problems, and I feel that, to become better at math, they need to build stamina and build the ability to struggle with math in order to learn it" (Turn. 18). Alicia's stated goals represent SMP 1. Problem-based learning, hands-on learning, and stamina match the values that SMP 1 emphasizes. SMP 1 provides the suggestions for students to understand a mathematical problem, figure out how to solve it, and keep working on it until the problem is solved. It recommends various pedagogical strategies to teachers such as "using concrete objects or pictures to help conceptualize and solve a problem" (NGA & CCSSO, 2010a, p. 6).

Betsy instills her love of learning to her students. When asked her goals of teaching mathematics, she answered, "I love learning, so I guess my goal is teach kids to love learning as well" (Turn 16). Similar to Alicia, Betsy valued problem solving, productive struggles, cognitively demanding tasks, and student explanations. Betsy's teaching goals correspond to some of the SMPs. For example, she mentioned several strategies to "teach kids to love learning" such as using writing prompts to reason, differentiating lessons, and providing problem-solving opportunities where they have to struggle to solve a problem. First, Betsy divulges her beliefs in SMP 1. She mentioned that she would "give [her students] problem-solving opportunities where they have to *struggle through* something [emphasis added]" (Betsy, Turn 20). Also, she stated her focus on "problem solving and being able to explain how they get to an answer" (Betsy, Turn 24). She emphasized writing prompts to reason and student discussion throughout the interview. This aligns closely with SMP 3 as it explicitly states that "mathematically proficient students … justify their conclusions, communicate them to others, and respond to the arguments of others" (NGA & CCSSO, 2010a, pp. 6-7). Betsy explained how she focused on students' reasoning and thinking stating, "I have a whole thing of *math writing prompts* that I like, and I also have *problem-solving games* that the kids—they loved those games. But they were *reasoning and thinking*, and I was reflecting on these…they would get in arguments…but it was fun to watch because when they would argue with each other about it, they were thinking about it. You know and *discussing* it" (Turn 196).

Cecilia focuses on basic mathematics, the pedagogical move of "I do, we do, you do," and cultivating students' independence and confidence. She believes that elementary school students' acquiring of basic math skills (adding, subtracting, multiplying, and dividing) was essential. For that purpose, she facilitates the teaching strategy of "I do, we do, you do." Known as gradual release of responsibility, this model of teaching has been the major teaching strategy in all fields of education. In this model, a teacher demonstrates how to solve a problem (I do) first, provides a guided practice (We do) next, then provides unprompted practice to the students (You do).

In more recent years, a movement of reversing this order has taken a place in student-centered education. This reverse order ("You do, We do, I do") lets students attempt to solve a problem first without teachers' guidance (You do). This requires a high level of cognitive demand of mathematical tasks that may provide multiple entry points and allow for varied solution paths (McCaffrey, 2016). In this sense, the reverse model represents the idea of the SMP better than gradual release of responsibility model of teaching.

Cecilia mentioned that her focus of teaching practice was students' independent learning. She said, "Sometimes, they instantly wanna put their hand up. 'I can't do this. I don't know how to do this.' They don't even try… 'No, go. What's the worst thing that could happen? You could be wrong. We'll fix it.'" (Cecilia, Turn 26). This focus on students' independent learning is relevant to perseverance that SMP 1 emphasizes. She continued, "So, getting them to learn they don't need to have somebody standing beside them telling them step by step, trying to think for themselves, and to not be afraid to be wrong" (Cecilia, Turn 26). This statement, however, contradicts her pedagogical focus of gradual release of responsibility (I do, We do, You do).

Jane values her students to love mathematics, to encounter productive struggles, to be challenged, and to be prepared for the next grade. She believes in students' learning from "confusion." She called it confusion and I interpreted it as *productive struggle*. It is the idea that "students grapple with the issues and are able to come up with a solution themselves, developing persistence and resilience in pursuing and attaining the learning goal or understanding" (Jackson & Lambert, 2010). In this manner, her goal for the students to "feel the confusion and be challenged" fits in SMP 1.

Jane mentioned another teaching goal as having students being ready for the next level. For this goal, she stated several things such as "a ton of modeling in mathematics, being able to explain, being able to teach someone else the strategy that they've used and what it actually means … [and to] explain why it works best for them" (Jane, Turn 8). The list of teaching approaches she mentioned assimilates SMP 1, SMP 3, and SMP 4.

Daisy strives to develop her students' number sense, to instill her love of mathematics, to help students overcome their unconstructive view of their mathematical ability, and to incorporate hands-on learning that cultivates students' problem-solving ability. Daisy teaches kindergarten. The CCSSM recognizes two instructional focal points for kindergarten mathematics teachers: "(1) representing, relating, and operating on whole numbers, initially with sets of objects; and (2) describing shapes and space. More learning time in kindergarten should be devoted to number than to other topics" (NGA & CCSSO, 2010a, p. 9). Thus, it is not surprising that Daisy's teaching goal is to build a foundation (e.g., number sense). She stated, "Now that I'm a kindergarten teacher, it is to build that foundation and love of mathematics. I want to begin that beginning of number sense. That's my goal every year is to make sure they have a solid foundation of number sense" (Turn 18). The key sentence for SMP 4 (Model with mathematics) would be "apply the mathematics they know to solve problems arising in everyday life (p. 7). Daisy's emphasis on hands-on activities and application of mathematics to everyday life, thus, are linked with SMP 4.

Grace desires her students to like mathematics, to have a good attitude toward learning, and to be willing to learn and make mistakes. Furthermore, she valued differentiated instruction, student engagement, and rigor of the selected tasks. As Grace was serving as a mathematics specialist and co-teaching K-5 in her school, her responses were more general than grade-specific. She mentioned the importance of students' positive attitude towards mathematics. For example, she stated, "I want kids to like math. I want them to think that's it's fun. I want them to like to do math and try math because there are a lot of careers they could have with math. So, I guess, a positive attitude towards math first and foremost is important to me, especially at the elementary level" (Turn 30). Grace thought students with more optimistic attitude towards math could have better opportunity to be successful in school and in their career trajectories.

The key shifts of the CCSSM can be summarized into three words: focus, coherence, and rigor (NGA & CCSSO, 2010b). Academic rigor, in particular, is the extent to which students are being intellectually challenged. Through well-planned instruction such as inquiry-based, problem-solving based, differentiated, and student-centered instruction, teachers and students can strive for academic rigor (NGA & CCSSO, 2010a). Grace's focus on students' positive attitude, differentiation in teaching, and high cognitive demanding tasks aligned with the rigor that CCSSM emphasizes. In particular, students' "willing to make mistakes" aligns with SMP 1.

Olivia values her students to love mathematics, to feel confidence in themselves, and to understand mathematics conceptually, and to sustain a mathematically productive discourse environment. She said that her biggest goals are "for children, number one, to love math as a subject area. Too often at a very young age, children decide they are not good at math and they don't like it already. And we know how important mathematics is in the world for their future careers and things like that. So, first to instill a love, and also a confidence in themselves that everybody's a mathematician. So, after that, for them to conceptually understand why math works and the patterns and things like that. I think too often children are taught that tricks and shortcuts. And they work, but the kids don't know why they work" (Turn 6). Olivia aspires for her students' love of mathematics, and instilling confidence in doing mathematics is her highest goal in teaching mathematics. She said, "everybody's a mathematician." Helping students think like a mathematician, who think more critically and understand conceptually is a primary focus of the CCSSM. Such rigor is one of the key shifts the CCSSM called for (NGA & CCSSO, 2010b).

When asked about her focus of teaching practice, Olivia expressed her passion for using mathematical discourse to improve students' reasoning. This seems to match with SMP 3, as it describes that "[students] justify their conclusions, communicate them to others, and respond to the arguments of others" (NGA & CCSSO, 2010a, pp. 6-7). She reiterated her dedication for classroom discourse later, when talking about her textbook usage. Olivia stated, "I am a huge fan of that math discourse" (Olivia, Turn 144).

Rachel focuses on her students to see mathematics as applicable and to experience hands-on activities. She said that her mathematical instructional goal was "to help children see mathematics as an everyday useful tool. Not just something to learn in school, but something they would use in everyday life. And to become comfortable with it" (Turn 8). SMP 4 articulates application of the mathematics to solve problems arising in everyday life. This is one of the mathematical proficiencies that students should develop. She also values students' number sense and conceptual understanding away from memorization. One example is her teaching of two-digit numbers. She indicated her effort to help the students understand the base-10 number system by using a calendar. "if today is December 3rd, then the children would understand everyday that's three 1s. But if it's the 23rd or the 13th, then they need one 10 and three 1s or two 10s and three 1s" (Turn 16). She explained that she taught 13 as one ten and three ones. In this specific example, Rachel's instruction corresponds with a Grade 1 content standard in Number and Operations in Base Ten: CCSS.MATH.CONTENT.1.NBT.B.2

(Understand that the two digits of a two-digit number represent amounts of tens and ones). This simple example also aligns with SMP 2, as it suggests students to attend to the meaning of quantities.

The SMPs that seemed to align with the participants' teaching goals are SMP 1, SMP 2, SMP 3, and SMP 4. Four of the eight teachers' (Alicia, Betsy, Cecilia, and Jane) goals are closely related with the substance of SMP 1. They emphasized their focus of their students' "stamina," "struggle to solve mathematics," "independence," and "learning from confusion." There was one teacher (Rachel) whose teaching goal aligns with SMP 2, as she attends to the meaning of the quantities. Three teachers (Betsy, Jane, and Olivia) highly valued the idea of SMP 3. The key words that stood out for SMP 3 are "reasoning," "explaining the strategy," "justifying the conclusion," "communicating," "responding to the arguments of others," and "math discourse." Lastly, three teachers' (Jane, Daisy, and Rachel) responses are linked to SMP 4, as they highlighted the application of mathematics in daily life (Table 2).

**Table 2.** Teachers' overarching goals and the SMPs

| SMP | Teacher | Key Words |
|---|---|---|
| SMP 1 | Alicia | Stamina |
| | Betsy | Struggle through problems |
| | Cecilia | Independence |
| | Jane | Learning from confusion |
| SMP 2 | Rachel | The meaning of the quantities |
| SMP 3 | Betsy | Reasoning |
| | Jane | Explain the strategy |
| | Olivia | Justifying, communicating, math discourse |
| SMP 4 | Jane | Modeling in mathematics |
| | Daisy | Apply mathematics … in everyday life |
| | Rachel | Mathematics as an everyday useful tool |

## 2. ELEMENTARY SCHOOL TEACHERS' PRODUCTIVE BELIEFS

To answer the research question, "How do the participating elementary school teachers' beliefs align with the SMP?" I not only probed teachers' overarching teaching goals, but I also examined their productive and unproductive beliefs. Productive beliefs, as delineated in *Principles to Actions* (NCTM, 2014), undergird effective instructional practice and allow students' access to a set of important practices such as the eight SMPs. During the interview, each participant sorted twelve cards with descriptions of productive beliefs and unproductive beliefs (Figure 1) on the sorting sheet. The twelve beliefs statements either describe teachers' beliefs that may affect teachers' instructional practices to teach mathematics or students' beliefs that may influence their learning of mathematics. Six of the beliefs represent the productive beliefs about teaching and learning mathematics and the other six beliefs statements are their counterparts.

The productive statements are closely related with the SMP. Again, it is not meaningful to distinguish one SMP from another, because of the complexity of mathematical teaching and learning. However, this alignment implies which SMPs the participating elementary school teachers might pay more attention to than others. A productive beliefs statement explains a more than one SMP as shown in Table 3. The connecting SMPs for most of the productive beliefs are the first four SMPs. It is interesting that the SMPs that align with both the participating teachers' overarching goals and the productive beliefs are SMP 1, SMP 2, SMP3, and SMP 4. In particular, SMP 1 seemed to be imbedded in each of the six productive beliefs.

**Table 3.** Relationships between productive beliefs and the SMPs

| **Productive Beliefs** | **SMPs** |
|---|---|
| Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse. | SMP 1 Make sense of problems and persevere in solving them.<br>SMP 2 Reason abstractly and quantitatively.<br>*SMP 3 Construct viable arguments and critique the reasoning of others.* |
| All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures. | SMP 1 Make sense of problems and persevere in solving them.<br>SMP 2 Reason abstractly and quantitatively.<br>*SMP 3 Construct viable arguments and critique the reasoning of others.* |
| Students can learn mathematics through exploring and solving contextual and mathematical problems. | SMP 1 Make sense of problems and persevere in solving them.<br>SMP 2 Reason abstractly and quantitatively.<br>SMP 4 Model with mathematics.<br>*SMP 6 Attend to precision.* |
| The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics. | SMP 1 Make sense of problems and persevere in solving them.<br>SMP 2 Reason abstractly and quantitatively.<br>*SMP 3 Construct viable arguments and critique the reasoning of others.* |
| The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others. | SMP 1 Make sense of problems and persevere in solving them.<br>SMP 2 Reason abstractly and quantitatively.<br>SMP 3 Construct viable arguments and critique the reasoning of others.<br>*SMP 4 Model with mathematics.* |
| An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics. | SMP 1 Make sense of problems and persevere in solving them. |

When asked to sort the beliefs cards into two piles (agree and disagree piles), teachers read each card carefully and placed it on either agree or disagree section. Of eight, six teachers (Alicia, Betsy, Cecilia, Jane, Grace, and Olivia) exhibited their agreement to all six productive belief statements and disagreement to all six unproductive belief statements. However, two teachers (Daisy and Rachel) agreed with one or two unproductive beliefs.

Both Daisy and Rachel agreed with the unproductive belief statement, "Students can learn to apply mathematics only after they have mastered the basic skills." In contrast, the productive beliefs statement expresses that "Students can learn mathematics through exploring and solving contextual and mathematical problems." The National Council of Teachers of Mathematics (NCTM) affirmed problem solving as "the focus" of mathematics education in schools (1989, p. 6). The NCTM Standards (1989, 1991) asserted problem solving in mathematics instruction as an essential aspect for students' learning of mathematical concepts and skills. Hiebert and his colleagues (1996) argued that students should be engaged in problem solving, rather than mastering skills to learn mathematics. They emphasized problem solving in mathematical instruction in which "students should be allowed and encouraged to problematize what they study, to define problems that elicit their curiosities and sense-making skills" (p. 12). Furthermore, Carpenter and others (1999) acknowledged that learning computational skills and conceptual understanding of mathematics go hand-in-hand. The CCSSM continued to advocate for the balance of conceptual understanding through problem solving and procedural skills. *Principles to Actions* reiterated that a teacher's strong beliefs about learning of mathematics through problem solving-related tasks enhance their implementation of effective instructional practice (NCTM, 2014).

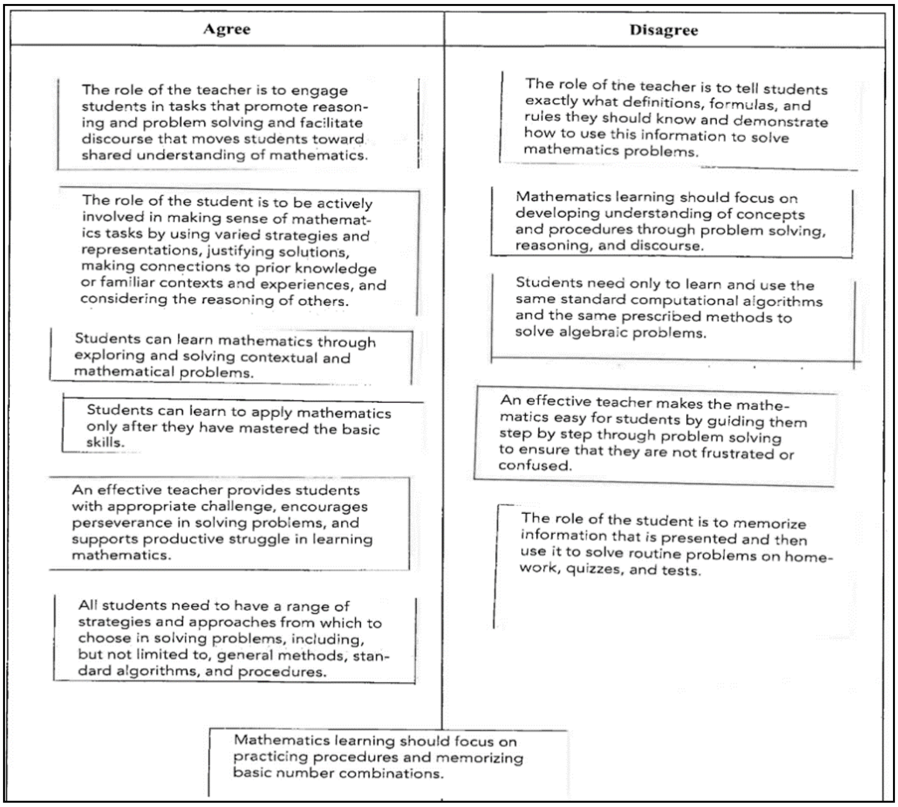


**Figure 4.** Daisy's agreement and disagreement on productive beliefs

Especially, Daisy partially agreed with the productive belief statement, "Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse." She said, "I agree that you definitely should focus on understanding the concepts through problem solving and reasoning, but I feel like math can be learned in so many other ways as well. I mean, sometimes you're learning math just by chance. We learn math through play, through … So, I felt like the word 'should' was the strong word in there, and it doesn't always have to be that way. Sometimes we're learning math …" (Daisy, Turn 36). During this exchange, she re-read the statement. This made her change her mind to agree with the belief statement. She said, "I don't know, now that I read it more, I feel it could be an agreement" (Daisy, Turn 36). Daisy might not have paid attention to the meaning of the whole phrase initially, clinging to the word, "should," rather than the phrase, "focus on developing understanding of concepts and procedures." She became more open to agreeing with the statement later, but not completely. She placed the belief statement card in-between agree and disagree, slightly more toward agree (See Figure 4).

When asked about their overall thinking about the SMP, the teachers chorused that they loved the standards. For example, Alicia exhibited her positive beliefs and attitude about the SMP as follows:

> Overall, I love the Standards for Mathematical Practice. I think it's something teachers need to be more aware of. I think I'm gonna go home and write these on a poster and stick them in my classroom so that I see them more often. It's a very good guideline to guide teachers teaching, and it's a very good guideline to guide students thinking. It's just necessary to be a well-rounded mathematician to be able to do all eight of these (Turn 266)

Jane showed her favor toward the SMP, but she stressed out the importance of the standards for students' career and lifelong learning opportunity. In fact, the CCSSM (CCSSM, 2010) emphasizes the knowledge, skills, and habits that students need to develop for college, career, and life.

As a mathematics liason for her school district, Grace affirmed how SMP could impact the students to become stronger mathematics thinkers, stating, "I like them (the SMP). I think that they're great. I think that if we can get kids to do them successfully, we're gonna have stronger math thinkers" (Turn 140). She further reflected the importance of teachers' experiences and belief system to facilitate the SMP despite the challenges of learning and implementing the standards as follows:

> It is challenging, and it takes a lot of thought. It isn't challenging like it can't be done, but I think that the thought and the preparation and planning it takes, there is a need for that. But I think that when you have the perspective on it of what you're looking for and where you're trying to go, it just becomes a habit. But I think that if someone's not in that habit, it can be a shift (Turn 140).

Even the teachers who have never read the SMP before the interviews expressed their positive attitude towards the SMP. For example, Cecilia indicated the changes of her beliefs and attitude towards the SMP as follws:

> But the more I do it, the more I start to see that that's because there's lots of kids who can't do it the old way. There are some that need to do it this way. So, I have

> gotten a whole lot more open-minded about the number of different approaches (Turn 122).

Rachel also mentioned that her practices aligned well with the SMP because her teaching philosophy associated closely with the SMP. Further, she stated her optimistic impression about the SMP after her reading of the descriptions of the SMP, "it's very appropriate. I can see where it is appropriate for different grade levels" (Turn 160).

## V. CONCLUSION

This study analyzed a case of elementary school teachers' beliefs and the aligning SMPs with their beliefs. This study is significant as it bridges the gap between teacher conceptions studies of NCTM Standards and the CCSSM. As identified through the review of literature, numerous studies around the CCSSM have focused on the content of the standards, curriculum alignment, professional development and implementation, and the effects of the CCSSM on instructional practices and student outcomes. Studies investigating elementary school teachers' beliefs about the SMP are limited. Moreover, most of these studies used survey methods to examine teachers' beliefs on a large scale. The in-depth interviews I employed for this case study was an efficient tool for deep scrutiny into eight teachers' beliefs of the SMP.

The findings of this study portrayed how elementary school teachers' beliefs were aligned with some the SMPs. The examples through their own voices supported the findings. The findings contribute to the line of the studies that have investigated teachers' beliefs about standards. Additionally, the findings of such literature indicate that teachers' beliefs are the critical factors for teachers' implementation of the SMP, which may ultimately affect students' learning of mathematics. It is likely that examining teachers' beliefs of the SMP could be found interesting not only to teachers, schools, and school districts, but also to other stakeholders such as mathematics educators.

The findings reported that the participating teachers had a fairly coherent set of beliefs which predicted their SMP-aligned instructional practices. However, the alignment of the SMP was limited to the first few SMPs. Colen (2019) asserted elementary school teachers understood SMP 1 (Make sense of problems and persevere in solving them) and SMP 3 (Construct viable arguments and critique the reasoning of others) more than other SMPs. She surmised some reasons for teachers' understanding of SMP 1. One of the reasons is due to the teachers' perception that SMP 1 is "relevant" for their elementary school students. For example, Daisy remarked: "I want to say I'm a good teacher in making sense of problems. But really, the kids do a lot of hands-on. I want them to be confident and to be able to: "Does this make sense?" and "Why does this make sense?" They do use concrete objects and pictures, but I think we may able to move a little higher than that (Daisy, Turn 160)." Grace added: "Perseverance through problem solving is that the kid won't give up, but I also think that it means you're willing to try as well. If you see something challenging or new, you're willing to take it on. And if it's something that's difficult or long or doesn't have a clear end in sight or a precise solution, they're open to keep working on it, and they have a desire to keep trying (Grace, Turn 90)." In fact, the SMP 1 aligned well with the participating teachers' overarching goals in teaching mathematics. Another reason can be that when encountering the CCSSM, the teachers may have paid the most attention to the first few standards of the SMP. It is plausible that the teachers have read SMP 1 the most frequently because it appears at the beginning of the CCSSM document. Both Betsy and Grace utilized the phrase, "problem solving" in mathematics, and the other teachers might also be familiar with the phrase as it has been accentuated not only in the previous standards (e.g., PSSM [NCTM, 2000]), but also in numerous research (Billstein, Libeskind, Lott, & Boschmans, 2004; Carpenter et al., 1999; Goldin, 2000; Hart, 1989; Hiebert et al., 1996; Lester, Garofalo, & Kroll, 1989). However, the focus on the "perseverance" is almost new to the teachers. Olivia (personal communication, December 3, 2018) stated that while it was very important in their learning, it was not easy to foster in students. The last plausible reason can be that the first four SMPs address less subject-specific standards. Often elementary school teachers are assigned to teach multiple subjects. The participating elementary school teachers might feel more comfortable with the SMPs that address less mathematics-specific standards.

To extend this line of study, I propose future studies that examine the relationship between teachers' beliefs about the SMP and their practice. The previous research has shown that teachers' beliefs and their practices were not always consistent (Cooney, 1985; Raymond, 1997; Thompson, 1984). Studying the relationship between the teachers' beliefs and their practices about the SMP could provide to the teachers, mathematics educators, and professional development program developers.

I call for a collaboration of the work from every stakeholder including teachers, mathematics educators, professional development providers, and standards writers to improve not only the standards themselves, but also teachers' awareness and understanding of the standards. It was hopeful to hear the participating teachers chorusing their changes of instructional practices over time. Many of the teachers appreciated far-reaching teacher education programs (specifically post-baccalaureate programs in mathematics education), abundant on-line resources from reputable organizations such as NCTM and PDE SAS, professional development programs offered by their schools or school districts as well as some universities, and the advice from peer teachers. Most of all, I value teachers' efforts and self-motivation to learn more about the SMP to improve their own mathematical instructions.

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## APPENDIX A

Descriptions of the Standards for Mathematical Practice (NGA & CCSSO, 2010a)

**1. Make sense of problems and persevere in solving them**: Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**2. Reason abstractly and quantitatively:** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; knowing and flexibly using different properties of operations and objects.

**3. Construct viable arguments and critique the reasoning of others:** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

**4. Model with mathematics:** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**5. Use appropriate tools strategically:** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**6. Attend to precision:** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7. Look for and make use of structure:** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

**8. Look for and express regularity in repeated reasoning:** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.