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CERTAIN FORMULAS INVOLVING A MULTI-INDEX MITTAG-LEFFLER FUNCTION

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Abstract. Since Mittag-Leffler introduced the so-called Mittag-Leffler function, a number of its extensions have been investigated due mainly to their applications in a variety of research subjects. Shukla and Prajapati presented a lot of formulas involving a generalized Mittag-Leffler function in a systematic manner. Motivated mainly by Shukla and Prajapati's work, we aim to investigate a generalized multi-index Mittag-Leffler function and, among possible numerous formulas, choose to present several formulas involving this generalized multi-index Mittag-Leffler function such as a recurrence formula, derivative formula, three integral transformation formulas. The results presented here, being general, are pointed out to reduce to yield relatively simple formulas including known ones.

1. Introduction and Preliminaries

Gösta Mittag-Leffler [5] (see also [4]) introduced and investigated the following so-called Mittag-Leffler function (M-L function)

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)} \quad (z \in \mathbb{C};\ \Re(\alpha) > 0). \tag{1}$$

Here and in the following, let $\mathbb{C}$, $\mathbb{R}^+$, $\mathbb{Z}_0^-$, and $\mathbb{N}$ be the sets of complex numbers, positive real numbers, non-positive integers, and positive integers, respectively, and let $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. Since then, the function (1) has been extended in a variety of ways. The function (1) and its various extensions have been observed to play an important role in applications of diverse research areas in, for example, physics, statistics, chemistry, social science, and engineering (see, e.g., [3], [6], [7], [9], [10], [11], [12], [15], [17] and the references cited therein).

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Among numerous extensions of (1), we recall some of them. Wiman [19] presented the following extension

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)} \quad (z,\, \beta \in \mathbb{C};\, \Re(\alpha) > 0). \tag{2}$$

The function (2) was extended by Prabhakar [7] as follows:

$$E_{\alpha,\beta}^{\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\alpha n + \beta)} \frac{z^n}{n!} \quad (z, \beta, \gamma \in \mathbb{C};\, \Re(\alpha) > 0), \tag{3}$$

where $(\lambda)_\nu$ is the Pochhammer symbol defined (for $\lambda,\, \nu \in \mathbb{C}$) by (see, e.g., [13, p. 2 and p. 5]):

$$\begin{aligned} (\lambda)_\nu :&= \frac{\Gamma(\lambda + \nu)}{\Gamma(\lambda)} \quad (\lambda + \nu \in \mathbb{C} \setminus \mathbb{Z}_0^-) \\ &= \begin{cases} 1 & (\nu = 0) \\ \lambda(\lambda+1)\cdots(\lambda+n-1) & (\nu = n \in \mathbb{N}) \end{cases} \end{aligned} \tag{4}$$

and $\Gamma(\lambda)$ is the familiar Gamma function whose Euler's integral is given by (see, e.g., [13, Section 1.1])

$$\Gamma(\lambda) = \int_0^\infty t^{\lambda-1}\, e^{-t}\, dt \quad (\Re(\lambda) > 0). \tag{5}$$

Srivastava and Tomovski [17] gave the following further extension of (3)

$$E_{\alpha,\beta}^{\gamma,\kappa}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{\kappa n}}{\Gamma(\alpha n + \beta)} \frac{z^n}{n!} \quad (z, \beta, \gamma \in \mathbb{C};\, \Re(\alpha) > \max\{0, \Re(\kappa) - 1\};\, \Re(\kappa) > 0). \tag{6}$$

In a systematic manner, Shukla and Prajapati [12] presented a number of interesting formulas involving the special case of (6) when

$$\kappa = q \ \ (q \in (0,1) \cup \mathbb{N}) \quad \text{and} \quad \min\{\Re(\beta), \Re(\gamma)\} > 0. \tag{7}$$

Motivated essentially by the work [12], we aim to investigate the generalized multi-index M-L function defined by (see [10, 9, 11])

$$E_{(\alpha_j,\beta_j)_m}^{\gamma,\kappa}[z] = E_{\gamma,\kappa}\left[(\alpha_j, \beta_j)_{j=1}^m; z\right] = \sum_{n=0}^{\infty} \frac{(\gamma)_{\kappa n}}{\prod_{j=1}^m \Gamma(\alpha_j n + \beta_j)} \frac{z^n}{n!} \tag{8}$$

$$\Big(z,\, \gamma,\, \kappa,\, \beta_j \in \mathbb{C},\ \Re(\alpha_j) > 0 \ (j = 1, \ldots, m,\, m \in \mathbb{N}); \quad \Re\Big(\sum_{j=1}^m \alpha_j\Big) > \Re(\kappa) - 1\Big)$$

and, in view of [12], among numerous formulas, choose to establish several formulas such as a recurrence formula, derivative formula, integral transformations of beta, Laplace, and Whittaker.

Remark 1. Under the given conditions, the function in (8) is an *entire* function of z. Indeed, recall the following known asymptotic formula (see, e.g., [13, p. 7])

$$\frac{\Gamma(z+\alpha)}{\Gamma(z+\beta)} = z^{\alpha-\beta}\left[1+\frac{(\alpha-\beta)(\alpha+\beta-1)}{2z}+O\left(z^{-2}\right)\right] \tag{9}$$

$$(|z|\to\infty;\ |\arg z|<\pi;\ |\arg(z+\alpha)|<\pi;\ \alpha,\,\beta\in\mathbb{C}).$$

Let a_n be the coefficient of z^n in the series (8). Then we have

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{n+1}\left|\frac{\Gamma(\kappa n+\gamma+\kappa)}{\Gamma(\kappa n+\gamma)}\prod_{j=1}^{m}\frac{\Gamma(\alpha_j n+\beta_j)}{\Gamma(\alpha_j n+\beta_j+\alpha_j)}\right|. \tag{10}$$

Using the formula (9) in (10) and noting $1/(n+1) = n/(n+1)\cdot 1/n$, we find

$$\left|\frac{a_{n+1}}{a_n}\right| \sim \left|\kappa^{\kappa}\Big/\prod_{j=1}^{m}\alpha_j^{\alpha_j}\right| n^{\Re(\kappa)-1-\Re\left(\sum_{j=1}^{m}\alpha_j\right)} \qquad (n\to\infty). \tag{11}$$

Let R be the radius convergence of the series in (8). Taking the limit in (11) with the condition in (8), we have

$$\frac{1}{\mathsf{R}} = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0.$$

Hence $\mathsf{R}=\infty$ and the function in (8) is an entire function of z. □

A generalization of the generalized hypergeometric series ${}_pF_q$ (see, e.g., [13, Section 1.5]; see also [8]) is due to Fox [1] and Wright [20, 21, 22] who had studied the asymptotic expansion of the generalized (Wright) hypergeometric function defined by (see, e.g., [16, p. 21] and [2, p. 67, Eq.(1.12.68)])

$${}_p\Psi_q\left[\begin{matrix}(\alpha_1,\,A_1),\,\ldots,\,(\alpha_p,\,A_p)\,;\\ (\beta_1,\,B_1),\,\ldots,\,(\beta_q,\,B_q)\,;\end{matrix}\ z\right] = \sum_{k=0}^{\infty}\frac{\prod\limits_{j=1}^{p}\Gamma\left(\alpha_j+A_j\,k\right)}{\prod\limits_{j=1}^{q}\Gamma\left(\beta_j+B_j\,k\right)}\,\frac{z^k}{k!}, \tag{12}$$

where the coefficients $A_1,\,\ldots,\,A_p\in\mathbb{R}^+$ and $B_1,\,\ldots,\,B_q\in\mathbb{R}^+$ such that

$$1+\sum_{j=1}^{q}B_j-\sum_{j=1}^{p}A_j \geqq 0. \tag{13}$$

A special case of (12) is

$${}_p\Psi_q\left[\begin{matrix}(\alpha_1,\,1),\,\ldots,\,(\alpha_p,\,1)\,;\\ (\beta_1,\,1),\,\ldots,\,(\beta_q,\,1)\,;\end{matrix}\ z\right] = \frac{\prod_{j=1}^{p}\Gamma\left(\alpha_j\right)}{\prod_{j=1}^{q}\Gamma\left(\beta_j\right)}\,{}_pF_q\left[\begin{matrix}\alpha_1,\,\ldots,\,\alpha_p\,;\\ \beta_1,\,\ldots,\,\beta_q\,;\end{matrix}\ z\right]. \tag{14}$$

The beta function $B(\alpha, \beta)$ is defined by (see, e.g., [13, Section 1.5])

$$B(\alpha,\,\beta) = \begin{cases} \displaystyle\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt & (\Re(\alpha)>0;\ \Re(\beta)>0) \\ \dfrac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)} & \left(\alpha,\,\beta \in \mathbb{C}\setminus\mathbb{Z}_0^-\right). \end{cases} \tag{15}$$

Euler (Beta) transform of the function f is defined by

$$B\{f;a,b\} = \int_0^1 t^{a-1}(1-t)^{b-1}\,f(t)\,dt. \tag{16}$$

The Laplace transform of f is given by

$$F(s) = \mathcal{L}(f) = \mathcal{L}(f(t)) = \int_0^\infty e^{-st}\,f(t)\,dt = \lim_{\tau\to\infty}\int_0^\tau e^{-st}\,f(t)\,dt \tag{17}$$

whenever the limit exits (as a finite real or complex number).

2. Certain properties of the generalized multi-index Mittag-Leffler function (8)

We begin by presenting a recurrence formula of the function (8) regarding the parameter γ, which is asserted in the following theorem.

Theorem 2.1. *Let $z,\,\gamma,\,\kappa,\,\beta_j \in \mathbb{C}$, and $\Re(\alpha_j)>0$ $(j=1,\ldots,m,\ m\in\mathbb{N})$. Also let*

$$\Re\Big(\sum_{j=1}^m \alpha_j\Big) > \Re(\kappa)-1.$$

Then

$$E^{\gamma,\kappa}_{(\alpha_j,\beta_j-\alpha_j)_m}[z] - E^{\gamma-1,\kappa}_{(\alpha_j,\beta_j-\alpha_j)_m}[z] = (\gamma)_{\kappa-1}\,z\,E^{\gamma-1+\kappa,\kappa}_{(\alpha_j,\beta_j)_m}[z]. \tag{18}$$

Proof. Using definition of the function (8) with the aid of

$$(\gamma)_{\kappa n} - (\gamma-1)_{\kappa n} = \frac{\kappa n}{\gamma-1}\,(\gamma-1)_{\kappa n} \quad (n\in\mathbb{N}_0)$$

and

$$(\gamma)_{\kappa(n+1)} = \frac{\Gamma(\gamma+\kappa)}{\Gamma(\gamma)}\,(\gamma+\kappa)_{\kappa n} = (\gamma)_\kappa\,(\gamma+\kappa)_{\kappa n} \quad (n\in\mathbb{N}_0), \tag{19}$$

we prove (18).

□

We give higher order derivative formulas for the function (8) in the following theorem.

Theorem 2.2. *The following formulas hold.*

(**i**) *Let* z, γ, κ, $\beta_j \in \mathbb{C}$, *and* $\Re(\alpha_j) > 0$ $(j = 1, \ldots, m,\ m \in \mathbb{N})$. *Also let*

$$\Re\Big(\sum_{j=1}^{m} \alpha_j\Big) > \Re(\kappa) - 1.$$

Then

$$\left(\frac{d}{dz}\right)^{\ell} E^{\gamma,\kappa}_{(\alpha_j,\beta_j)_m}[z] = (\gamma)_{\ell\kappa}\, E^{\gamma+\ell\kappa,\kappa}_{(\alpha_j,\ell\alpha_j+\beta_j)_m}[z] \quad (\ell \in \mathbb{N}). \tag{20}$$

(**ii**) *Let* ℓ, $m \in \mathbb{N}$, z, w, γ, λ, $\beta_j \in \mathbb{C}$ *with* $-\pi < \arg z < \pi$, *and* κ, μ, $\alpha_j \in \mathbb{R}^+$ $(j = 1, \ldots, m)$. *Also let*

$$\sum_{j=1}^{m} \alpha_j > \kappa - 1.$$

Then

$$\begin{aligned}\left(\frac{d}{dz}\right)^{\ell} &\left\{z^{\lambda-1} E^{\gamma,\kappa}_{(\alpha_j,\beta_j)_m}[wz^{\mu}]\right\} \\ &= \frac{z^{\lambda-\ell-1}}{\Gamma(\gamma)}\ {}_2\Psi_{m+1}\left[\begin{array}{r} (\gamma,\kappa), (\lambda,\mu); \\ (\beta_1,\alpha_1), \ldots, (\beta_m,\alpha_m),\ (\lambda-\ell,\mu); \end{array}\ wz^{\mu}\right].\end{aligned} \tag{21}$$

Proof. In view of Remark 1, we can differentiate the series in (8) term by term, ℓ times in succession and use (19) in each step, we obtain (20).

Using

$$\left(\frac{d}{dz}\right)^{\ell} z^{\mu n+\lambda-1} = \frac{\Gamma(\mu n+\lambda)}{\Gamma(\mu n+\lambda-\ell)}\, z^{\mu n+\lambda-\ell-1},$$

and (12), similarly as in (**i**), we get (21). □

3. Integral transforms of the function (8)

Among various integral transforms (see, e.g., [12]), here, we choose to present three integral formulas involving the function (8), which are Euler, Laplace, and Whittaker transforms asserted in Theorems 3.1, 3.2, and 3.3, respectively.

Theorem 3.1. *Let* $m \in \mathbb{N}$, κ, σ, $\alpha_j \in \mathbb{R}^+$, z, $\beta_j \in \mathbb{C}$ $(j = 1, \ldots, m)$, *and* $\Re(a) > 0$, $\Re(b) > 0$. *Also let*

$$\sum_{j=1}^{m} \alpha_j > \kappa - 1.$$

Then

$$\begin{aligned}\int_0^1 &t^{a-1}(1-t)^{b-1} E^{\gamma,\kappa}_{(\alpha_j,\beta_j)_m}(zt^{\sigma})\, dt \\ &= \frac{\Gamma(b)}{\Gamma(\gamma)}\,{}_2\Psi_{m+1}\left[\begin{array}{r} (\gamma,\kappa), (a,\sigma); \\ (\beta_1,\alpha_1), \ldots, (\beta_m,\alpha_m),\ (a+b,\sigma); \end{array}\ z\right].\end{aligned} \tag{22}$$

Proof. Using the series (8) in the integrand of the integral (22), in view of Remark 1, under the given conditions here, we can integrate the resulting integral term by term. Then, using (15), we obtain (22). □

Theorem 3.2. *Let $m \in \mathbb{N}$, $\kappa, \sigma, \alpha_j \in \mathbb{R}^+$, $z, \ \beta_j \in \mathbb{C}$ $(j = 1, \ldots, m)$, and $\Re(a) > 0$, $\Re(b) > 0$, $\Re(s) > 0$. Also let*

$$\sum_{j=1}^{m} \alpha_j > \kappa - 1.$$

Then

$$\begin{aligned} &\int_0^\infty t^{a-1} e^{-st} E^{\gamma,\kappa}_{(\alpha_j,\beta_j)_m}(zt^\sigma)\, dt \\ &\quad = \frac{s^{-a}}{\Gamma(\gamma)} \, {}_2\Psi_m \left[\begin{array}{c} (\gamma,\kappa), (a,\sigma); \\ (\beta_1,\alpha_1), \ldots, (\beta_m,\alpha_m); \end{array} zs^{-\sigma} \right]. \end{aligned} \tag{23}$$

Proof. Using (5), similarly as in the proof of Theorem 3.1, we can establish the result here. We omit the details. □

We recall a known integral formula involving the Whittaker function $W_{\lambda,\mu}$ (see, e.g., [12, p. 809] ; see also [18, Chapter XVI])

$$\int_0^\infty e^{-t/2}\, t^{\nu-1}\, W_{\lambda,\mu}(t)\, dt = \frac{\Gamma\left(\frac{1}{2}+\mu+\nu\right) \Gamma\left(\frac{1}{2}-\mu+\nu\right)}{\Gamma\left(1-\lambda+\nu\right)} \quad \left(\Re(\nu \pm \mu) > -\frac{1}{2}\right). \tag{24}$$

Theorem 3.3. *Let $m \in \mathbb{N}$, $\delta, \kappa, \alpha_j \in \mathbb{R}^+$, $z, \ \beta_j \in \mathbb{C}$ $(j = 1, \ldots, m)$, and $\Re(\rho) > 0$, $\Re(p) > 0$, $\Re(\rho \pm \mu) > -\frac{1}{2}$. Also let*

$$\sum_{j=1}^{m} \alpha_j > \kappa - 1.$$

Then

$$\begin{aligned} &\int_0^\infty t^{\rho-1} e^{-\frac{1}{2}pt}\, W_{\lambda,\mu}(pt) E^{\gamma,\kappa}_{(\alpha_j,\beta_j)_m}(zt^\delta)\, dt \\ &= \frac{p^{-\rho}}{\Gamma(\gamma)} \, {}_3\Psi_{m+1} \left[\begin{array}{c} (\gamma,\kappa), (\frac{1}{2}+\mu+\rho,\delta), (\frac{1}{2}-\mu+\rho,\delta); \\ \\ (\beta_1,\alpha_1), \ldots, (\beta_m,\alpha_m),\ (1-\lambda+\rho,\delta); \end{array} p^{-\delta}\, z \right]. \end{aligned} \tag{25}$$

Proof. Using (24), similarly as in the proof of Theorem 3.1, we can establish the result here. We omit the details. □

4. Special cases and remarks

If κ, $\alpha_j \in \mathbb{R}^+$ $(j = 1, \ldots, m)$ in (8), then

$$E^{\gamma,\kappa}_{(\alpha_j,\beta_j)_m}[z] = \frac{1}{\Gamma(\gamma)}\, {}_1\Psi_m \left[\begin{array}{r} (\gamma, \kappa); \\ (\beta_1, \alpha_1), \ldots, (\beta_m, \alpha_m); \end{array} z \right]. \tag{26}$$

Further, if κ, $\alpha_j \in \mathbb{N}$ $(j = 1, \ldots, m)$ in (8), by using the multiplication formula for the gamma function (see, e.g., [13, p. 6, Eq. (31)]), we can express the series (8) in terms of the generalized hypergeometric function ${}_pF_q$. So can the results in Theorems 3.1, 3.2 and 3.3 when κ, σ, δ, $\alpha_j \in \mathbb{N}$ $(j = 1, \ldots, m)$.

The results presented here, being general, can be reduced to yield relatively simple formulas. For example, the particular cases of (22), (23), and (25) when $m = 1$ yield, respectively, the results in [12, p. 806, Eq. (5.1.1)], [12, p. 807, Eq.(5.2.1)], and [12, p. 809, Eq. (5.4.1)].

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