

ON EXTREMALLY DISCONNECTED SPACES VIA m -STRUCTURES

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ABSTRACT. In this paper, we introduce a modification of extremally disconnected spaces which is said to be m -extremally disconnected. And we obtain many characterizations of m -extremally disconnected spaces. The concepts of $*$ -extremally disconnected spaces, $*$ -hyperconnected spaces, and generalized hyperconnectedness are as examples for this paper.

1. Introduction

In this paper, we said a topological space (X, τ) with a minimal structure m_X [11] to be m -extremally disconnected if $mCl(U)$ is open for every open set U of (X, τ) . We obtain many characterizations of m -extremally disconnected spaces. It follows from simple examples that m -extremal disconnectedness and extremal disconnectedness are independent. However, if $m_X = SO(X, \tau)$ or $SPO(X, \tau)$, then the mixed space (X, τ, m_X) is m -extremally disconnected for every topological space (X, τ) . Moreover, if $m_X = \alpha(X, \tau)$, $PO(X, \tau)$ or $BO(X, \tau)$, then (X, τ, m_X) is m -extremally disconnected for every extremally disconnected space (X, τ) . The concepts of $*$ -extremally disconnected spaces, $*$ -hyperconnected spaces, and generalized hyperconnectedness are as examples for this paper. Recently papers [2–5] have introduced some new classes of sets via m -structures.

2. Minimal structures

Definition 2.1. Let X be a nonempty set and $\mathcal{P}(X)$ the power set of X . A subfamily m_X of $\mathcal{P}(X)$ is called a *minimal structure* (briefly *m -structure*) on X [17] if $\emptyset \in m_X$ and $X \in m_X$. Each member of m_X is said to be m_X -open and the complement of an m_X -open set is said to be m_X -closed.

Definition 2.2. Let (X, τ) be a topological space. A subset A of X is said to be

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- (1) α -open [16] if $A \subset \text{Int}(\text{Cl}(\text{Int}(A)))$,
- (2) semi-open [13] if $A \subset \text{Cl}(\text{Int}(A))$,
- (3) preopen [15] if $A \subset \text{Int}(\text{Cl}(A))$,
- (4) b -open [7] if $A \subset \text{Int}(\text{Cl}(A)) \cup \text{Cl}(\text{Int}(A))$,
- (5) β -open [1] or semi-preopen [6] if $A \subset \text{Cl}(\text{Int}(\text{Cl}(A)))$.

The family of all α -open (resp. semi-open, preopen, b -open, semi-preopen) sets in (X, τ) is denoted by $\alpha(X)$ (resp. $\text{SO}(X)$, $\text{PO}(X)$, $\text{BO}(X)$, $\text{SPO}(X)$).

Definition 2.3. Let X be a nonempty set and m_X an m -structure on X . For a subset A of X , the m_X -closure of A and the m_X -interior of A are defined in [14] as follows:

- (1) $\text{mCl}(A) = \cap\{F : A \subset F, X \setminus F \in m_X\}$,
- (2) $\text{mInt}(A) = \cup\{U : U \subset A, U \in m_X\}$.

Remark 2.4. Let (X, τ) be a topological space and A a subset of X . If $m_X = \tau$ (resp. $\text{SO}(X)$, $\text{PO}(X)$, $\text{BO}(X)$, $\text{SPO}(X)$), then we have

- (1) $\text{mCl}(A) = \text{Cl}(A)$ (resp. $\text{sCl}(A)$, $\text{pCl}(A)$, $\text{bCl}(A)$, $\text{spCl}(A)$),
- (2) $\text{mInt}(A) = \text{Int}(A)$ (resp. $\text{sInt}(A)$, $\text{pInt}(A)$, $\text{bInt}(A)$, $\text{spInt}(A)$).

Lemma 2.5 (Maki et al. [14]). *Let X be a nonempty set and m_X a minimal structure on X . For subsets A and B of X , the following properties hold:*

- (1) $\text{mCl}(X \setminus A) = X \setminus \text{mInt}(A)$ and $\text{mInt}(X \setminus A) = X \setminus \text{mCl}(A)$,
- (2) If $(X \setminus A) \in m_X$, then $\text{mCl}(A) = A$ and if $A \in m_X$, then $\text{mInt}(A) = A$,
- (3) $\text{mCl}(\emptyset) = \emptyset$, $\text{mCl}(X) = X$, $\text{mInt}(\emptyset) = \emptyset$ and $\text{mInt}(X) = X$,
- (4) If $A \subset B$, then $\text{mCl}(A) \subset \text{mCl}(B)$ and $\text{mInt}(A) \subset \text{mInt}(B)$,
- (5) $A \subset \text{mCl}(A)$ and $\text{mInt}(A) \subset A$,
- (6) $\text{mCl}(\text{mCl}(A)) = \text{mCl}(A)$ and $\text{mInt}(\text{mInt}(A)) = \text{mInt}(A)$.

Lemma 2.6 (Popa and Noiri [17]). *Let X be a nonempty set with an m -structure m_X and A a subset of X . Then $x \in \text{mCl}(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x .*

Definition 2.7. An m -structure m_X on a nonempty set X is said to have *property \mathcal{B}* [14] if the union of any family of subsets belong to m_X belongs to m_X .

Remark 2.8. Let (X, τ) be a topological space. Then the families $\alpha(X)$, $\text{SO}(X)$, $\text{PO}(X)$, $\text{BO}(X)$ and $\text{SPO}(X)$ are m -structures on X with property \mathcal{B} .

Lemma 2.9 (Popa and Noiri [17]). *Let X be a nonempty set and m_X an m -structure on X satisfying property \mathcal{B} . For a subset A of X , the following properties hold:*

- (1) $A \in m_X$ if and only if $\text{mInt}(A) = A$,
- (2) A is m_X -closed if and only if $\text{mCl}(A) = A$,
- (3) $\text{mInt}(A) \in m_X$ and $\text{mCl}(A)$ is m_X -closed.

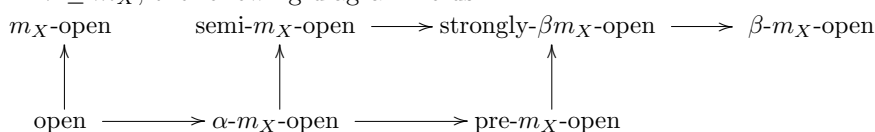
A topological space (X, τ) with an m -structure m_X on X is called a mixed space and is denoted by (X, τ, m_X) .

Definition 2.10. A subset A of a mixed space (X, τ, m_X) is said to be:

- (1) m_X -dense if $mCl(A) = X$.
- (2) m_X -nowhere dense if $Int(mCl(A)) = \phi$.
- (3) α - m_X -open if $A \subseteq Int(mCl(Int(A)))$.
- (4) semi- m_X -open if $A \subseteq mCl(Int(A))$.
- (5) pre- m_X -open if $A \subseteq Int(mCl(A))$.
- (6) β - m_X -open if $A \subseteq Cl(Int(mCl(A)))$.
- (7) semi- m_X^* -open if $A \subseteq Cl(mInt(A))$.
- (8) strongly- βm_X -open if $A \subseteq mCl(Int(mCl(A)))$.

Lemma 2.11. If $\tau \subseteq m_X$, then every semi- m_X -open set is semi- m_X^* -open.

If $\tau \subseteq m_X$, the following diagram holds:



Lemma 2.12. For a subset A of a mixed space (X, τ, m_X) , the following properties hold:

- (1) A is semi- m_X -open if and only if there exists $B \in \tau$ such that $B \subseteq A \subseteq mCl(B)$.
- (2) If there exists $B \in m_X$ such that $B \subseteq A \subseteq Cl(B)$, then A is semi- m_X^* -open.
- (3) A is semi- m_X^* -open if and only if $Cl(A) = Cl(mInt(A))$.

3. Characterizations of m -extremally disconnected spaces

Definition 3.1. A mixed space (X, τ, m_X) is said to be m -extremally disconnected (resp. m -hyperconnected) if $mCl(A) \in \tau$ (resp. $mCl(A) = X$) for each $A \in \tau$.

Example 3.2. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $m_X = \{X, \phi, \{a\}, \{b\}, \{c\}\}$. Then the topological space (X, τ) is not extremally disconnected and the mixed space (X, τ, m_X) is m -extremally disconnected.

Example 3.3. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $m_X = \{X, \phi, \{a\}, \{b\}, \{a, c\}\}$. Then the topological space (X, τ) is extremally disconnected and the mixed space (X, τ, m_X) is not m -extremally disconnected.

A subfamily \mathcal{I} of the power set $\mathcal{P}(X)$ of a nonempty set X is called an ideal if the following properties are satisfied: (1) $A \in \mathcal{I}$ and $B \subseteq A$ imply $B \in \mathcal{I}$; (2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$. A topological space (X, τ) with an ideal \mathcal{I} on X is called an ideal topological space and is denoted by (X, τ, \mathcal{I}) . For an ideal topological space and a subset A of X , $A^*(\mathcal{I})$ is defined as follows: $A^*(\mathcal{I}) = \{x \in X : U \cap A \notin \mathcal{I} \text{ for every open set } U \text{ containing } x\}$. In [12], $A^*(\mathcal{I})$ (briefly A^*) is called the local function of A with respect to \mathcal{I} and τ

and $Cl^*(A) = A^* \cup A$ defines a Kuratowski closure operator for a topology τ^* which is finer than τ . A subset A is \star -closed if and only if $A^* \subseteq A$. Naturally, the complement of a \star -closed set is said to be \star -open. A is said to be \star -dense if $Cl^*(A) = X$.

By setting $m_X = \tau^*$, as a special case of Definition 3.1 we obtain the following definitions:

Definition 3.4 ([10]). An ideal space (X, τ, \mathcal{I}) is said to be \star -extremally disconnected if the \star -closure of every open subset A of X is open.

Definition 3.5 ([11]). An ideal space (X, τ, \mathcal{I}) is said to be \star -hyperconnected if A is \star -dense for every open subset $A \neq \phi$ of X .

Let X be a nonempty set and let $\mathcal{P}(X)$ be the power set of X . Then $\mu \subseteq \mathcal{P}(X)$ is called a generalized topology (briefly GT) [8] on X if $\emptyset \in \mu$ and $G_i \in \mu$ for $i \in I \neq \emptyset$ implies $G = \cup_{i \in I} G_i \in \mu$. We call the pair (X, μ) a generalized topological space (briefly GTS) on X .

For a GTS (X, μ) , the elements of μ are called μ -open sets and the complements of μ -open sets are called μ -closed sets. For $A \subseteq X$, we denote by $c_\mu(A)$ the intersection of all μ -closed sets containing A , i.e., the smallest μ -closed set containing A .

By setting $m_X = \mu$, where m_X has property \mathcal{B} , as a special case of Definition 3.1 we obtain the following definition:

Definition 3.6 ([9]). Let (X, μ) be a GTS and G is a subset of X .

- (1) G is said to be μ -dense if $c_\mu(G) = X$,
- (2) (X, μ) is said to be hyperconnected if G is μ -dense for every μ -open set $G \neq \phi$ of (X, μ) .

Lemma 3.7. *Let (X, τ, m_X) be a mixed space. Then, the following properties hold:*

- (1) *If X is m -hyperconnected, then X is m -extremally disconnected.*
- (2) *If $m_X = SO(X, \tau)$ or $SPO(X, \tau)$, then (X, τ, m_X) is m -extremally disconnected.*
- (3) *Let (X, τ) be extremally disconnected. If $m_X = \alpha(X)$, $PO(X, \tau)$ or $BO(X, \tau)$, then (X, τ, m_X) is m -extremally disconnected.*

Proof. (1) This is obvious.

(2) It is known in [3] that $sCl(A) = A \cup Int(Cl(A))$ and $spCl(A) = A \cup Int(Cl(Int(A)))$ for every subset A of X . Therefore, $sCl(V)$ and $spCl(V)$ are open for every open set V and hence (X, τ, m_X) is m -extremally disconnected for $m_X = SO(X, \tau)$ or $SPO(X, \tau)$.

(3) It is known in [3] and [5] that $\alpha(A) = A \cup Cl(Int(Cl(A)))$, $pCl(A) = A \cup Cl(Int(A))$ and $bCl(A) = sCl(A) \cap pCl(A)$ for every subset A of X . Therefore, $\alpha(V)$, $pCl(V)$ and $bCl(V)$ are open for every open set V of an extremally disconnected space (X, τ) and hence (X, τ, m_X) is m -extremally disconnected for $m_X = \alpha(X, \tau)$, $PO(X, \tau)$, or $BO(X, \tau)$. \square

The following example shows that the converse of each statement of Lemma 3.7 are not true.

Example 3.8. Consider the mixed space (X, τ, m_X) , where $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}\}$ and $m_X = \{\phi, \{a\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. If $A = \{a\}$, then A is open and $mCl(A) = \{a\} \neq X$. Hence (X, τ, m_X) is not m -hyperconnected. Since m_X -closure of every open set is open, X is m -extremally disconnected. Moreover, since $\{b, d\}$ is not β -open, m_X is not $SPO(X, \tau)$.

Theorem 3.9. Let (X, τ, m_X) be a mixed space, the following properties are equivalent:

- (1) X is m -extremally disconnected;
- (2) $mInt(A)$ is closed for every closed subset A of X ;
- (3) $mCl(Int(A)) \subseteq Int(mCl(A))$ for every subset A of X ;
- (4) Every semi- m_X -open set is pre- m_X -open;
- (5) $mCl(A) \in \tau$ for every strongly- βm_X -open set A ;
- (6) Every strongly- βm_X -open set is pre- m_X -open;
- (7) A is α - m_X -open if and only if it is semi- m_X -open for every $A \subseteq X$.

Proof. (1) \Rightarrow (2): Let A be a closed set in X . Then $X - A$ is open. By (1) $mCl(X - A) = X - mInt(A)$ is open. Thus, $mInt(A)$ is closed.

(2) \Rightarrow (3): Let A be any set of X . Then $X - Int(A)$ is closed in X and by (2) $mInt[X - Int(A)]$ is closed in X . Therefore, $mCl(Int(A))$ is open in X and hence $mCl(Int(A)) \subseteq Int(mCl(A))$.

(3) \Rightarrow (4): Let A be semi- m_X -open. By (3), we have $A \subseteq mCl(Int(A)) \subseteq Int(mCl(A))$. Thus, A is pre- m_X -open.

(4) \Rightarrow (5): Let A be a strongly- βm_X -open set. Then $mCl(A)$ is semi- m_X -open. By (4), $mCl(A)$ is pre- m_X -open. Thus, $mCl(A) \subseteq Int(mCl(A))$ and hence $mCl(A)$ is open.

(5) \Rightarrow (6): Let A be a strongly- βm_X -open set. By (5), $mCl(A) = Int(mCl(A))$. Thus, $A \subseteq mCl(A) = Int(mCl(A))$ and hence A is pre- m_X -open.

(6) \Rightarrow (7): Let A be a semi- m_X -open set. Since a semi- m_X -open set is strongly- βm_X -open, then by (6) it is pre- m_X -open. Since A is semi- m_X -open and pre- m_X -open, it is α - m_X -open.

(7) \Rightarrow (1): Let A be an open set of X . Then $mCl(A)$ is semi- m_X -open and by (7) $mCl(A)$ is α - m_X -open. Therefore, $mCl(A) \subseteq Int(mCl(Int(mCl(A)))) = Int(mCl(A))$ and hence $mCl(A) = Int(mCl(A))$. Hence $mCl(A)$ is open and X is m -extremally disconnected. \square

Corollary 3.10. Let (X, τ, m_X) be a mixed space. Then, the following properties are equivalent:

- (1) X is m -extremally disconnected;
- (2) $mCl(A) \in \tau$ for every α - m_X -open set A of X ;
- (3) $mCl(A) \in \tau$ for every semi- m_X -open set A of X ;

(4) $mCl(A) \in \tau$ for every pre- m_X -open set A of X .

Proof. This follows from Theorem 3.9 and Diagram. \square

Theorem 3.11. *Let (X, τ, m_X) be a mixed space and m_X have property \mathcal{B} . Then, the following properties are equivalent:*

- (1) X is m -extremally disconnected;
- (2) For any $A \in \tau$ and $B \in m_X$ such that $A \cap B = \phi$, there exist disjoint a m_X -closed set U and a closed set V such that $A \subseteq U$ and $B \subseteq V$;
- (3) $mCl(U) \cap Cl(V) = \phi$ for every $U \in \tau$ and $V \in m_X$ with $U \cap V = \phi$;
- (4) $mCl[Int(mCl(U))] \cap Cl(V) = \phi$ for every $U \subseteq X$ and $V \in m_X$ with $U \cap V = \phi$.

Proof. (1) \Rightarrow (2): Let X be m -extremally disconnected. Let A and B be two disjoint open and m_X -open sets, respectively. Then $mCl(A)$ and $X - mCl(A)$ are disjoint m_X -closed and closed sets containing A and B , respectively.

(2) \Rightarrow (3): Let $U \in \tau$ and $V \in m_X$ with $U \cap V = \phi$. By (2), there exist disjoint an m_X -closed set F and a closed set G such that $U \subseteq F$ and $V \subseteq G$. Therefore, $mCl(U) \cap Cl(V) \subseteq F \cap G = \phi$. Thus, $mCl(U) \cap Cl(V) = \phi$.

(3) \Rightarrow (4): Let $U \subseteq X$ and $V \in m_X$ with $U \cap V = \phi$. Since $Int(mCl(U)) \in \tau$ and $Int(mCl(U)) \cap V = \phi$, by (3) $mCl[Int(mCl(U))] \cap Cl(V) = \phi$.

(4) \Rightarrow (1): Let U be any open set. Then $[X - mCl(U)] \cap U = \phi$. Since m_X has property \mathcal{B} , $X - mCl(U) \in m_X$ and by (4) $mCl[Int(mCl(U))] \cap Cl(X - mCl(U)) = \phi$. Since $U \in \tau$, we have $mCl(U) \cap [X - Int(mCl(U))] = \phi$. Therefore, $mCl(U) \subseteq Int(mCl(U))$ and $mCl(U)$ is open. This shows that X is m -extremally disconnected. \square

Definition 3.12. A subset A of a mixed space (X, τ, m_X) is called an R_m -open set if $A = Int(mCl(A))$. The complement of an R_m -open set is said to be R_m -closed.

Theorem 3.13. *Let (X, τ, m_X) be a mixed space and m_X have property \mathcal{B} . Then, the following properties are equivalent:*

- (1) X is m -extremally disconnected;
- (2) Every R_m -open set of X is m_X -closed in X ;
- (3) Every R_m -closed set of X is m_X -open in X .

Proof. (1) \Rightarrow (2): Let X be m -extremally disconnected. Let A be an R_m -open set of X . Then $A = Int(mCl(A))$. Since A is an open set, then $mCl(A)$ is open. Thus, $A = Int(mCl(A)) = mCl(A)$ and hence A is m_X -closed.

(2) \Rightarrow (1): Suppose that every R_m -open subset of X is m_X -closed in X . Let A be an open subset of X . Since $Int(mCl(A))$ is R_m -open, then it is m_X -closed. This implies that $mCl(A) \subseteq mCl(Int(mCl(A))) = Int(mCl(A))$ since $A \subseteq Int(mCl(A))$. Thus, $mCl(A)$ is open and hence X is m -extremally disconnected.

(2) \Leftrightarrow (3): It is obvious. \square

Theorem 3.14. *Let (X, τ, m_X) be a mixed space. Then the following properties are equivalent:*

- (1) X is m -extremally disconnected;
- (2) $mCl(A) \in \tau$ for every R_m -open set A of X .

Proof. (1) \Rightarrow (2): Let A be an R_m -open set of X . Then A is open and $mCl(A) \in \tau$.

(2) \Rightarrow (1): Suppose that $mCl(A) \in \tau$ for every R_m -open set A of X . Let V be any open set of X . Then $Int(mCl(V))$ is an R_m -open set and $mCl(V) = mCl(Int(mCl(V))) \in \tau$. Thus $mCl(V) \in \tau$ and hence X is m -extremally disconnected. \square

Theorem 3.15. *Let (X, τ, m_X) be a mixed space and m_X have property \mathcal{B} . Then, the following properties are equivalent:*

- (1) X is m -extremally disconnected;
- (2) If A is semi- m_X -open, B is semi- m_X^* -open and $A \cap B = \phi$, then $mCl(A) \cap Cl(B) = \phi$.

Proof. (1) \Rightarrow (2): Let A be semi- m_X -open, B semi- m_X^* -open and $A \cap B = \phi$. Since m_X is property \mathcal{B} , $mInt(B)$ is m_X -open and $mCl(A) \cap mInt(B) = \phi$. By Corollary 3.10, $mCl(A)$ is open and $mCl(A) \cap Cl(mInt(B)) = \phi$. Since B is semi- m_X^* -open, $Cl(B) = Cl(mInt(B))$ and hence $mCl(A) \cap Cl(B) = \phi$.

(2) \Rightarrow (1): Let A be a semi- m_X -open set. Since A and $X - mCl(A)$ are disjoint semi- m_X -open and semi- m_X^* -open, respectively, by (2) we have $mCl(A) \cap Cl[X - mCl(A)] = \phi$. This implies that $mCl(A) \subseteq Int(mCl(A))$. Thus $mCl(A)$ is open. Hence, by Corollary 3.10, X is m -extremally disconnected. \square

Theorem 3.16. *Let (X, τ, m_X) be a mixed space and m_X have property \mathcal{B} . Then X is m -extremally disconnected if and only if for every open set G and every m_X -closed set F with $G \subseteq F$, there exist an open set G_1 and an m_X -closed set F_1 such that $G \subseteq F_1 \subseteq G_1 \subseteq F$.*

Proof. Suppose X is m -extremally disconnected. Let G be an open set and F an m_X -closed set in X such that $G \subseteq F$. Then $G \cap (X - F) = \phi$. Then by Theorem 3.11 $mCl(G) \cap Cl(X - F) = \phi$, that is, $mCl(G) \subseteq X - Cl(X - F)$. Using the fact that $X - Cl(X - F) \subseteq F$ and writing $mCl(G) = F_1$, $X - Cl(X - F) = G_1$, we get $G \subseteq F_1 \subseteq G_1 \subseteq F$.

Conversely, let the condition hold. Let U be an open set and V be an m_X -open set in X such that $U \cap V = \phi$. Then, $U \subseteq X - V$ and $X - V$ is m_X -closed. Accordingly, there exist an open set G and an m_X -closed set F such that $U \subseteq F \subseteq G \subseteq X - V$. This implies that $mCl(U) \cap X - [Int(X - V)] = \phi$. But $X - [Int(X - V)] = Cl(V)$. That is, $mCl(U) \cap Cl(V) = \phi$ and by Theorem 3.11 X is m -extremally disconnected. \square

Definition 3.17. Let (X, τ, m_X) be a mixed space and R be the real line with the usual topology. A mapping $f : X \rightarrow R$ is said to be upper semicontinuous

or *u.s.c.* in brief (resp. lower minimal semi-continuous or *l.m.s.c.* in brief) if for each $a \in R$, the set $\{x : x \in X, f(x) < a\}$ is open (resp. $\{x : x \in X, f(x) > a\}$ is m_X -open).

Theorem 3.18. *Let (X, τ, m_X) be an m -extremally disconnected mixed space and m_X have property \mathcal{B} . Let U be an open set, V be an m_X -open set in X such that $U \cap V = \emptyset$. Then there exists a real-valued *u.s.c.* and *l.m.s.c.* mapping $f : X \rightarrow [0, 1]$ such that $f(U) = \{0\}$ and $f(V) = \{1\}$.*

Proof. Since $U \subseteq X - V$, by Theorem 3.16, there exist an open set $G_{1/2}$ and an m_X -closed set $F_{1/2}$ such that $U \subseteq F_{1/2} \subseteq G_{1/2} \subseteq X - V$. Again, since $U \subseteq F_{1/2}$ and $G_{1/2} \subseteq X - V$, by the same reasoning, there exist open sets $G_{1/4}, G_{3/4}$ and m_X -closed sets $F_{1/4}, F_{3/4}$ such that

$$U \subseteq F_{1/4} \subseteq G_{1/4} \subseteq F_{1/2} \subseteq G_{1/2} \subseteq F_{3/4} \subseteq G_{3/4} \subseteq X - V.$$

We continue this process for each dyadic rational number of the form $t = \frac{m}{2^n}$ (where $n = 1, 2, 3, \dots$ and $m = 1, 2, \dots, 2n - 1$). We find that for $t_1 < t_2$, there exist open sets G_{t_1} and G_{t_2} and m_X -closed sets F_{t_1} and F_{t_2} such that

$$U \subseteq F_{t_1} \subseteq G_{t_1} \subseteq F_{t_2} \subseteq G_{t_2} \subseteq X - V.$$

Now, we define a mapping f on X as follows:

$$f(x) = \begin{cases} 0, & x \in G_t \text{ for some } t; \\ \sup\{t : t \notin G_t\}, & \text{otherwise.} \end{cases}$$

Clearly, the values of f lies in $[0, 1]$. Also, $f(U) = \{0\}$ and $f(V) = \{1\}$.

(1) f is *u.s.c.*: We show that $f^{-1}([0, a])$ is open for each a , where $0 < a < 1$. Let $x \in f^{-1}([0, a])$. Then $f(x) < a$, hence there must be some dyadic rational $t < a$ such that $x \in G_t$. Thus $f^{-1}([0, a]) \subseteq \cup_{t < a} G_t$. Again, if $x \in \cup_{t < a} G_t$, then $x \in G_{t_0}$ for some $t_0 < a$, so that $x \in f^{-1}([0, a])$. Thus, $f^{-1}([0, a]) = \cup_{t < a} G_t$, which is open. Hence, f is *u.s.c.*

(2) f is *l.m.s.c.*: We show that $f^{-1}((a, 1])$ is m_X -open for each a , where $0 < a < 1$. Let $y \in f^{-1}((a, 1])$. Then there exists a dyadic rational $t > a$ such that $y \notin G_t$. Then $y \notin F_t$. Thus $f^{-1}((a, 1]) \subseteq \cup_{t > a} (X - F_t)$. Also $y \in X - F_t$ for some $t > a$ implies that $y \notin G_{t_0}$ for some t_0 with $t > t_0 > a$. Hence $f(y) > a$, that is, $y \in f^{-1}((a, 1])$. Thus we get, $f^{-1}((a, 1]) = \cup_{t > a} (X - F_t)$, which is m_X -open. Hence f is *l.m.s.c.* \square

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