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Haidao Suanjing in Joseon Mathematics

海島算經과 朝鮮 算學

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Haidao Suanjing was introduced into Joseon by discussion in Yang Hui Suanfa (楊輝算法) which was brought into Joseon in the 15th century. As is well known, the basic mathematical structure of Haidao Suanjing is perfectly illustrated in Yang Hui Suanfa. Since the 17th century, Chinese mathematicians understood the haidao problem by the Western mathematics, namely an application of similar triangles. The purpose of our paper is to investigate the history of the haidao problem in the Joseon Dynasty. The Joseon mathematicians mainly conformed to Yang Hui's verifications. As a result of the influx of the Western mathematics of the Qing dynasty for the study of astronomy in the 18th century Joseon, Joseon mathematicians also accepted the Western approach to the problem along with Yang Hui Suanfa.

Keywords: Chongcha (重差), Haidao Suanjing (海島算經), Yang Hui Suanfa (楊輝算法, 1274–1275), Joseon (朝鮮) mathematics, Jihe Yuanben (幾何原本, 1607), Celiang Fayi (測量法義, 1608), Celiang Yitong (測量異同, 1608), Shuli Jingyun (数理精蘊, 1723), Gyeong Seon-jing (慶善徵, 1616–1690), Hong Jeong-ha (洪正夏, 1684–1727), Jo Tae-gu (趙泰耉, 1660–1723)

MSC: 01A13, 01A25, 01A27, 11D09, 12–03, 26–03

1 Introduction

It is well known that the surveying in the traditional East Asian mathematics had been related with right triangles. The surveying or measuring is strongly related with geometrical structures. Right triangles in the East Asian mathematics are not defined by the right angles but by the Pythagorean relations, i.e., the equation $a^2 + b^2 = c^2$ between their three sides a, b, c. Moreover, most of right triangles in the traditional East Asian mathematical books were indicated by the Pythagorean triple

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$3, 4, 5$. Furthermore, the right angle in the East Asia was designated by ju (矩) or juchi (矩尺), presumably designed by the triple $3, 4, 5$.

Jiuzhang Suanshu (九章算術) with the commentary by Liu Hui (劉徽, fl. the 3rd Century) is the most complete mathematical work in China which is extant in the present [2, 11].

Liu Hui made great contributions to Jiuzhang Suanshu in his commentary. We just note an example concerning with the length of a circle. Liu Hui mentioned it in the *first* book, Fangtian (方田) of Jiuzhang Suanshu. In order to obtain a good approximation $\pi \approx \frac{157}{50} = 3.14$, he used the Pythagorean equation $a^2 + b^2 = c^2$ of the right triangle and square roots. The former is dealt in the *last* book Gougu (句股) and the latter in the *fourth* book Shaoguang (少廣) in Jiuzhang Suanshu. He also used the perpendicular bisector and sagitta implicitly.

Furthermore, Liu Hui added one more chapter, called chongcha (重差) and he must have been very much satisfied with his work, because he allotted more than half of his preface of the commentary of Jiuzhang Suanshu. When Li Chenfeng (李淳風, 602-670) published Shibu Suanjing (十部算經, 656), the collection of ten mathematical works, he included chongcha as a separate book and named it Haidao Suanjing (海島算經). Chongchashu (重差術), or chongbiaofa (重表法) in Haidao Suanjing (海島算經) is a method of surveying with double poles (表). As in the other traditional mathematical works in China, Liu Hui did not include the mathematical proof for his chongchashu in his Haidao Suanjing.

The mathematical proof of chongchashu was well illustrated in Xugu Zhaiqi Suanfa (續古摘奇算法, 1275) in Yang Hui Suanfa (楊輝算法, 1274–1275) [11, 13]. Since the translation of the first six books of Euclid's Elements was published by Matteo Ricci (利瑪竇, 1552–1610) and Xu Guangqi (徐光啓, 1562–1633), called Jihe Yuanben (幾何原本, 1607), the Western mathematics books were translated in China for the study of Western astronomy and mathematics. We should point out that Tianxue Chuhan (天學初函, 1629) and Chongzen Lishu (崇禎曆書, 1643) were published for the study of the Western astronomy and mathematics. The former was compiled by Li Zhizao (李之藻, 1565–1630) and the latter by Xu Guangqi and Li Tianjing (李天經, 1579–1659). Chongzen Lishu was revised by Adam Schall von Bell(湯若望, 1591–1666) with his corrections and arrangement, and the collection was called Xiyang Xinfa Lishu (西洋新法曆書, 1645), later called Xinfa Suanshu (新法算書). The Qing Dynasty adopted a new calendar system, Shixianli (時憲曆) based on Xinfa Suanshu in 1645.

As an application of Jihe Yuanben, Celiang Fayi (測量法義, 1608) was published by Ricci and Xu. It deals with chongchashu based on the similarity of *right* triangles and a property of proportionality introduced in Jihe Yuanben. It is also quoted in Tongwen Suanzhi (同文算指, 1613) but its verification in Celiang Fayi is omitted in Tongwen Suanzhi. Xu Guangqi noticed that various methods of surveying in Celiang Fayi were already presented in the traditional mathematics books in China and then wrote Celiang Yitong (測量異同, 1608) to relate them. The above three books can be found in Tianxue Chuhan.

The haidao problem, or chongchashu was dealt in the another huge collection, Shuli Jingyun (數理精蘊, 1723). They include its verification in Shuli Jingyun which is based on the similarity of *arbitrary* triangles.

All of the books mentioned above except Jiuzhang Suanshu were brought into Joseon (1392–1910) [10]. Jiuzhang Suanshu was imported to Joseon in the mid-19th century.

The purpose of this paper is to study the history of Haidao Suanjing in Joseon. It divides into three sections. We briefly compare the proofs for haidao problem in the Chinese literatures in the second section, and then reveals its history in the Joseon Dynasty with a conclusion.

For the Chinese books included in [1], they will not be numbered as an individual reference.

2 Mathematical structures of Haidao Suanjing

The algorithms in Haidao Suanjing are well known, and we just quote references [2, 11, 13] for them. In this section, we exhibit and compare the mathematical principles of haidao problems practiced in the Chinese literatures.

We first discuss Yang Hui's proof for haidao problems, or chongchashu (重差術) in Yang Hui Suanfa as mentioned above. Indeed, Yang Hui dealt with these problems at the section, haidao tijie (海島題解) in the end of Xugu Zhaiqi Suanfa (續古摘奇算法, 1275). He began with the exact proof for surveying with a single pole and then that for the double poles in Haidao Suanjing. It begins with the quote of the first problem of Haidao Suanjing, which contains just an algorithm without any mathematical verifications.

Yang Hui put the problem of the single pole and then applied the method (九章以表望山術) in Problem 23 in the chapter Gougu in Jiuzhang Suanshu. The problem is as follows:

> 假如竿不知高 從竿腳量遠二十五尺立一丈表 表後退行五尺用窺穴望表與竿齊平
> 其人目窺穴高四尺 問竿高幾何

Yang Hui then added its verification (解術) with the following diagram, Figure 1. We first note that Yang Hui drastically reduced the dimensions of the problem

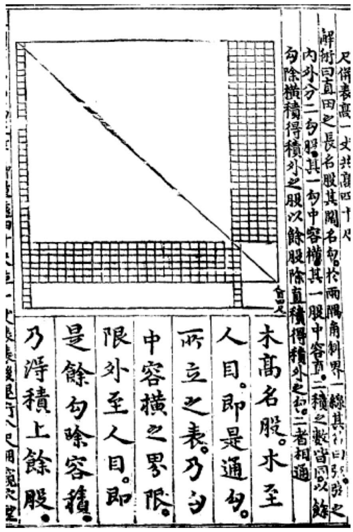

Figure 1. single pole in Yang Hui Suanfa

from the original problem in Jiuzhang Suanshu so that he could specifically indicated the lengths and areas in its proof by the diagram. He then showed that the areas of two rectangles involved are exactly the same. The proof is precisely a special case of the proof of Proposition 43 in Book 1 of Jihe Yuanben because the whole rectangle is clearly a parallelogram and the indicated rectangles are exactly its complements about the diagonal [11, 14]. We also note that the similarity of triangles is introduced in Book 6 of Jihe Yuanben and properties of proportions in Book 5. In the Proposition 43 (第四十三題) of Jihe Yuanben, the word, parallel (平行) is missing in 凡(平行)方形對角線旁兩餘方形自相等.

Having the above single pole problem, Yang Hui showed the verification for the haidao problem, or the double poles problem as before.

> 隔水有竿不知其高 立二表各高一丈 前後相去一十五尺 自前表退行五尺
> 於窺穴內望表與竿齊平 又從後表退行八尺 亦窺穴望表與竿齊平 問竿高幾何

One can easily point out that the dimensions for the front pole are exactly the same with the previous problem for a single pole without the distance from the pole to the bamboo rod (竿) and that the heights of the surveyor's eye in the both problems are the same (4尺) but indicated in the following diagram, Figure 2.

In the diagram in Figure 2, Yang Hui also omitted the detail of the front pole part for it is already given in Figure 1 and then the upper rectangle in the front part is *translated* into the upper rectangle of the rear part so that the area of the difference of two rectangles involving the height equals that of the lower rectangle given by the length between two poles. We should emphasize that Yang Hui's proof is really

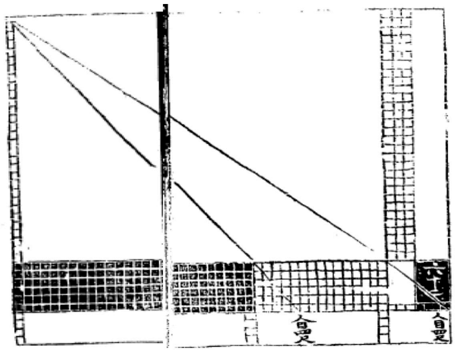

Figure 2. double poles in Yang Hui Suanfa

pedagogical.

We now discuss the haidao problems influenced by the Western mathematics in the 17th century China.

The problem is included in the Problem 10, yibiaocegao (以表測高) (also see the Problem 6, yimucegao (以目測高)) in Celiang Fayi. The problem includes also the surveying with a single pole.

They used the similarity of *right* triangles as discussed above. Using the properties of proportion, namely

i) $a : b = c : d, a' : b = c' : d$ and $a > a'$ imply $c > c'$;

ii) $a : b = c : d$ implies $a : b = (a - c) : (b - d)$, they have the desired height of the haidao problem, where the proportions are obtained by the similarity of right triangles.

By the influences of Jihe Yuanben, the problems with single pole and double poles in Celiang Fayi are not given specific numbers for their dimensions contrary to the discussions in the Eastern mathematics books but they were illustrated with diagrams. The diagram for the haidao problem is similar to that in Yang Hui Suanfa as they indicated the distance between the viewer's eye to the front pole from the right end (see Figure 2 and 3). We will discuss later the difference between those in Celiang Fayi and Shuli Jingyun.

Xu Guangqi completed Celiang Yitong to claim that basic results of Celiang Fayi were already included in Jiuzhang Suanshu and they were well practiced throughout its history. Using the same problems, namely those for a single pole and double poles of the haidao tijie in Yang Hui Suanfa, Xu tried to convince that the basic structures of surveying by poles are the same in the Eastern and Western mathematics. Although he quoted the problems of haidao tijie, he just applied the algorithms obtained in Celiang Fayi to the problems given dimensions of the specific numbers. Thus, he missed that the structures of its mathematical basis are completely different from each other.

Presumably, Xu didn't fully understand Yang Hui's diagrams which are precisely their verifications as we claimed above. Furthermore, he could not relate the diagrams with Proposition 43 of Book 1 in Jihe Yuanben. We should add that Xu Guangqi added the algorithm and its proof for the distance between the front pole and the bamboo pole as in Haidao Suanjing but used again the similarity and the height.

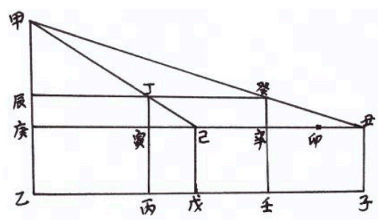

Figure 3. double poles in Celiang Fayi and Celiang Yitong

Due to the unfounded accusation to the new calendar system, Shixianli and Schall von Bell by Yang Guangxian (楊光先, 1597–1669) in 1664, the calendar system was abolished until the Jesuit astronomers proved that their astronomical estimate was much more accurate in 1669 (see [12]). As is well known, by the order of Kangxidi (康熙帝, 1654–1722, r. 1661–1722) in 1713, Lüli Yuanyuan (律曆淵源, 1723) was completed in 1722 and it contains Lixiang Kaocheng (曆象考成), Lülü Zhengyi (律呂正義) and Shuli Jingyun.

Shuli Jingyun was intended to supply the mathematical basis for others, in particular Lixiang Kaocheng. Its authors tried to extend and improve the mathematics dealt in Xinfa Suanshu. Further, they have to fill out the missing parts of the Elements in Jihe Yuanben. The basic style of presentations in Shuli Jingyun is as follows: they first give propositions, or algorithms for the given problems and then illustrate their verifications, or proofs in detail. Indeed, the latter begin with the word ru (如) and convert the problems with specific dimensions into those with the general symbols given by ganzhi (干支). Thus, their problems become universal one.

In Shuli Jingyun, haidao problems were dealt in Book 18, Celiang (測量). There are two sets, namely chongjufa (重矩法) in Problem 3 and 4, and liangbiaofa (兩表法), double poles method in Problem 8 and 9. The mathematical structures of the two sets are almost the same and hence we first discuss the second set. Problem 8 is the usual haidao problem and Problem 9 is to use different heights of poles. One can easily transform Problem 9 into the type of Problem 8.

We relabel the diagram for Problem 8 in Shuli Jingyun and then designate them with alphabets instead of ganzhi in Figure 4, because it will be again used in the next section. We note that the point J is taken by $EF = GJ$ and hence the length of the interval JH is precisely $GH - EF$.

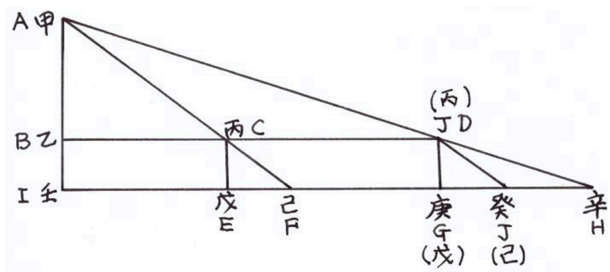

Figure 4. double poles in Shuli Jingyun

One can easily have $\triangle ACD \sim \triangle DJH$ and $\triangle ABD \sim \triangle DGH$. We note that the former similarity relates to the non-right triangles and that the latter is used in the previous verifications. These facts imply the following proportions:

$AD : DH \simeq CD : JH$ and $AD : DH \simeq AB : DG$,
which imply the desired proportion for the height

$AB : DG \simeq CD : JH$.

In all, the verification in Shuli Jingyun for the haidao problem used the similarity of non-right triangles for the sake of the algebraic properties of proportions. Since the given dimensions of Problem 3 are slightly different, it involves more calculations, namely the calculation of square roots but the geometrical verification is the same with that of Problem 8. Problem 4 deals with the unknown distance in the haidao problem.

3 Haidao Suanjing in Joseon Mathematics

The Silla dynasty (新羅, 57 BCE–935) established its educational system, called Gughag (國學) in 682 which included the department of mathematics. The subjects in the department are Cheolgyeong (Jhuijing, 綴經), Samgae (三開), Gujang (Jiuzhang, 九章), and Yugjang (Liuzhang, 六章). They took the system along that in the Tang Dynasty (618–907). Thus, Gujang and Cheolgyeong should be Jiuzhang Suanshu and Jhuishu (綴術) and hence those books were brought into the Korean peninsular during the 7th century.

The next dynasty, the Goryeo dynasty (高麗, 918–1392) after Silla also established the institution similar to Gughag of Silla, called Gugjagam (國子監) in 992. The institution also contained the department of mathematics. The dynasty adopted the system of national examination for officials of mathematics among others in the mid-10th century. The subjects for the examination included Gujang (九章), Cheolsul (綴

術), Samgae (三開) and Saga (謝家). The system for mathematics was retained in Injong (仁宗, r. 1122–1146) and hence Jiuzhang Suanshu was available up to the 12th century in the Korean peninsular.

Except the above records in Goryeosa (高麗史, 1451), or the history of Goryeo, we don't have any records on the history of mathematics up to the Goryeo dynasty.

For the study of the astronomy for Joseon, King Sejong (世宗, 1397–1450, r. 1419–1450) ordered to import mathematical books along with the calendar systems. This fact with a comment on the double poles can be found in Joseon Wangjo Sillok (朝鮮王朝實錄), or The Annals of the Joseon Dynasty (King Sejo (世祖, r. 1455–1468), June 16, 1460) as follows [15].

> 惟我世宗慨念曆法之未明, 博求曆算之書, 幸得 大明曆, 回回曆, 授時曆, 通軌及啓蒙, 楊輝全集, 捷用九章 等書。...
> 度高, 測深, 重表, 累矩, 三望, 四望, 句股, 重差之法乎

Indeed, Suanxue Qimeng (算學啓蒙, 1299), Yang Hui Suanfa (楊輝算法, 1274–1275) and a version of Jiuzhang Suanshu were brought into Joseon, where they were briefly denoted by 啓蒙, 楊輝全集, 捷用九章. We do not have any information on Cheobyong Gujang (捷用九章, Jieyong Jiuzhang). Further, the Annals includes surveying dealt in the Book Gougu in Jiuzhang and Haidao Suanjing. Thus, Haidao Suanjing must be imported into Joseon in the reign of King Sejong. In the Annals, one can find that King Sejong himself studied Suanxue Qimeng in 1430 and that Yang Hui Suanfa was republished in 1433. Further, King Sejong also chose Suanxue Qimeng, Yang Hui Suanfa and Xiangming Suanfa (詳明算法, 1373) for the subjects to select mathematical officials. Thus, Suanxue Qimeng and Yang Hui Suanfa had become the major references throughout the Joseon Dynasty. These are all the history of the Joseon mathematics before the 17th century.

After the devastating Japanese invasion (1592–1598), the Joseon government had to reconstruct the governmental systems and officials. Thus, they must train new mathematical officials among others and they needed a basic text book for mathematics.

The first result is the book, Mugsajib Sanbeob (默思集算法) [3]. It was completed by a Hojo (戶曹) official, Gyeong Seonjing (慶善徵, 1616–1690). Mugsa is his pseudonym (號) and he belongs to the jung-in (中人) class. He passed the examination (取才) for the mathematical officials in 1640. Mugsajib Sanbeob is the oldest Joseon mathematics book handed down to the present. His references include Suanxue Qimeng, Yang Hui Suanfa, Xiangming Suanfa. Haidao problems were dealt in the section cheuglyang gowonmun (celiang gaoyuanmen, 測量高遠門) in the second book and included 9 problems. The first problem is as follows:

今有立竹 不知其長 只云其影量之 得一丈八尺 別立一表 長一尺五寸 其影則六寸 問竹長幾何

The problem is slightly different from the single pole problem in Yang Hui Suanfa but a variation of the last problem just before the section haidao tijie (海島題解). Yang Hui might notice that these problems should be differentiated. The position of the given pole was not clear as the one in Yang Hui Suanfa is situated and hence its verification becomes difficult. We note that the similarity of right triangles were not introduced in the 17th century Joseon (also see the similar problem in Duying Lianggan (度影量竿) of Suanxue Baojian (算學寶鑑, 1524) by Wang Wensu (王文素). Joseon scholar Hwang Yun-seog (黃胤錫, 1729–1791) also dealt with a similar problem in his Sanhag Ibmun (算學入門) and said that the problem was taken in Zhiming Suanfa (出指明算法). Gyeong Seon-jing's next 3 problems are the usual single pole problems and Problems 5-8 are given by the formation of the double poles problem but given conditions imply that they were solved by the couple of steps of the single pole problem. The final problem is an example of Haidao problem.

As another effort for the revival of Joseon mathematics, Suanxue Qimeng was republished in 1660 by Kim Si-jin (金始振, 1618–1667). Kim Si-jin strongly emphasized tianyuanshu (天元術) over the other subjects in his preface. Further, Kim added the section haidao tijie (海島題解) in Yang Hui Suanfa as an appendix in his republication. Thus, haidao problem has remained as an interesting subject in the history of Joseon mathematics.

As a consequence, we have now the most important book in the history of Joseon mathematics, namely Gu-il Jib (九一集, 1713–1724) by Hong Jeong-ha (洪正夏, 1684–1727) [6]. He fully understood the power of tianyuanshu and applied it to every subject which can be solved by equations. He also studied Yang Hui Suanfa whose Tianmu Bulei Chengchu Jiefa (田畝比例乘除捷法) discussed the theory of equation in Yigu Genyuan (議古根源, ca. 12th Century) by Liu Yi (劉益). We recall that Jia Xian (賈憲) and Liu Yi established a basic method for solving polynomial equations and that Yang Hui transmitted them in his Xiangjie Jiuzhang Suanfa (1261) along with Yang Hui Suanfa but its Shaoguang (少廣) is missing. Hong Jeong-ha showed the mathematical structure for zengcheng kaifangfa (增乘開方法) by the binomial expansions [8]. He also dealt with haidao problems in the section, manghaedosulmun (望海島術門) with 6 problems, 3 problems each for single pole and double poles problem and he just followed the discussions in Yang Hui Suanfa but omitted the verifications.

Choe Seog-jeong (崔錫鼎, 1646–1715) also quoted the haidao problem in Tongwen Suanzhi and also mentioned that in Yang Hui Suanfa in his Gusuryag (九數略).

We have discussed the haidao problem in our paper, Jo Tae-gu's Juseo Gwan-gyeon and Jihe Yuanben [7]. We also refer [7] for Jo Tae-gu (趙泰耇, 1660–1723) and his book, Juseo Gwan-gyeon (籌書管見, 1718) (also see [9]). We could not fully understand his verification of the problem, because we can't figure out his diagram and his word sangcheob (相疊, xiangdie in Chinese) for congruent triangles, or superpositions (重疊, chongdie).

Surprisingly, Jo's diagram for the proof of the haidao problem is exactly the same with that in Shuli Jingyun (1723).

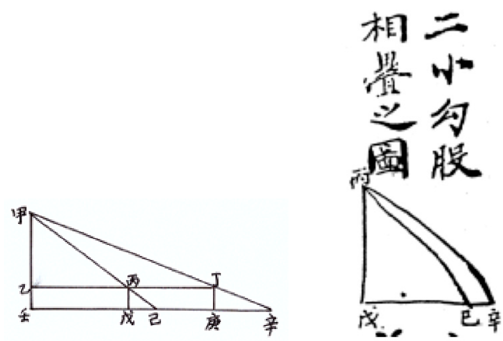

Figure 5. double poles and sangcheob in Juseo Gwan-gyeon

As is well known, the congruences of triangles are not defined in Jihe Yuanben but the similarities of triangles are well defined. Congruences are called deng (等) [4], or equal and played very important roles to reveal geometrical structures in Book One. We just recall the proof of Pythagorean theorem in Propostion 47 in Book One, which might give the most important impact to traditional East Asian mathematicians. The similarities, or xiangshi (相似) are defined in the Book Six of Jihe Yuanben, which also revealed the importance of angles to them.

For the congruence of triangles, Jo Tae-gu introduced the terminology, sangcheob as 二小勾股相疊之圖 in the above Figure 5, namely the figure obtained by the translation, one of the isometry. It is indicated by the same labels of the original one. Thus, adding the diagram of sangcheob into the original diagram for double poles, one can identify Jo's diagram with Figure 4 in Shuli Jingyun. Jo Tae-gu did not recognize the similarity between two obtuse triangles in Shuli Jingyun but included the proofs given in Celiang Fayi. Jo Tae-gu did not study mathematics in Xinfa Suanshu and he could not recognize the importance of angles except the right angle. Thus, he did have the proportions involved in the obtuse triangles in Shuli Jingyun but missed their similarity.

Shuli Jingyun was imported into Joseon in 1730 [10] but it was mainly studied by astronomical officials in the national observatory, Gwansang-gam (觀象監) and hence we do not have any mathematical works on Shuli Jingyun except Juhae Suyong (籌解需用) written by Hong Dae-yong (洪大容, 1731–1783) [5]. Juhae Suyong is a part of his book Damheonseo (湛軒書) written in the period 1765–1775. Hong Dae-yong had been to Beijing as an envoy in 1765 and met many scholars including Jesuit priests August von Hallerstein (劉宋齡, 1703–1774) and Anton Gogeisi (鮑友管, 1701–1771), high officials in the national observatory, Qintianjian (欽天監). He studied Shuli Jingyun, in particular Book 16–18, Geyuan (割圜), Sanjiaoxing bianxian jiaodu xiangqiu (三角形邊線角度相求) and Celiang (測量). These Books contain the enormous basic mathematical structures for surveying influenced by the Western mathematics and astronomy. We will discuss Hong Dae-yong's contributions in his Juhae Suyong to these subjects together with those of the 19th century in a separate paper later.

4 Conclusions

Liu Hui's chongchashu (重差術) or haidao problems in Haidao Suanjing is one of the most important contributions in the history of Chinese mathematics. Yang Hui put its verification in his book, Yang Hui Suanfa (楊輝算法, 1274–1275). It is quoted in Suanfa Tongzong (算法統宗, 1592) of Cheng Dawei (程大位, 1533–1606). We recall that mathematics of the Song dynasty (宋, 960–1279) and the Yuan dynasty (元, 1271–1368) was mostly neglected in the Ming dynasty (1368–1643) except Yang Hui's works. In the 17th century, the Western mathematics was brought into China by the Jesuit priests. The chongchashu had been familiar to the European mathematicians and hence it should be a common subject for Chinese and Jesuit mathematicians. Since then, chongchashu in China was mainly understood by the Western geometry in Celiang Fayi (測量法義) and Shuli Jingyun (數理精蘊) among others.

Yang Hui Suanfa and Sanxue Qimeng have been basic references throughout the Joseon Dynasty so that the haidao problem in Yang Hui Suanfa was well comprehended by Joseon mathematicians. As discussed in this paper, there were a few efforts paid to the Western approach to the problem. Unlike Chinese counterparts, Joseon mathematicians always kept the mathematical structure of chongchashu illustrated in Yang Hui Suanfa. We note that the Joseon dynasty (1392–1910) and the Ming Dynasty were founded almost same times. Countries in the history of Korea had much longer history than those in China. Thus, Joseon mathematicians might be much more conservative.

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