Modified adaptive complementary sliding mode control for the longitudinal motion stabilization of the fully-submerged hydrofoil craft

Sheng Liu a, Hongmin Niu a,*, Lanyong Zhang a, Changkui Xu b

a College of Automation, Harbin Engineering University, Harbin, 150001, China
b Beijing Electro-Mechanical Engineering Institute, Beijing, 100074, China

ABSTRACT

This paper presents a Modified Adaptive Complementary Sliding Mode Control (MACSMC) system for the longitudinal motion control of the Fully-Submerged Hydrofoil Craft (FSHC) in the presence of time varying disturbance and uncertain perturbations. The nonlinear disturbance observer is designed with less conservatism that only boundedness of the derivative of the disturbance is required. Then, a complementary sliding mode control system combined with adaptive law is designed to reduce the bound of stabilization error with fast convergence. In particularly, the modified complementary sliding mode surface which contains the estimation of the disturbance can reduce the switching gain and retain the normal performance of the system. Moreover, a hyperbolic tangent function contained in the control law is utilized to attenuate the chattering of the actuator. The global asymptotic stability of the closed-loop system is demonstrated utilizing the Lyapunov stability theory. Ultimately, the simulation results show the effectiveness of the proposed approach.

1. Introduction

The Fully-Submerged Hydrofoil Craft (FSHC) is a nonlinear, strong coupling system with uncertain disturbance and unknown parameter perturbations. Comparing with traditional ships, the FSHC possesses characteristics of less resistance, higher speed and outstanding sea-keeping performance. When the vessel cruises at a high speed, the lift force of the hydrofoils generated by the high-speed fluid can elevate the hull above the sea, which is different from the underwater vehicles. Thus, the wetted surface area, wave resistance and viscous force will be attenuated dramatically. Nevertheless, since the craft hull is totally above the sea level, the hull cannot provide restoring forces or moments and the stability cannot be guaranteed correspondingly. The drastic motion would result in uncomfortable and unsafe. Therefore, advanced control system of the FSHC is required to improve the robustness and reduce the longitudinal motion simultaneously. The operation conditions of the FSHC contain the taking off process, floating mode and platform mode. The taking off process is transient and it is mainly depending on the forces generated by the propeller. When the FSHC is under the floating mode, the resistance force is greatly increased as the hull is partially submerged in the water as the traditional vessels. The platform mode is generally studied in this paper as the FSHC cruises at high speed in the sea with less resistance. Considering that the craft is left-right symmetric and the FSHC operates under the platform mode at a straight line with small angle maneuver in the open sea, the longitudinal motion of the craft is slightly coupled with lateral motion, thus the longitudinal motion can be analyzed independently (Fossen, 1994) (Kim and Yamato, 2004a; b).

Some efforts have been devoted to the motion control of the FSHC (Kim and Yamato, 2004a). studied the variation of foil’s lift and redesigned the control law of the fully-submerged hydrofoil based on Kalman filtering. While, the method is only effective in still water as it is unable to reject the disturbances of the waves (Kim and Yamato, 2004b). designed feedback controller combined

* Corresponding author.
E-mail address: niuhongmin@hrbeu.edu.cn (H. Niu).
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with feed-forward controller based on optimal preview servo system to decrease the influence of the wave disturbance. While, the prediction of ocean wave is not contained in the preview system. Gradually (Bai and Kim, 2010), applied the PID controller for the motion control of the hydrofoil craft. The PID controller has large overshoot and long settling time although it is easy to apply (Liu et al., 2016), and (Liu et al., 2017) studied the course keeping of the fully-submerged hydrofoil craft with parameter perturbations and disturbances using observers (Liu et al., 2011), utilized the Least Squares Support Vector Machine (LSSVM) for the course keeping of ship. Other control strategies such as feedback control, Linear Quadratic Regulator (LQR) control and linear robust theory are also utilized for the hydrofoil craft control in (Bidikli et al., 2017) and (Zhang, 2016). (Zhang, 2009) designed a robust controller using MIMO gain shaping algorithm for the longitudinal motion control of the hydrofoil craft. These methods are designed based on the simplified linear function of the craft and the disturbance of waves are neglected. Thus, these methods are lack of practical application. Recently, various advanced methods are introduced for their better performance. Recently, the predictive control is used for ship dynamic positioning in (Li et al., 2017). (Djavareshkian and Esmaeili, 2013) used the fuzzy control method to analyze the hydrofoil performance. Other advanced methods such as neural network control, adaptive strategy, sliding mode control have been proposed for motion control of different rigid objects. Sliding Mode Control (SMC) is attractive as its simplicity and insensitivity to the external disturbances. The sliding mode control and its modified forms are widely studied and applied for spacecraft in (Li et al., 2016) and (Cong et al., 2013). While, the procedure is complex for higher order system and the robustness of the sliding mode control is always guaranteed by using the large switching gain which results in substantial chattering of the controller. The Complementary Sliding Mode Control (CSMC) is introduced which can reduce the steady-state error and improve the dynamic response of the system with fast convergence characteristic compared with traditional sliding mode control (Lin et al., 2013), and (Lin et al., 2012) applied it to the linear motors with intelligent algorithm (Zhao and Zhao, 2015), used it for the Permanent Magnet Synchronous Motor (PMSM) speed regulations due to its fast convergence and robustness to the disturbance. While, there is still less research about its applications on the FSHC with uncertain disturbances. Thus, further study on how to apply it to the FSHC is significant.

As is known, the submerged hydrofoil craft is inherently under the influence of various environmental disturbances caused by tides, waves and so on (Neves, 2016), analyzed and summarized the dynamic stability of ships in waves. To reduce the influence of the uncertain external disturbance, the Disturbance Observer (DOB) is broadly used to estimate and reject the external disturbance without influencing the control accuracy. The DOB possesses fast and excellent performance as it compensates disturbances straightforward through the feed forward channel (Yang et al., 2012). It is worth mentioning that the DOB is extensively used such in spacecraft (Gao et al., 2016), robotic manipulators (Chen et al., 2000) and ship motion (Hu et al., 2014) and (Song et al., 2016) for disturbance rejection. For the underwater vehicle system, the flow of deep water varies slowly corresponding to wave equation. In robotic systems, the disturbance is always assumed as time invariant or constant in finite time that the derivative of the disturbance is considered to be zero. As the wave equation is complicated, uncertain and the frequency is higher than the system, the assumption that the first derivative of the disturbance is zero is not suitable for the FSHC system.

Furthermore, the high switching gain of the sliding mode controller which is used to offset the disturbance is inevitable to result in the chattering problem. There are some methods presented to eliminate the chattering phenomenon (Cui et al., 2016), used the saturation function to replace the signal function. While, it brings about the steady-state error depending on the selection of boundary layer. Thus, the hyperbolic tangent function is utilized to attenuate the chattering of the actuator in this paper. Besides, the adaptive update law is widely utilized to estimate parametric uncertainties for controller design (Mondal and Mahanta, 2014), introduced an adaptive tuning method to deal with the system uncertainties. Thus, design adaptive update law is proposed to estimate the robust gain of the controller in this paper.

Inspired by the approaches presented in the past, this paper introduces a modified adaptive complementary sliding mode controller with disturbance observer particularly for the MIMO system of the FSHC. The uncertain disturbance and unknown perturbations of the system compose the generalized disturbances. The nonlinear longitudinal dynamic model of the FSHC with generalized disturbances is established and the uncertain disturbance caused by ocean waves is analyzed. The proposed disturbance observer is applied for the estimation of generalized disturbances without strict conditions which is less conservatism and the estimation error is ultimately bounded. The modified sliding mode surface which contains the disturbance estimation is designed that it can attenuate the switching gain without causing an adverse effect on the system. The continuous hyperbolic tangent function is used to replace the discontinuous sign function and the adaptive law is designed to approximate the switch gain that the chattering phenomenon can be attenuated. The designed continuous controller guarantees the global asymptotic stability of system in the presence of generalized disturbances utilizing the Lyapunov theory.

In this paper, a modified adaptive complementary sliding mode control method with nonlinear disturbance observer is implemented for the MIMO system of the FSHC. The proposed methodology guarantees global asymptotic stability of the closed-loop system and improves the disturbance rejection performance of the FSHC. The main contributions are as follows:

1. The nonlinear dynamics of the longitudinal motion of the FSHC is proposed and the controller is designed directly based on the MIMO nonlinear model without any linearization. The external wave disturbances are accurately calculated and simulated based on stochastic process theory.

2. Nonlinear disturbance observer is designed considering the generalized disturbance generated by ocean waves and perturbations. Only boundedness of the derivative of the disturbance is required in the disturbance observer design rather than the upper bound of the disturbance itself, which decreases the system conservatism during the control design.

3. A modified adaptive complementary sliding mode control technique is presented for the stabilization of the longitudinal motion of the FSHC with less stabilization error and fast response. The modified sliding surface contains the estimation of disturbance which can attenuate the switch gain and retain the disturbance rejection performance. A continuous hyperbolic tangent function is utilized instead of the discontinuous sign function to reduce the chattering of the actuator.

4. The proposed modified continuous controller with robust disturbance compensation guarantees global asymptotic stability of the MIMO closed-loop system of the FSHC.

This paper is organized as follows. Section 2 introduces the construction of the control objective including the dynamic model of the FSHC and the analysis of external disturbance. The nonlinear disturbance observer (NDOB) based modified adaptive
complementary sliding mode controller and the stability analysis of
the MIMO closed-loop system are presented in section 3. The
related formulas are derived in detail. Section 4 shows the validity
of the proposed method by simulating on the FSHC. The numerical
results are analyzed and discussed. The conclusions of the study are
given in section 5.

2. Problem formulation

2.1. Dynamic model of the FSHC

As the craft is left-right symmetric and the longitudinal motion
couples slightly with lateral motion, the longitudinal motion of the
FSHC is considered independently in this paper. The typical
configuration of the FSHC is shown in Fig. 1. The longitudinal mo-
tion of the FSHC are composed of two parts, the pitch motion about
y-rotation and the heave motion along the z-axis. Other motions
such as roll, sway, yaw and surge are neglected. The constant for-
ward speed is defined as $U_e$. The coordinate system and the lon-
gitudinal motion of the FSHC are shown in Fig. 2 according to (Kim
and Yamato, 2004a).

The FSHC is equipped with T-shaped bow foil and aft foil. The T-
shaped bow foil is equipped with two controlled flaps, acting
together which are shown in Fig. 3(a). The aft foil has a pair of
synchronous central flaps acting together and two ailerons with
differential motion which are shown in Fig. 3(b). Each pair has in-
dependent drive system. The central flaps of the aft foil and the bow
foils are working together for the longitudinal motion control of
the FSHC. The rudders equipped in the vertical struts together with
the ailerons are used for roll and yaw dynamics which are different
from the nominal vessels. Thus, the rudder system would not be
used in the longitudinal motion control of the FSHC when the FSHC
cruises at a straight line and there is no coupling motion effect. The
schematic of T-shaped bow foil and the aft foil are shown in Fig. 3.
The schematic of hydrofoil section profile and coupling shaft are
shown in Fig. 4.

The right-hand coordinates are defined as Fig. 2. The earth-fixed
frame $O-XYZ$ is an inertial coordinate frame, with the $X$-axis
points to the north, the $Y$-axis points towards the east and the $Z$-
axis is directed to the center of the earth. The body-fixed frame is
fixed to the gravity center $G$ of the FSHC, which $x$-axis points to the
bow, the $y$-axis points to the starboard and the $z$-axis directs to the
downwards. The direction of the arrow points the positive direction
of the translation and rotation.

The fully submerged hydrofoil craft is a kind of marine vessel
and it is a rigid body. The model of the FSHC which contains the
translational motion and the rotational motion is established based
on the rigid body dynamics (Fossen, 1994; Kim and Yamato, 2004b).
The $p, q, r$ are the angle velocity in the body-fixed frame and $\xi, \eta, \zeta$
are the position coordinate in the earth-fixed frame. The $\psi, \theta, \phi$
are the azimuth angle between earth-fixed frame and body-fixed frame
which is shown in Fig. 2, and the relation between earth-fixed and
body-fixed coordinate is as follows (Fossen, 2011).

$$\begin{bmatrix}
\xi \\
\eta \\
\zeta \\
\end{bmatrix} = \begin{bmatrix}
\cos \psi \cos \theta & \sin \psi \cos \theta & -\sin \theta \\
\cos \psi \sin \theta \sin \psi - \sin \psi \cos \phi & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \cos \theta \sin \phi \\
\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi & \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi & \cos \theta \cos \phi \\
\end{bmatrix} \begin{bmatrix}
\xi \\
\eta \\
\zeta \\
\end{bmatrix}.$$ (2)

The relation between the $\begin{bmatrix}
\dot{z} \\
\dot{\theta} \\
\end{bmatrix}$ and $\begin{bmatrix}
w \\
q \\
\end{bmatrix}$ of the longitudinal
motion can be obtained as follows

$$\begin{bmatrix}
\dot{z} \\
\dot{\theta} \\
\end{bmatrix} = \begin{bmatrix}
\cos \theta & 0 \\
0 & 1 \\
\end{bmatrix} \begin{bmatrix}
w \\
q \\
\end{bmatrix} + \begin{bmatrix}
-U_e \sin \theta \\
0 \\
\end{bmatrix}.$$ (3)

where $\begin{bmatrix}
\dot{z} \\
\dot{\theta} \\
\end{bmatrix}$ are the heave velocity and pitch angle velocity in the
earth-fixed frame, $\begin{bmatrix}
w \\
q \\
\end{bmatrix}$ are the same meaning in the body-fixed
coorinate. $U_e$ is the constant speed of the FSHC.

In this study, the origin of earth fixed frame is assumed to
coincide with the center of the body-fixed coordinate. According to
the theorem of momentum and dynamics, the longitudinal dy-
namic equations of the FSHC can be written as

$$(Z_w - m)\ddot{w} + Z_{ww}w + Z_{zz}z + Z_{q}q + (Z_{q} + U_em)q + Z_{\theta}q = Z_{\theta} \delta_{e} - Z_{\theta} \delta_{f} - Z_{S}.$$ (4)
where $Z_{w} - m, Z_{q}, M_{w}, M_{q} - I_{y}$ are inertia matrices. $Z_{o}, M_{o}, Z_{q}, M_{q}$ are the restoring matrices, $Z_{w}, M_{w}, Z_{q}, M_{q}$ are the damping matrices. $Z_{o}, M_{o}, Z_{q}, M_{q}$ are control force and moment coefficient matrices. $Z_{S}$ and $M_{S}$ are external disturbances acting on the FSHC. $m$ is the mass of the FSHC and $w$ is the heave velocity. $\theta$ is the pitch angle and $q$ is the pitch angular velocity. $Z, M$ are the force of z-axis and moment around y-axis respectively. $\delta_{e}$ and $\delta_{f}$ are the flap deflection angles of bow foil and aft foil. $Z_{w}$ represents the derivative of hydrodynamic force in z-axis about heave velocity as $dz/dw$. $M_{w}$ means the derivative of hydrodynamic moment in y-axis about heave velocity, other coefficients such as $Z_{o}, Z_{q}, Z_{q}, M_{w}, M_{o}, M_{q}, M_{q}, M_{q}, M_{o}$ are defined of the similar meaning.

Transforming the dynamic function of the FSHC in Eq. (4) and Eq. (5), one can obtain

\[
\begin{align*}
M_{w} \dot{w} + M_{w} w + M_{z} \dot{z} + (M_{q} - I_{y}) \dot{q} + M_{q} q + M_{q} \theta = & -M_{b_{e}} \delta_{e} - M_{b_{f}} \delta_{f} - M_{S},
\end{align*}
\]

(5)

\[
\begin{align*}
(M_{q} - I_{y}) \dot{q} + (M_{q} - I_{y}) q = & \frac{M_{w}}{(M_{q} - I_{y})} \dot{w} + \frac{M_{w}}{(M_{q} - I_{y})} w + \frac{M_{z}}{(M_{q} - I_{y})} \dot{z} + \frac{M_{q} + U_{w} m}{(M_{q} - I_{y})} q + \frac{M_{q}}{(M_{q} - I_{y})} \theta
\end{align*}
\]

(6)

where $Z_{w}' = \frac{Z_{w}}{(Z_{w} - m)}, M_{w}' = \frac{M_{w}}{(M_{q} - I_{y})}$. They are equal to the disturbance exerts to the system.

For the convenience of illustration, we define the parameters in Eq. (6) and Eq. (7) such as follows according to (Liu, 1991) and (Aghababa and Akbari, 2012).

\[
\begin{align*}
\alpha_{1} = \frac{Z_{w}}{(Z_{w} - m)} \quad \alpha_{2} = \frac{Z_{o}}{(Z_{w} - m)} \quad \alpha_{3} = \frac{Z_{q}}{(Z_{w} - m)} \quad \alpha_{4} = \frac{(Z_{q} + U_{w} m)}{(Z_{w} - m)} \quad \alpha_{5} = \frac{Z_{o}}{(Z_{w} - m)} \quad \alpha_{6} = \frac{Z_{o}}{(Z_{w} - m)} \quad \alpha_{7} = \frac{Z_{q}}{(Z_{w} - m)} \quad \alpha_{8} = 1,
\end{align*}
\]

(7)

\[
\begin{align*}
\beta_{1} = \frac{M_{w}}{(M_{q} - I_{y})} \quad \beta_{2} = \frac{M_{w}}{(M_{q} - I_{y})} \quad \beta_{3} = \frac{M_{z}}{(M_{q} - I_{y})} \quad \beta_{4} = \frac{M_{z}}{(M_{q} - I_{y})} \quad \beta_{5} = \frac{M_{w}}{(M_{q} - I_{y})} \quad \beta_{6} = \frac{M_{w}}{(M_{q} - I_{y})} \quad \beta_{7} = \frac{M_{w}}{(M_{q} - I_{y})} \quad \beta_{8} = 1,
\end{align*}
\]

(8)

where $\alpha_{8}, \beta_{8}$ are the parameters of $Z_{S}, M_{S}$.

Combining Eqs. (3), (6) and (7) and translating them to the state-space model by Laplace transform, the second-order system representation of FSHC is given by

\[
\begin{align*}
\dot{x}_{1} = x_{2},
\end{align*}
\]

(8)

\[
\begin{align*}
x_{2} = f_{1}(x_{1}, x_{2}) + f_{2}(x_{1}, x_{2}) + Bu + DW.
\end{align*}
\]

(9)

The system states are $x_{1} = [\dot{z}, \dot{\theta}]^{T} \in \mathbb{R}^{2 \times 1}$, $x_{2} = [\dot{z}, \dot{\theta}]^{T} \in \mathbb{R}^{2 \times 1}$, $f_{1}(x_{1}, x_{2}) \in \mathbb{R}^{2 \times 2}$, $f_{2}(x_{1}, x_{2}) \in \mathbb{R}^{2 \times 2}$ are the nonlinear terms of the system.
The parameters are illustrated as follows. $a_i$ is the amplitude, $\omega_{ai}$ is the encounter frequency, $\psi_i$ is the phase and $\epsilon_i$ is the random variable of $[0,2\pi]$. $S(\omega_i)$ is the P-M spectrum, $H$ is the significant wave height. $\Delta \omega$ is the strip breadth of the spectral density function and is equal to 0.04. The calculating frequency bounds are chosen as $\omega_1 = 0.1$ and $\omega_2 = 1.5$ according to the ITTC single parameter spectral density function. $\omega_i$ is the angular frequency of ocean wave, $\lambda_i$ is the wave length, $c$ is the wave velocity and $\chi$ is the intersection angle from the direction of $U_e$ to the wave direction based on the right-hand rule.

### 3. Disturbance observer based adaptive complementary sliding mode control

In this section, the nonlinear disturbance observer is designed to estimation the external unknown disturbance. Then the modified adaptive complementary sliding mode control based on disturbance observer is designed. The control configuration of the closed-loop system is given by Fig. 5.

#### 3.1. Disturbance observer design

The generalized disturbances exert great influence on the FSHC and it is difficult to know the exact form of the disturbance. The nonlinear disturbance observer is developed to estimate the disturbance and the disturbance is compensated through the feed forward channel. In this section, we introduce the modified nonlinear disturbance observer which only needs the boundedness of the derivative of the disturbance and is convenient to apply. The stability of the observer system is illustrated by Lyapunov function.

Considering the system established by Eqs. (8), (12) and (13), a nonlinear disturbance observer is designed as follows

$$\dot{d} = p + h$$

$$\dot{p} = -LP + L(-f_1(x_1,x_2) - f_2(x_1,x_2) - Bu - h)$$

The vector $h$ can be determined from the gain matrix of observer that $h = L\dot{x}_2$. Where $\dot{d}, L \in \mathbb{R}^{2 \times 2}$ are the estimation of disturbance and the gain of observer respectively.

Then, the derivative of estimated disturbance is presented by

$$\dot{\delta} = \dot{p} + h = -LP + L(-f_1(x_1,x_2) - f_2(x_1,x_2) - Bu - h) + \dot{h} = \dot{L}d$$

Define $\dot{\delta} = d - \dot{d}$ as the disturbance estimation error. According to Eq. (21), the $\dot{d}$ can be shown as $\dot{d}$.

It is worth noting that the convergence rate of the estimation error, to a great extent, is related to the matrix $L$. Thus, the matrix $L$ should be determined seriously by trial and error. As the disturbance varies relatively fast to the system dynamics, we assume that the derivative of disturbance is bounded rather than assumed to be...
Considering the nonlinear disturbance designed by Eqs. (19), (20) and Theorem 1, the disturbance observer is ultimately stable with the bound of the estimation error as follows.

**Theorem 1.** The disturbance estimation error is bounded if Assumption 2 is held. For any $Q = Q^T > 0$, there is a $P = P^T > 0$ that subject to the Riccati function $A^TP + PA = -Q$, since $A$ is Hurwitz.

**Proof** Choose the Lyapunov function of observer system as follows.

$$V_1 = \tilde{d}^T P \tilde{d}. \tag{22}$$

By differentiating Eq. (22), pointing the Rayleigh Inequality $\lambda_{\min}(Q) \| \tilde{d} \|^2 \leq \tilde{d}^T Q \tilde{d} \leq \lambda_{\max}(Q) \| \tilde{d} \|^2$ and defining $-L = A \in \mathbb{R}^{2 \times 2}$, we can obtain that

$$V_1 = \tilde{d}^T P \tilde{d} + \tilde{d}^T \tilde{d} = \tilde{d}^T P \tilde{d} + \tilde{d}^T (A + A^T) \tilde{d} = \tilde{d}^T (A + A^T) \tilde{d} \leq -\lambda_{\min}(Q) \| \tilde{d} \|^2 + 2 \| \tilde{d} \| P \| M \|, \tag{23}$$

Considering the Eq. (23), the estimation error $\tilde{d}$ is uniformly bounded with the bounded norm $\gamma = 2 \| P \| M / \lambda_{\min}(Q)$, which denotes the smallest eigenvalue of $Q$.

**Remark 1.** Considering the nonlinear disturbance designed by Eqs. (19), (20) and Theorem 1, the disturbance observer is ultimately stable with the bound of the estimation error as $\gamma = 2 \| P \| M / \lambda_{\min}(Q)$. A great advantage of the proposed disturbance observer is that it does not need exact information of the states. It is able to estimate the disturbance only required the boundedness of the derivative of the disturbance that the conservatism of the system is reduced.

### 3.2. Adaptive complementary sliding mode control design

In this part, the modified complementary sliding surface with disturbance estimation is introduced. A continuous hyperbolic tangent function is utilized instead of the discontinuous sign function and an adaptive update law is designed to estimate the robust gain.

In order to guarantee the states stabilize to the equilibrium point accurately under the ocean disturbances and parameter perturbations, the tracking error and its derivation are defined respectively as follows.

$$e = x_1 - x_d = [e_1 \ e_2 \ e_3]^T, \quad \dot{e} = \dot{x}_2 - x_{2d} = [e_2 \ e_3]^T. \tag{24}$$

where $x_d = [0 \ 0]^T$ and $x_{2d} = [0 \ 0]^T$ are the equilibrium points.

The procedure of the complementary sliding mode controller design contains two steps. The first step is to select the sliding surface which contains the disturbance estimation of disturbance observer. The modified generalized sliding surface with estimation of disturbance is designed in the following (Han et al., 2017).

$$S_m = \left( \frac{d}{d\tau} + \eta \right) \int_0^\tau e(\tau)d\tau + \lambda \tilde{d} = \dot{e} + 2\eta e + \eta^2 \int_0^\tau e(\tau)d\tau + \lambda \tilde{d}, \tag{25}$$

where $\eta = \text{diag}(\eta_1, \eta_2)$, $\eta_1$, $\eta_2$, $\eta$ are positive constants to be determined. $\dot{\tilde{d}}$ is the disturbance estimation given by NDOB. Taking the derivative of Eq. (25) and using Eq. (12), the following Eq. (26) can be obtained

$$\dot{S}_m = \dot{\tilde{d}} + 2\eta \dot{e} + \eta^2 \ddot{e} = \dot{f}_1(x_1, x_2) + \ddot{f}_2(x_1, x_2) + Bu + d - \ddot{x}_{2d} + 2\eta \dot{e} + \eta^2 \ddot{e} + \lambda \tilde{d}. \tag{26}$$

Then, the complementary sliding surface with disturbance estimation is designed as follows

$$S_h = \left( \frac{d}{d\tau} + \eta \right) \left( \frac{d}{d\tau} - \eta \right) \int_0^\tau e(\tau)d\tau + \lambda \tilde{d} = \dot{e} - \eta^2 \int_0^\tau e(\tau)d\tau + \lambda \tilde{d}. \tag{27}$$

Corresponding to the same positive diagonal matrix $\eta$, the relation between $S_m$ and $S_h$ can be gained in the following

$$\sigma(t) = S_h + S_m = 2\dot{e} + 2\eta e + 2\lambda \tilde{d}, \tag{28}$$

$$S_h = S_m - \eta \sigma + 2\lambda \eta \tilde{d}. \tag{29}$$

The second step is to determine an adaptive complementary sliding mode control law. The continuous hyperbolic tangent
function is utilized to eliminate the chattering phenomenon caused by discontinuous sign function.

\[ u_{CSMC} = u_{eq} + u_c, \quad (30) \]

\[ u_{eq} = -B^{-1} \left( f_1(x_1, x_2) + f_2(x_1, x_2) + 2\eta \hat{e} + \eta^2 e + \sigma^+ \left( S_m \eta \sigma + 2\lambda S_h \hat{d} \right) \right) + \hat{d} + \sigma^+ \hat{\mu} (||\hat{L}|| \cdot ||\sigma|| + 2k_1 \epsilon + 2||P|| \cdot M), \quad (31) \]

\[ u_c = -B^{-1} \beta \tanh \left( \frac{\theta}{\epsilon} \right), \quad (32) \]

where \( \hat{d} \in \mathbb{R}^{2 \times 1} \) is the output of disturbance observer, \( \sigma^+ = \sigma(\sigma^T \sigma)^{-1} \) is the Moore-Penrose pseudoinverse, \( \hat{\mu} \) is the adaptive law of controller designed in the following subsection and \( \beta \) is the hyperbolic tangent function.

Appropriate adaptation law is designed as Eq. (33) to estimate the robust gain of the controller to attenuate the chattering phenomenon.

\[ \hat{\mu} = \beta (||L + \hat{L}|| \cdot ||\sigma|| + 2M \cdot ||P||). \quad (33) \]

where \( \hat{\mu} \) is the adaptive law to undertake the unknown controller parameter \( \mu \) of hyperbolic tangent function. \( \beta \) is the positive parameter to be determined. Define the error of adaptive law as \( \hat{\mu} = \mu - \hat{\mu} \). As \( \mu \) is a constant parameter, we can get

\[ \hat{\mu} = \hat{\mu} - \mu = -\hat{\mu}. \quad (34) \]

3.3. Stability analysis

In this section, the stability of the overall system is analyzed. The global asymptotic stability of the closed system is derived based on Lyapunov stability arguments. In order to illustrate the stability, the reference lemma is firstly given.

**Lemma 1.** (Aghababa and Akbari, 2012): According to hyperbolic tangent function, the following inequality holds for any \( \epsilon > 0 \) and for any \( \chi \in \mathbb{R}^n \)

\[ ||\chi||_1 - \chi^T \tanh (\chi/\epsilon) \leq n k_1 \epsilon, \quad k_1 = 0.2785. \quad (35) \]

\[ V_2 = \sigma^T \left[ f_1(x_1, x_2) + f_2(x_1, x_2) - B \cdot B^{-1} \left( f_1(x_1, x_2) + f_2(x_1, x_2) + \sigma^+ \left( S_m \eta \sigma + 2\lambda S_h \hat{d} \right) \right) \hat{d} + 2\eta \hat{e} + \eta^2 e + \sigma^+ \hat{\mu} (||\hat{L}|| \cdot ||\sigma|| + 2k_1 \epsilon + 2M \cdot ||P||) + \hat{\mu} \tanh \left( \frac{\theta}{\epsilon} \right) + 2\eta \hat{e} + \eta^2 e + \lambda \hat{d} \right] \]

\[ -S_m \eta \sigma + 2\lambda S_h \hat{d} - \frac{\hat{\mu}}{\beta} + 2\hat{d} \hat{P} \]

\[ = \sigma^T \left( -\hat{\mu} \tanh \left( \frac{\theta}{\epsilon} \right) \right) - \hat{d} \left( S_m \eta \sigma + 2\lambda S_h \hat{d} \right) - S_m \eta \sigma - 2\lambda S_h \hat{d} - S_m \eta \sigma + 2\lambda S_h \hat{d} \]

\[ -\hat{\mu} (||\hat{L}|| \cdot ||\sigma|| + 2k_1 \epsilon + 2M \cdot ||P||) - \frac{\hat{\mu}}{\beta} \hat{d} + 2\hat{d} \hat{P} - \hat{d} \hat{Q} \hat{d}. \]

Now considering the Lemma 1 and \( \sigma \in \mathbb{R}^2 \), we can get

\[ -\sigma^T \tanh (\sigma/\epsilon) \leq -||\sigma||_2 + 2k_1 \epsilon, \quad k_1 = 0.2785. \quad (36) \]

Introducing the adaptive law in Eq. (33), it is clear that
\[ V_2 \leq -\sigma^T \sigma + 2k_1 \tilde{e} \tilde{u} + \|I + \lambda L\| \cdot \| \tilde{d} \| \cdot \| \sigma \| - \tilde{u} \mu(\|I + \lambda L\| \cdot \| \sigma \| + 2k_1 \epsilon + 2M \cdot \| P \|) \\
- \tilde{u} \mu(\|I + \lambda L\| \cdot \| \sigma \| + 2M \cdot \| P \|) - \lambda_{\text{min}}(Q) \| \tilde{d} \|^2 \]

\[ \leq -\sigma^T \sigma - \tilde{u} \mu(\|I + \lambda L\| \cdot \| \sigma \| + 2M \cdot \| P \|) - \lambda_{\text{min}}(Q) \| \tilde{d} \|^2 \]

\[ \leq -\sigma^T \sigma - \lambda_{\text{min}}(Q) \| \tilde{d} \|^2 \]

\[ \leq -\sigma^T \sigma \leq 0. \tag{41} \]

**Remark 2.** The discontinuity sign function in the control law is replaced by the continuous hyperbolic tangent function, thus the chattering problem of controller is attenuated. On the other hand, the proportion of the hitting control law influences the magnitude of the chattering, the adaptive law designed can reduce the gain of the tanh function and eliminate the chattering problem further.

**Remark 3.** As the sliding surface satisfies the reaching condition simultaneously, the control variables \( e, \dot{e} \) contained in the sliding surface will move onto the sliding surface and slide to the origin in finite time. Compared with other discontinuous functions contained sign function, the continuous controller with tanh function in this paper guarantees the global asymptotic stability of the closed loop system and has much strong stability that the states converge to the desired reference point with less error.

### 4. Simulation results

In order to analyze the control strategy, the designed modified

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal value</th>
<th>Unit</th>
<th>Parameter</th>
<th>Nominal value</th>
<th>Unit</th>
</tr>
</thead>
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<tr>
<td>( m )</td>
<td>2.62 \times 10^7</td>
<td>kg</td>
<td>( I_p )</td>
<td>3.9 \times 10^7</td>
<td>kg \cdot m^2</td>
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<td>( U_i )</td>
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<td>m/s</td>
<td>( I_i )</td>
<td>17.86</td>
<td>m</td>
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<td>( a_1 )</td>
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<td>1/s</td>
<td>( a_2 )</td>
<td>0.338</td>
<td>1/s^2</td>
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<tr>
<td>( a_3 )</td>
<td>3.41</td>
<td>m</td>
<td>( a_4 )</td>
<td>42.4</td>
<td>m/s</td>
</tr>
<tr>
<td>( a_5 )</td>
<td>454</td>
<td>m/s^2</td>
<td>( a_6 )</td>
<td>51.5</td>
<td>m/s^2</td>
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<tr>
<td>( a_7 )</td>
<td>62.9</td>
<td>m/s^2</td>
<td>( b_1 )</td>
<td>0.016</td>
<td>1/m</td>
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<tr>
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<td>1/(m \cdot s)</td>
<td>( b_3 )</td>
<td>0.069</td>
<td>1/(m \cdot s^2)</td>
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<tr>
<td>( b_4 )</td>
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<td>1/s</td>
<td>( b_5 )</td>
<td>0.654</td>
<td>1/s^2</td>
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<tr>
<td>( b_6 )</td>
<td>-4.58</td>
<td>1/s^2</td>
<td>( b_7 )</td>
<td>1.88</td>
<td>1/s^3</td>
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### Table 2

<table>
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<tr>
<th>Parameters of MACSMC</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>Input parameters of observer</th>
<th>Parameter values</th>
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<td>Gain1</td>
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<td>( L_1 )</td>
<td>diag [18 9]</td>
</tr>
<tr>
<td>Gain2</td>
<td>0.4</td>
<td>2.4</td>
<td>( L_2 )</td>
<td>diag [2 3]</td>
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<tr>
<td>Gain3</td>
<td>1.5</td>
<td>5.8</td>
<td>( L_3 )</td>
<td>diag [30 20]</td>
</tr>
</tbody>
</table>

**Fig. 6.** Estimation of disturbance force of encounter angle 45°.

**Fig. 7.** Estimation of disturbance moment of encounter angle 45°.
adaptive complementary sliding mode control law based on NDOB is implemented to the FSHC system through the MATLAB/Simulink platform. The nominal parameter values are shown in Table 1 and the input parameters are shown in Table 2 based on (Liu, 1991) and (Fossen, 2011).

To demonstrate the effectiveness of the proposed controller, the simulations are arranged as following steps.

1) The disturbance observer with different parameters are compared to obtain the disturbance gain by trial and error.
2) The performance and the control output of different control gains of the nonlinear disturbance observer based modified adaptive complementary sliding mode control (MACSMC-NDOB) method are analyzed for the optimal performance.
3) The longitudinal motion of complementary sliding mode control (CSMC), nonlinear disturbance observer based complementary sliding mode control (CSMC-NDOB) and the MACSMC-NDOB method are compared based on the simulations and the error statistics to illustrate the effectiveness of the proposed approach.

Assuming that the FSHC cruises at a constant speed, the course direction of the craft is not changed. The parameter perturbations are about ±10% over the nominal parameter. The significant wave height is 1.5 m and the angles of ocean wave are selected as 45°, 135° respectively.

To verify the performance of the disturbance observer, the estimation results of different parameter combinations are compared with real disturbance in the condition that the wave encounter angles are 45° and 135° respectively. The observer of different parameters L1, L2, L3 are compared in Figs. 6–9. From the experiment results in Figs. 6–9, one can gain that the performance of the observer with parameter L1 is better than that of L2 and L3. The estimation error of the observer with L2 and L3 are larger than that of L1 as they can’t track the disturbance precisely and there are some time delays. Then we choose the parameter L1 of observer for further study. The difference between estimation results of the force and moment is affected by the bandwidth of the observer system and the disturbance. The estimation errors of observer with L1 under the encounter angle of 45°, 135° are given in Fig. 10 and Fig. 11.

The estimation errors are bounded in all cases and much smaller...
According to the disturbance. Besides, the value of estimation error is similar under different encounter angles. The disturbance observer has ability of excellent performance and reduction of conservatism in different cases.

Comparing the disturbance forces and moments of different encounter angles respectively in Figs. 6-11, we can get that the disturbance generated by ocean wave has slight difference. Thus, we take one condition into account for further study.

In order to illustrate the effectiveness of the chosen controller gain, the performance of different combination of the controller parameter are shown in Figs. 12 and 13. The control output of different gains are shown in Figs. 14-16. The control output of the CSMC-NDOB is shown in Fig. 17.

The parameters $\eta_1$, $\eta_2$ that in the complementary sliding mode surface effect the rate of the error convergence and the boundary of the sliding mode surface. They are selected by trail and error. According to Figs. 12 and 13, we can get that the heave motion and pitch angle of Gain1 is obviously smaller than that of Gain2.

Although the performance of Gain3 is better than that of Gain1, control output of Gain3 exists some chattering phenomenons, thus it may cause abrasion and is not optimal to apply. As the control output of Gain1 is much smooth, the foil deflection angles are in the limitation of the actuator and the performance is much better that the Gain1 is the optimal parameter of the controller. The $\eta_1$, $\eta_2$ in MACSMC-NDOB method are set as Gain1 that $\eta_1 = 0.4$, $\eta_2 = 5.2$ and the parameter of the hyperbolic tangent function is $\varepsilon = 0.2$. The parameter of the hyperbolic tangent function is related to the inflection point, thus it effects the smoothness of the control output. If the parameter is selected too small, the inflection point approaches to zero and the hyperbolic tangent function would be like the sign function, the chattering problem can’t be solved. Thus, the parameter is selected moderately as 0.2.

Figs. 14 and 17 show the experimental results of the controllers of MACSMC-NDOB method and CSMC approach to identify that the tanh function and adaptive law adopted in MACSMC-NDOB are capable to solve the chattering problem.
The controller’s switching frequency of CSMC-NDOB is higher than that of MACSMC-NDOB according to Figs. 14 and 17. Then, it can be observed that the chattering of proposed MACSMC-NDOB method is reduced much using the hyperbolic tangent function and adaptive law. Besides, the energy consumes less and the mechanical abrasion can be reduced for facility life prolonging.

To illustrate the effectiveness of the proposed method, the results of heave motion, pitch angle of CSMC, CSMC based on NDOB, MACSMC based on NDOB are compared in Figs. 18 and 20. The performance of the heave and pitch of MACSMC-NDOB with $L_1$ are plotted in Figs. 19 and 21, separately.

The comparative results in Figs. 18 and 20 show that the longitudinal motion of CSMC without NDOB changes sharply than that with NDOB, which illustrate that the nonlinear disturbance observer can be used to estimate and compensate the generalized disturbance through the feed forward channel. Thus, the influence of the external disturbance is reduced. Simulation results indicate that the MACSMC with disturbance observer has better performance than the CSMC approach as the steady-state error is much reduced. The heave motion error of the MACSMC-NDOB is about...
80% of the CSMC method and the pitch angle error of the MACSMC-NDOB is about 20% of the CSMC method. Besides, the proposed continuous control law can guarantee the global asymptotic stability of overall system with less error and reject the uncertain disturbance influence.

In order to assess the robustness of proposed method under generalized disturbances, the control results under encounter angle 45° between nominal and perturbation conditions are plotted in Figs. 22 and 23. Considering Figs. 22 and 23, it is obvious that the states of the system between nominal system and perturbation conditions have little difference. The states are bounded and the system is stable under nominal and perturbation conditions. Thus, we can easily get the point that the proposed method has strong robustness and is able to reject the parameter perturbations.

In order to measure the performance of the controllers explicitly, we define performance criterion as root mean square error (RMSE) and average value of steady-state error. The RMSE value of the steady-state is defined as 
\[ E_{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} |e(i)|^2} \]
and the average value of the steady-state error is defined as 
\[ E_{av} = \frac{1}{N} \sum_{i=1}^{N} |e(i)| \]. The desired value of states are zero.

Considering the parameters in Tables 3 and 4, it is obvious that the MACSMC-NDOB has the best performance as the RMSE and average of steady-state error are smaller than the CSMC. The performance of the CSMC-NDOB is a little bit worse than that of the MACSMC-NDOB. Besides, there exists some large values of the states in the CSMC-NDOB which may influence the stability of the FSHC. Moreover, the slew rate of the controller of the CSMC-NDOB is much higher than it is not applicable. Thus, to sum up, the MACSMC-NDOB method is better than the other two approaches. Furthermore, comparing the criteria between nominal and perturbation conditions in the same condition, one can get that criteria have little difference that the proposed method has the strong robustness ability.

It is obvious the modified adaptive complementary sliding mode controller based on disturbance observer can attenuate the effect of disturbance and reduce the longitudinal motion of the FSHC a lot. Besides, the chattering phenomenon of the controllers can be solved by hyperbolic tangent function and adaptive law and the designed continuous controller ensures the global asymptotic stability of the overall system with strong robustness. Thus, the NDOB based MACSMC is effective in both theoretical and practical applications for the FSHC longitudinal motion control.

5. Conclusions

This paper has presented a modified adaptive complementary sliding mode controller based on disturbance observer to reduce the longitudinal motion of a fully-submerged hydrofoil craft. The nonlinear dynamic model of the FSHC is established considering the generalized disturbances. Then, the MACSMC based on NDOB strategy is designed to reduce the influence of the generalized
disturbances and guarantee the global asymptotic stability of the closed loop system. From the experimental results, the heave motion error of the MACSMC-NDOB is about 80% of the CSMC method and the pitch angle error of the MACSMC-NDOB is about 20% of the CSMC method. The RMSE and average of steady-state error of proposed method is much lower than the other two approaches. Furthermore, the chattering phenomenon is resolved by the hyperbolic tangent function and adaptive law for mechanical life prolonging. Thus, the MACSMC-NDOB exhibits better performance, such as fast convergence, least steady-state error of the longitudinal motion of the FSHC. Then, the security and comfort of the FSHC are guaranteed.

While the proposed method has better performance, there is a plenty room for development such as practical applications. The state observer would be studied for the states estimation which are used in the controller for the FSHC. Furthermore, the actuator limitation would be considered in practice for the engineering applications in the future work to make the controller much practicability.

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