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A HYPOTHESIS TESTING PROCEDURE OF ASSESSMENT FOR THE LIFETIME PERFORMANCE INDEX UNDER A GENERAL CLASS OF INVERSE EXPONENTIATED DISTRIBUTIONS WITH PROGRESSIVE TYPE I INTERVAL CENSORING†

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Abstract. One of the main objective of manufacturing industries is to assess the capability performance of different processes. In this paper, we use the lifetime performance index C_L as a criterion to measure larger-the-better type quality characteristic for evaluating the product performance. The lifetimes of products are assumed to follow a general class of inverted exponentiated distributions. We use maximum likelihood estimator to estimate the lifetime performance index under the assumption that data are progressive type I interval censored. We also obtain asymptotic distribution of this estimator. Based on this estimator, a new hypothesis testing procedure is developed with respect to a given lower specification limit. Finally, two numerical examples are discussed in support of the proposed testing procedure.

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1. Introduction

One of the main objective in life testing experiments is to provide inference that are useful in predicting reliability of manufactured products, systems, devices, machines etc. Data collected using such experiments are normally censored in nature. A series of censoring methodologies for deriving statistical inference

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on unknown quantities of interest have been discussed in literature. To investigate performance capabilities of a process, among others, process capability indices (PCIs) such as C_p, C_{pk}, C_{pm} and C_{pmk} have found wide applications in practice. One may refer to Montgomery [11] and Kane [9] for several examples and important applications of PCIs in reliability analysis. In particular these references describe in detail C_L and C_{pl} indices also which measure the larger-better-type quality characteristics such as lifetime of product or system, tensile hardness, durability, etc. In many applications these procedures are investigated using the normal distribution. However lifetime characteristics are usually modeled using distributions such as exponential, lognormal, gamma and Weibull, among others. Tong et al. [14] obtained the uniformly minimum variance unbiased estimator (UMVUE) of C_l and further constructed a hypothesis testing procedure assuming exponential lifetime under complete sample situation. Recently Laumen and Cramer [12] investigated the lifetime performance index under progressive type II censoring for a gamma distribution. As mentioned life testing experiments are conducted under cost and time constraints and so it is difficult to record failure times of all products subjected to a test. To overcome such restrictions failure times are commonly recorded using different censoring procedures. One may refer to Balakrishnan and Aggarwala [2], Balakrishnan and Cramer [3], Balakrishnan and Kundu [4] for a detailed review of work done various censoring schemes.

In this work we consider the progressive type I interval censoring scheme to estimate the lifetime performance index. Aggarwala [1] initially introduced this censoring in literature. Recently, Wu and Lin [16] obtained the maximum likelihood estimate for C_L and suggested a hypothesis testing procedure for the one-parameter exponential distribution under progressive type I interval censoring. Dey et al. [5] obtained similar results for the Weibull distribution under progressive type II censoring. Wu and Lin [17], Wu and Lu [18], Wu et. al. [19], Wu et. al. [20] have also derived inferences for lifetime performance index based on progressive type I interval censoring using different lifetime models. Progressive type I interval censoring can briefly be described as follows. Suppose n items are put on a life test at time zero. Let $(T_1, T_2, \ldots, T_m)$ and $(p_1, p_2, \ldots p_m)$ respectively denote prefixed inspection times and removal percentages of the surviving units at time $T_i,\ i = 1, 2, \ldots, m$, where $p_m = 1$. During the test let X_1 number of failures be observed in the interval $(0, T_1]$ and then $r_1 = \lfloor (n - X_1)p_1 \rfloor$ units are randomly removed from the remaining $n - X_1$ surviving units. Where $\lfloor b \rfloor$ denotes the greatest integer less than or equal to b. Similarly at time T_2 the X_2 number of failure times are observed during the interval $(T_1, T_2]$ and then $r_2 = \lfloor (n - X_1 - r_1 - X_2)p_2 \rfloor$ units are again randomly removed from the remaining $n - X_1 - r_1 - X_2$ surviving units. Finally at time T_m the X_m number of failure is observed during $(T_{m-1}, T_m]$ and the remaining surviving units $r_m = n - \sum_{i=1}^{m-1}(X_i + r_i) - X_m$ units are all removed. Here we consider making inference for the lifetime performance index under progressive type I interval

censoring based on a general class of inverted exponentiated model. In particular, we develop a hypothesis testing procedure to study the lifetime performance of the inverted exponentiated exponential distribution where the shape parameter is predetermined by maximum p-value method (see for details, Lee [13]).

We have organized the rest of this paper as follows. In Section 2, we discuss some properties of the lifetime performance index under a general class of inverse exponentiated distributions and the relationship between lifetime performance index and conforming rate is illustrated. The maximum likelihood estimator of lifetime performance index is computed in Section 3 and the corresponding asymptotic normal distribution under progressive type I interval censored samples is obtained as well. In Section 4, we provide an algorithm for developing hypothesis testing procedure for the lifetime performance index. In this section, we also discuss the power function of the proposed test. In Section 5, we provide two numerical examples in support of the proposed testing procedure. Finally, a brief conclusion is given in Section 6.

2. The lifetime performance index and the conforming rate

Suppose that lifetime V of items follow a exponential class of distributions with cumulative density function (CDF) given as

$$F_V(v;\alpha,\beta) = (1-e^{-\beta Q(v)})^{\alpha},\ v>0,\ \alpha>0,\ \beta>0, \tag{1}$$

where $Q(.)$ is an increasing function such that $Q(0)=0$ and $Q(\infty)=\infty$. This family of distributions includes generalized exponential, generalized Rayleigh and generalized Pareto distributions, see Ghitany et. al. [6] for details. The inverse class of distributions is constructed by using the transformation $Y=\frac{1}{V}$. The corresponding CDF is obtained as

$$F_Y(y;\alpha,\beta) = 1-(1-e^{-\beta Q(1/y)})^{\alpha},\ y>0,\ \alpha>0,\ \beta>0, \tag{2}$$

and the probability density function (PDF) is given by

$$f(y,\alpha,\beta) = \alpha\beta\frac{Q'(1/y)}{y^2}e^{-\beta Q(1/y)}(1-e^{-\beta Q(1/y)})^{\alpha-1}\ y>0,\ \alpha>0,\ \beta>0. \tag{3}$$

It is seen that the hazard rate function of this distribution is

$$h(y,\alpha,\beta) = \frac{f(y,\alpha,\beta)}{1-F_Y(y,\alpha,\beta)} = \frac{\alpha\beta\ Q'(1/y)e^{-\beta Q(1/y)}}{y^2(1-e^{-\beta Q(1/y)})}. \tag{4}$$

Here α is a shape parameter and β is a scale parameter. This family of distributions includes the inverted exponentiated exponential distribution when $Q(1/y)=\frac{1}{y}$, for $Q(1/y)=\frac{1}{y^2}$ it reduces to the inverted exponentiated Rayleigh distribution and $Q(1/y)=\ln(1+\frac{1}{y})$ provides the inverted exponentiated Pareto distribution. One may also refer to Tripathi and Rastogi [15], Kayal et.al. [10] for some inferential results on these distribution under different sampling situations. The larger-the-better lifetime characteristic is an indication of reliable products.

So it is desired that lifetime of such items in general be more than a prescribed time duration to satisfy consumers expectations. Here we use the approach of Montogomery [11] to measure the larger-the-better type quality characteristic. According to this method a process capability index C_L is given by

$$C_L = \frac{\mu - L}{\sigma}, \tag{5}$$

where μ, σ and L respectively represent process mean, process standard deviation and lower specification limit respectively. Now we use the transformation $W = -\ln(1 - e^{-\beta Q(1/Y)})$ and observe that W follows exponential distribution with rate α. The corresponding PDF and CDF are given by

$$f(w;\alpha) = \alpha e^{-\alpha w},\ w > 0,\ \alpha > 0, \tag{6}$$

and

$$F(w;\alpha) = 1 - e^{-\alpha w},\ w > 0,\ \alpha > 0. \tag{7}$$

It is seen that $\mu = E(W) = \frac{1}{\alpha}$ and variance $\sigma^2 = Var(W) = \frac{1}{\alpha^2}$. Let L_Y denotes the lower specification limit for lifetime Y then $L = -\ln(1-e^{-\beta Q(1/L_Y)})$ denotes the corresponding lower specification limit for W. The lifetime performance index C_L can now be written as

$$C_L = 1 - \alpha L. \tag{8}$$

We observe that $C_L > 0$ for $L < \frac{1}{\alpha}$ and for $L > \frac{1}{\alpha},\ C_L < 0$. It is also observed that smaller failure rate α indicates larger lifetime performance index C_L. Thus performance of a product can be adequately measured in terms of index C_L. Also when lifetime of an item or product exceeds the lower specification limit (i.e., $W \geq L$), then we identify the product as a conforming one which is defined as

$$P_r = P(W \geq L) = \exp[-\alpha L] = \exp[C_L - 1],\ -\infty < C_L < 1. \tag{9}$$

In sequel we observe that there is a strictly increasing connection between P_r and C_L (see, Table 1). As an example if a manufacturer wants P_r to exceed 0.8395 then the corresponding C_L needs to exceed 0.825.

3. Maximum likelihood estimator of the lifetime performance index

In this section we derive maximum likelihood estimator of the performance index. Let $X_1, X_2, \ldots, X_m$ denotes a progressive type I interval censored sample observed at prespecified time points $(T_1, T_2, \ldots, T_m)$ using the removal scheme $(r_1, r_2, \ldots, r_m)$ based on the percentage $(p_1, p_2, \ldots, p_m)$. The likelihood function of the observed data is given by

$$\begin{aligned} L(\alpha) &\propto \prod_{i=1}^{m} [F(T_i) - F(T_{i-1})]^{X_i} [1 - F(T_i)]^{r_i} \\ &= \prod_{i=1}^{m} \left[\left(1 - e^{-\beta Q(1/T_{i-1})}\right)^{\alpha} - \left(1 - e^{-\beta Q(1/T_i)}\right)^{\alpha} \right]^{X_i} \end{aligned}$$

TABLE 1. The lifetime performance index C_L and its corresponding conforming rates P_r

C_L	P_r	C_L	P_r	C_L	P_r	C_L	P_r	C_L	P_r	C_L	P_r
-∞	0.0000	-2.350	0.0351	-1.675	0.0689	-1.000	0.1353	-0.325	0.2658	0.350	0.5220
-3.000	0.0183	-2.325	0.0360	-1.650	0.0706	-0.975	0.1388	-0.300	0.2725	0.375	0.5353
-2.975	0.0188	-2.300	0.0369	-1.625	0.0724	-0.950	0.1423	-0.275	0.2794	0.400	0.5488
-2.950	0.0192	-2.275	0.0378	-1.600	0.0743	-0.925	0.1459	-0.250	0.2865	0.425	0.5627
-2.925	0.0197	-2.250	0.0388	-1.575	0.0762	-0.900	0.1496	-0.225	0.2938	0.450	0.5769
-2.900	0.0202	-2.225	0.0398	-1.550	0.0781	-0.875	0.1534	-0.200	0.3012	0.475	0.5916
-2.875	0.0208	-2.200	0.0408	-1.525	0.0801	-0.850	0.1572	-0.175	0.3088	0.500	0.6065
-2.850	0.0213	-2.175	0.0418	-1.500	0.0821	-0.825	0.1612	-0.150	0.3166	0.525	0.6219
-2.825	0.0218	-2.150	0.0428	-1.475	0.0842	-0.800	0.1653	-0.125	0.3247	0.550	0.6376
-2.800	0.0224	-2.125	0.0439	-1.450	0.0863	-0.775	0.1695	-0.100	0.3329	0.575	0.6538
-2.775	0.0229	-2.100	0.0450	-1.425	0.0885	-0.750	0.1738	-0.075	0.3413	0.600	0.6703
-2.750	0.0235	-2.075	0.0462	-1.400	0.0907	-0.725	0.1782	-0.050	0.3499	0.625	0.6873
-2.725	0.0241	-2.050	0.0474	-1.375	0.0930	-0.700	0.1827	-0.025	0.3588	0.650	0.7047
-2.700	0.0247	-2.025	0.0486	-1.350	0.0954	-0.675	0.1873	0.000	0.3679	0.675	0.7225
-2.675	0.0254	-2.000	0.0498	-1.325	0.0978	-0.650	0.1920	0.025	0.3770	0.700	0.7408
-2.650	0.0260	-1.975	0.0510	-1.300	0.1003	-0.625	0.1969	0.050	0.3867	0.725	0.7596
-2.625	0.0266	-1.950	0.0523	-1.275	0.1028	-0.600	0.2019	0.075	0.3965	0.750	0.7788
-2.600	0.0273	-1.925	0.0537	-1.250	0.1054	-0.575	0.2070	0.100	0.4066	0.775	0.7985
-2.575	0.0280	-1.900	0.0550	-1.225	0.1081	-0.550	0.2122	0.125	0.4169	0.800	0.8187
-2.550	0.0287	-1.875	0.0564	-1.200	0.1108	-0.525	0.2176	0.150	0.4274	0.825	0.8395
-2.525	0.0294	-1.850	0.0578	-1.175	0.1136	-0.500	0.2231	0.175	0.4382	0.850	0.8607
-2.500	0.0302	-1.825	0.0593	-1.150	0.1165	-0.475	0.2288	0.200	0.4493	0.875	0.8825
-2.475	0.0310	-1.800	0.0608	-1.125	0.1194	-0.450	0.2346	0.225	0.4607	0.900	0.9048
-2.450	0.0318	-1.775	0.0624	-1.100	0.1225	-0.425	0.2405	0.250	0.4724	0.925	0.9277
-2.425	0.0326	-1.750	0.0639	-1.075	0.1256	-0.400	0.2466	0.275	0.4843	0.950	0.9512
-2.400	0.0334	-1.725	0.0656	-1.050	0.1287	-0.375	0.2528	0.300	0.4966	0.975	0.9753
-2.375	0.0342	-1.700	0.0672	-1.025	0.1320	-0.350	0.2592	0.325	0.5092	1.000	1.0000

$$\left(1 - e^{-\beta Q(1/T_i)}\right)^{r_i \alpha}. \tag{10}$$

The corresponding log-likelihood function turns out to be

$$l(\alpha) = \sum_{i=1}^{m} X_i \ln\left[\left(1 - e^{-\beta Q(1/T_{i-1})}\right)^{\alpha} - \left(1 - e^{-\beta Q(1/T_i)}\right)^{\alpha}\right] + \alpha \sum_{i=1}^{m} r_i \ln\left(1 - e^{-\beta Q(1/T_i)}\right). \tag{11}$$

The likelihood equation of α is

$$\frac{dl(\alpha)}{d\alpha} = \sum_{i=1}^{m} x_i \frac{\mathcal{G}_{i-1}^{\alpha} \ln(\mathcal{G}_{i-1}) - \mathcal{G}_i^{\alpha} \ln(\mathcal{G}_i)}{\mathcal{G}_{i-1}^{\alpha} - \mathcal{G}_i^{\alpha}} + \sum_{i=1}^{m} r_i \ln(\mathcal{G}_i) = 0, \tag{12}$$

where $\mathcal{G}_c = 1 - e^{-\beta Q(1/T_c)}$. The MLE $\hat{\alpha}$ of α can be obtained by solving the equation (12) numerically. We also have

$$\begin{aligned}\frac{d^2l(\alpha)}{d\alpha^2} &= \sum_{i=1}^{m} x_i \Bigg\{ \frac{\mathcal{G}_{i-1}^{\alpha}[\ln(\mathcal{G}_{i-1})]^2 - \mathcal{G}_i^{\alpha}[\ln(\mathcal{G}_i)]^2}{\mathcal{G}_{i-1}^{\alpha} - \mathcal{G}_i^{\alpha}} - \\ &\quad \frac{[\mathcal{G}_{i-1}^{\alpha}\ln(\mathcal{G}_{i-1}) - \mathcal{G}_i^{\alpha}\ln(\mathcal{G}_i)]^2}{[\mathcal{G}_{i-1}^{\alpha} - \mathcal{G}_i^{\alpha}]^2} \Bigg\} \qquad (13)\end{aligned}$$

The Fisher information of α is given by $J(\alpha) = -E\left(\frac{d^2l(\alpha)}{d\alpha^2}\right)$. Now following Wu and Lin [17] it is observed that

$$X_i \mid X_{i-1}, \ldots, X_1, r_{i-1}, \ldots, r_1 \sim Binom\left(n - \sum_{j=1}^{i-1}(X_j + r_j), q_i\right), \qquad (14)$$

where $q_i = \frac{F(T_i)-F(T_{i-1})}{1-F(T_{i-1})} = 1 - \left(\frac{\mathcal{G}_i}{\mathcal{G}_{i-1}}\right)^{\alpha}$, $i = 1, 2, \ldots, m$.

Therefore we have

$$\begin{aligned}E(X_1) &= nq_1 \\ E(r_1) &= EE(r_1 \mid X_1) = E((n - X_1)p_1 \mid X_1) = n(1 - q_1)p_1 \\ E(X_2) &= EE(X_2 \mid X_1, r_1) = n(1 - q_1)(1 - p_2)q_2,\end{aligned}$$

continuing this process we finally finally have

$$E(X_i) = nq_i \textstyle\prod_{l=1}^{i-1}(1 - p_l)(1 - q_l),\ i = 1, 2, \ldots, m.$$

Now using the above computations we find that we have

$$\begin{aligned}J(\alpha) &= n\textstyle\sum_{i=1}^{m}\Bigg[\left(\frac{(\ln(\mathcal{G}_{i-1})-(1-q_i)\ln(\mathcal{G}_i))^2}{q_i} - \{(\ln(\mathcal{G}_{i-1}))^2 - (1 - q_i)(\ln(\mathcal{G}_i))^2\}\right) \\ &\qquad \textstyle\prod_{l=1}^{i-1}(1 - p_l)(1 - q_l)\Bigg]. \qquad (15)\end{aligned}$$

We presume that monitoring and censoring occur periodically $T_i - T_{i-1} = T$ and then $q_i = 1 - \left(\frac{\mathcal{H}_i}{\mathcal{H}_{i-1}}\right)^{\alpha}$, $i = 1, 2, \ldots, m$ and $\mathcal{H}_c = 1 - e^{-\beta Q(\frac{1}{cT})}$ Therefore the equation (12) can be rewritten as

$$\frac{dl(\alpha)}{d\alpha} = \sum_{i=1}^{m} x_i \frac{\mathcal{H}_{i-1}^{\alpha}\ln(\mathcal{H}_{i-1}) - \mathcal{H}_i^{\alpha}\ln(\mathcal{H}_i)}{\mathcal{H}_{i-1}^{\alpha} - \mathcal{H}_i^{\alpha}} + \sum_{i=1}^{m} r_i \ln(\mathcal{H}_i) = 0. \qquad (16)$$

Now the estimate $\hat{\alpha}$ can be obtained by solving the equation (16) numerically. Also in the case when percentages of the removals are same i.e., $p_1 = p_2 = \ldots = p_{m-1} = p$, equation (15) implies

$$J(\alpha) = n\sum_{i=1}^{m}\Bigg[\Big(\frac{(\ln(\mathcal{H}_{i-1})-(1-q_i)\ln(\mathcal{H}_i))^2}{q_i} -\{(\ln(\mathcal{H}_{i-1}))^2-(1-q_i)(\ln(\mathcal{H}_i))^2\}\Big)\prod_{l=1}^{i-1}(1-p_l)(1-q_l)\Bigg]. \quad (17)$$

By the invariance property the MLE of C_L can now be obtained as

$$\hat{C}_L = 1-\hat{\alpha}L. \quad (18)$$

Since $\hat{\alpha} \xrightarrow[m\to\infty]{D} N(\alpha, J^{-1}(\alpha))$, therefore

$$\hat{C}_L \xrightarrow[m\to\infty]{D} N(C_L, V(\hat{C}_L)), \quad (19)$$

where $V(\hat{C}_L)) = L^2J^{-1}(\hat{\alpha})$. Hence $\hat{C}_L$ is an unbiased estimator of C_L and variance of $\hat{C}_L$ can be calculated by $V(\hat{C}_L) = L^2J^{-1}(\hat{\alpha})$.

4. Testing procedure algorithm for the lifetime performance index

In this section we develop a hypothesis testing procedure using the maximum likelihood estimator of C_L to verify whether the lifetime performance index attains a prespecified level or not. Let d_0 be the prespecified level and lifetime performance index greater than d_0 will indicate the process is reliable. The hypothesis procedure is constructed as follows. Let the null hypothesis be $H_0 : C_L \le d_0$ (the process is not reliable) against the alternative hypothesis $H_1 : C_L > d_0$ (the process is reliable) using the test statistic $\hat{C}_L$ and the critical value C_L^0 for the one sided hypothesis testing is then given by

$$P\left(\hat{C}_L > C_L^0 \mid C_L \le d_0\right) = P\left(1-\hat{\alpha}L > C_L^0 \mid 1-\alpha L \le d_0\right)$$
$$= P\left(\hat{\alpha} < \frac{1-C_L^0}{L} \,|\alpha \ge \frac{1-d_0}{L}\right) = P\left(Z < \frac{\left(\frac{1-C_L^0}{L}-\alpha\right)}{\sqrt{J^{-1}(\alpha)}} \mid \alpha \ge \frac{1-d_0}{L}\right) \le \gamma,$$

where $Z = \frac{\hat{\alpha}-\alpha}{\sqrt{J^{-1}(\alpha)}} \xrightarrow[m\to\infty]{D} N(0,1)$. Thus we have

$$\sup_{\alpha\ge\frac{1-d_0}{L}} P\left(Z < \frac{\left(\frac{1-C_L^0}{L}-\alpha\right)}{\sqrt{J^{-1}(\alpha)}}\right) = \gamma$$

Note that Supremum of $P\left(Z < \frac{\left(\frac{1-C_L^0}{L}-\alpha\right)}{\sqrt{J^{-1}(\alpha)}}\right)$ occurs at $\alpha_0 = \frac{1-d_0}{L}$ since $\frac{\left(\frac{1-C_L^0}{L}-\alpha\right)}{\sqrt{J^{-1}(\alpha)}}$ is a decreasing function of α for $\alpha \ge \frac{1-d_0}{L}$. Thus we have,

$$P\left(Z < \frac{\left(\frac{1-C_L^0}{L} - \alpha_0\right)}{\sqrt{J^{-1}(\alpha_0)}} \mid \alpha_0 = \frac{1-d_0}{L}\right) = \gamma \Rightarrow Z_{1-\gamma} = \frac{\left(\frac{1-C_L^0}{L} - \alpha_0\right)}{\sqrt{J^{-1}(\alpha_0)}},$$

where $\alpha_0 = \frac{1-d_0}{L}$ and $Z_{1-\gamma}$ represents the lower 100γth percentile of a standard normal distribution. Thus critical value is obtained as

$C_L^0 = 1 - L\left(\alpha_0 + Z_{1-\gamma}\sqrt{J^{-1}(\alpha_0)}\right)$ and the critical region for this test is

$$\left\{\hat{C}_L \mid \hat{C}_L > C_L^0\right\}.$$

Wu and Lin [17], Wu and Lu [18] proposed an algorithmic procedure to construct C_L. We reproduce their algorithm for our case for the sake of completeness.

Algorithm:

Step 1: Given a lower specification limit L_Y observe that $L = -\ln(1 - e^{-\beta Q(1/L_Y)})$ denotes the corresponding lower specification limit for lifetime variable W. Now generate a progressive type I interval censored sample $X_1, X_2, \ldots, X_m$ from the one-parameter exponential distribution at the prespecified time points $T_1, T_2, \ldots, T_m$ under a censoring scheme $r_1, r_2, \ldots, r_m$.

Step 2: Determine the required level d_0 to achieve a pre-assigned conforming rate P_r from Table 1. Then testing procedure for null hypothesis $H_0 : C_L \leq d_0$ and alternative hypothesis $H_1 : C_L > d_0$ are constructed with d_0 denoting the target value.

Step 3: Compute the maximum likelihood estimate $\hat{\alpha}$ of α and then find $\hat{C}_L = 1 - \hat{\alpha}L$.

Step 4: For the level of significance γ, evaluate the critical value $C_L^0 = 1 - L\left(\alpha_0 + Z_{1-\gamma}\sqrt{J^{-1}(\alpha_0)}\right)$, where $\alpha_0 = \frac{1-d_0}{L}$. Also $J(\alpha)$ is given in (17).

Step 5: Check if $\hat{C}_L > C_L^0$, then conclude that the lifetime performance index attains the required level.

The power $h(d_1)$ of the proposed test procedure at the point $C_L = d_1 > d_0$ is computed as

$$\begin{aligned} h(d_1) &= P\left(\hat{C}_L > C_L^0 \mid d_1 = 1 - \alpha_1 L\right) \\ &= P\left(1 - \hat{\alpha}L > 1 - L\left(\alpha_0 + Z_{1-\gamma}\sqrt{J^{-1}(\alpha_0)}\right) \mid \alpha_1 = \frac{1-d_1}{L}\right) \end{aligned}$$

$$
\begin{aligned}
&= P\left(\frac{\hat{\alpha}-\alpha}{\sqrt{J^{-1}(\alpha_1)}} < \frac{\alpha_0-\alpha_1+Z_{1-\gamma}\sqrt{J^{-1}(\alpha_0)}}{\sqrt{J^{-1}(\alpha_1)}} \mid \alpha_1 = \frac{1-d_1}{L}\right) \\
&= \Phi\left(\frac{\alpha_0-\alpha_1+Z_{1-\gamma}\sqrt{J^{-1}(\alpha_0)}}{\sqrt{J^{-1}(\alpha_1)}}\right), \qquad (20)
\end{aligned}
$$

where $\Phi(.)$ is the CDF for the standard normal distribution. Also from equation (19), we get $Z = \frac{\hat{C}_L - C_L}{\sqrt{V(\hat{C}_L)}} \xrightarrow[m\to\infty]{D} N(0,1)$. Now using $Z = \frac{\hat{C}_L - C_L}{\sqrt{V(\hat{C}_L)}}$ as the pivotal quantity we see that the $100(1-\gamma)\%$ lower confidence bound for C_L can be obtained as $\hat{C}_L - Z_\gamma\sqrt{V(\hat{C}_L)}$.

We consider the case $Q(1/y) = \frac{1}{y}$, the inverted exponentiated exponential distribution, for illustrative purposes. The powers $h(d_1)$ for testing $H_0 : C_L \leq 0.825$ are given in Tables 2-4 at $\gamma = 0.01, 0.05, 0.1$ respectively. From Figure (1) we observe that the power $h(d_1)$ is non-decreasing in n for fixed m, p and γ. Also from Figure (2) we further observe that the power $h(d_1)$ is a non-decreasing function of m for fixed n, p and γ. Likewise Figure (3) suggests that the power $h(d_1)$ is non-increasing in p for fixed n, m and γ values. Finally visual analysis of Figure (4) suggests that the power $h(d_1)$ is a non-decreasing function of γ for fixed n, m and p values.

5. Two numerical examples

In this section we analyze the above algorithmic test procedure using a real data set and a simulated data set.

Example 1: The data set represents the operational lifetimes for components in hours. The data set is initially reported by Hand et al. [8] and is given below as:

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2398  2812  3113  3212  3523  5236  6125  6278  7725  8604  9003
9350  9460 11584 11825 12628 12888 13431 14266 17809.
```

The G-test based on Gini statistic (see, Gill and Gastwirth [7] and Lee [13]) is 0.4996 and the corresponding p-value is 0.9950 when $\beta = 0.1.66$ (using the maximum p-value method), which indicates that inverted exponentiated exponential distribution fits the data set preferably well. For computational convenience we divided each data point by 10000. Figure (5) represents the p-values for different values of the parameter β. These visual analysis suggest that an inverted exponentiated exponential distribution fits the data set quite good. Now we generate a progressive type I interval censored sample using the given data set. We take number of inspection as $m = 8$ with equal length of inspection interval $T = 0.4$ and the pre-specified percentages for the removal are considered as $(p_1, p_2, \ldots, p_8) = (\underbrace{0.3, \ldots, 0.3}_{7 \text{ times}}, 1)$. The testing procedure about C_L is illustrated

TABLE 2. The values of $h(d_1)$ at $\gamma = 0.01$ for $d_1 = 0.7, 0.75(0.025)0.95$, $m = 7(1)10$, $n = 70(15)100$ and $p = 0.05(0.025)0.1$ under $L = 0.05$, $T = 0.1$ and $d_0 = 0.825$

m	n	p	d_1									
			0.700	0.750	0.775	0.800	0.825	0.850	0.875	0.900	0.925	0.950
7	70	0.050	0.000089	0.000535	0.001381	0.003670	0.010000	0.027659	0.076224	0.201777	0.475635	0.854864
		0.075	0.000141	0.000707	0.001661	0.004020	0.010000	0.025424	0.065286	0.165269	0.391128	0.765146
		0.100	0.000217	0.000919	0.001976	0.004382	0.010000	0.023430	0.056003	0.134635	0.314831	0.656569
	85	0.050	0.000038	0.000321	0.000988	0.003119	0.010000	0.031965	0.098565	0.275646	0.622421	0.948994
		0.075	0.000064	0.000442	0.001219	0.003454	0.010000	0.029191	0.083938	0.227285	0.529615	0.897242
		0.100	0.000106	0.000596	0.001484	0.003805	0.010000	0.026727	0.071533	0.185881	0.439262	0.819385
	100	0.050	0.000016	0.000198	0.000722	0.002682	0.010000	0.036376	0.122901	0.353193	0.743157	0.984758
		0.075	0.000030	0.000283	0.000911	0.003001	0.010000	0.033033	0.104276	0.293996	0.653646	0.961039
		0.100	0.000053	0.000396	0.001135	0.003338	0.010000	0.030075	0.088460	0.242091	0.559038	0.916144
8	70	0.050	0.000042	0.000333	0.000999	0.003117	0.010000	0.032338	0.101564	0.288749	0.650963	0.962291
		0.075	0.000076	0.000472	0.001258	0.003490	0.010000	0.029211	0.084819	0.232882	0.546582	0.912206
		0.100	0.000128	0.000652	0.001560	0.003881	0.010000	0.026485	0.070958	0.186052	0.444593	0.829917
	85	0.050	0.000016	0.000185	0.000681	0.002591	0.010000	0.037882	0.132803	0.387627	0.791748	0.992619
		0.075	0.000031	0.000276	0.000886	0.002942	0.010000	0.033961	0.110403	0.317115	0.696177	0.975621
		0.100	0.000057	0.000400	0.001130	0.003314	0.010000	0.030561	0.091816	0.255935	0.591095	0.936785
	100	0.050	0.000006	0.000106	0.000475	0.002182	0.010000	0.043613	0.166687	0.485221	0.885124	0.998826
		0.075	0.000013	0.000166	0.000637	0.002510	0.010000	0.038848	0.138282	0.403721	0.809527	0.994370
		0.100	0.000026	0.000252	0.000836	0.002863	0.010000	0.034734	0.114599	0.330175	0.715201	0.980012
9	70	0.050	0.000025	0.000233	0.000781	0.002750	0.010000	0.036433	0.125703	0.369837	0.775343	0.991317
		0.075	0.000049	0.000351	0.001026	0.003142	0.010000	0.032402	0.102755	0.295527	0.667268	0.969192
		0.100	0.000090	0.000512	0.001317	0.003554	0.010000	0.028961	0.084153	0.232851	0.551179	0.917972
	85	0.050	0.000008	0.000122	0.000512	0.002246	0.010000	0.043122	0.165392	0.486384	0.889639	0.999064
		0.075	0.000018	0.000196	0.000699	0.002609	0.010000	0.038029	0.134784	0.397200	0.806902	0.994561
		0.100	0.000037	0.000301	0.000929	0.002997	0.010000	0.033706	0.109832	0.318131	0.702274	0.978260
	100	0.050	0.000003	0.000056	0.000344	0.001860	0.010000	0.050080	0.208122	0.594321	0.950823	0.999921
		0.075	0.000007	0.000112	0.000487	0.002195	0.010000	0.043853	0.169569	0.497044	0.896848	0.999229
		0.100	0.000016	0.000182	0.000669	0.002558	0.010000	0.038591	0.137868	0.405872	0.815694	0.995246
10	70	0.050	0.000017	0.000179	0.000650	0.002502	0.010000	0.039902	0.147456	0.439908	0.855147	0.998012
		0.075	0.000036	0.000285	0.000884	0.002908	0.010000	0.035004	0.118363	0.349587	0.753188	0.988926
		0.100	0.000070	0.000433	0.001169	0.003339	0.010000	0.030904	0.095202	0.272641	0.632064	0.958956
	85	0.050	0.000005	0.000090	0.000414	0.002014	0.010000	0.047604	0.194675	0.567102	0.941020	0.999879
		0.075	0.000013	0.000153	0.000588	0.002387	0.010000	0.041380	0.156034	0.463754	0.874891	0.998722
		0.100	0.000028	0.000248	0.000809	0.002789	0.010000	0.036199	0.124993	0.369779	0.778149	0.992016
	100	0.050	0.000002	0.000046	0.000270	0.001646	0.010000	0.055655	0.245082	0.677956	0.978573	0.999994
		0.075	0.000005	0.000084	0.000400	0.001986	0.010000	0.048009	0.196769	0.571174	0.942435	0.999885
		0.100	0.000011	0.000146	0.000572	0.002359	0.010000	0.041675	0.157474	0.466852	0.876601	0.998751

as follows:

Step 1: Let lower lifetime limit be $L_Y = 0.2537$, then we have $L = -\ln(1 - e^{-\beta/L_Y}) = -\ln(1 - e^{-1.66/0.2537}) = 0.001441$. Generate a progressive type I interval censored sample $(X_1, X_2, \ldots, X_8) = (0, 2, 5, 2, 1, 0, 0, 0)$ at prespeciified time points $(T_1, T_2, \ldots, T_8) = (0.2, 0.6, 1.0,$
$1.4, 1.8, 2.2, 2.6, 3.0)$ using the censoring scheme $(r_1, r_2 \ldots, r_8) = (6, 3, 1, 0, 0, 0, 0, 0)$.

Step 2: If the conforming rate P_r of items is required to exceed 0.8395, then the lifetime performance index value d_0 should be taken as 0.825 (see Table 1). So test procedures of the null hypothesis $H_0 : C_L \leq 0.825$ and the alternative

hypothesis $H_1 : C_L > 0.825$ are constructed.

Step 3: Compute the value of test statistic $\hat{C}_L = 1 - \hat{\alpha}L = 0.995454$.

Step 4: At $\gamma = 0.05$ level of significance, the critical value is given by $C_L^0 = 1 - L\left(\alpha_0 + Z_{1-\gamma}\sqrt{J^{-1}(\alpha_0)}\right) = 0.905932$

Step 5: Since $\hat{C}_L = 0.995454 > C_L^0 = 0.905932$ therefore we reject the null hypothesis $H_0 : C_L \leq 0.825$ and conclude that the lifetime performance index of the product attains the required level.

TABLE 3. The values of $h(d_1)$ at $\gamma = 0.05$ for $d_1 = 0.7, 0.75(0.025)0.95$, $m = 7(1)10$, $n = 70(15)100$ and $p = 0.05(0.025)0.1$ under $L = 0.05$, $T = 0.1$ and $d_0 = 0.825$

m	n	p	d_1									
			0.700	0.750	0.775	0.800	0.825	0.850	0.875	0.900	0.925	0.950
7	70	0.050	0.000564	0.003266	0.008078	0.020152	0.050000	0.120614	0.272220	0.538840	0.848142	0.991972
		0.075	0.000850	0.004148	0.009431	0.021697	0.050000	0.113433	0.245920	0.483714	0.791686	0.980849
		0.100	0.001245	0.005188	0.010907	0.023269	0.050000	0.106852	0.222093	0.431257	0.727573	0.959963
	85	0.050	0.000263	0.002109	0.006106	0.017664	0.050000	0.133883	0.320983	0.631932	0.919461	0.998583
		0.075	0.000424	0.002776	0.007284	0.019195	0.050000	0.125374	0.289586	0.573142	0.877579	0.995534
		0.100	0.000660	0.003589	0.008594	0.020764	0.050000	0.117591	0.260966	0.515312	0.825068	0.988094
	100	0.050	0.000126	0.001390	0.004688	0.015623	0.050000	0.146875	0.368516	0.710925	0.959309	0.999779
		0.075	0.000217	0.001895	0.005709	0.017124	0.050000	0.137046	0.332474	0.651898	0.931076	0.999069
		0.100	0.000359	0.002530	0.006867	0.018675	0.050000	0.128068	0.299374	0.591722	0.891812	0.996802
8	70	0.050	0.000284	0.002134	0.006089	0.017555	0.050000	0.135603	0.329362	0.650399	0.932454	0.999196
		0.075	0.000475	0.002886	0.007399	0.019253	0.050000	0.125983	0.293581	0.584349	0.889115	0.996818
		0.100	0.000763	0.003812	0.008865	0.020991	0.050000	0.117316	0.261443	0.519473	0.832432	0.989970
	85	0.050	0.000117	0.001286	0.004411	0.015104	0.050000	0.151832	0.388885	0.744709	0.972501	0.999932
		0.075	0.000214	0.001822	0.005510	0.016761	0.050000	0.140386	0.346799	0.679621	0.947194	0.999588
		0.100	0.000371	0.002510	0.006772	0.018473	0.050000	0.130093	0.308587	0.612346	0.908819	0.998133
	100	0.050	0.000050	0.000793	0.003251	0.013125	0.050000	0.167770	0.445781	0.817635	0.989484	0.999995
		0.075	0.000099	0.001175	0.004171	0.014727	0.050000	0.154510	0.398335	0.757628	0.976202	0.999954
		0.100	0.000186	0.001686	0.005252	0.016399	0.050000	0.142600	0.354711	0.692145	0.952718	0.999693
9	70	0.050	0.000170	0.001541	0.004894	0.015768	0.050000	0.148300	0.378360	0.732963	0.969939	0.999922
		0.075	0.000312	0.002203	0.006165	0.017587	0.050000	0.136298	0.333683	0.661009	0.939821	0.999459
		0.100	0.000541	0.003052	0.007619	0.019456	0.050000	0.125651	0.293786	0.587626	0.893804	0.997341
	85	0.050	0.000064	0.000879	0.003426	0.013365	0.050000	0.167113	0.446154	0.821200	0.990561	0.999997
		0.075	0.000130	0.001329	0.004462	0.015118	0.050000	0.152793	0.394421	0.755508	0.976562	0.999960
		0.100	0.000248	0.001936	0.005684	0.016942	0.050000	0.140110	0.347487	0.683675	0.950460	0.999681
	100	0.050	0.000025	0.000514	0.002443	0.011450	0.050000	0.185618	0.509653	0.883518	0.997250	1.000000
		0.075	0.000056	0.000821	0.003287	0.013125	0.050000	0.169002	0.452389	0.827700	0.991453	0.999998
		0.100	0.000116	0.001255	0.004310	0.014889	0.050000	0.154296	0.399501	0.762001	0.978176	0.999967
10	70	0.050	0.000117	0.001207	0.004144	0.014519	0.050000	0.158820	0.419041	0.791949	0.986049	0.999992
		0.075	0.000232	0.001810	0.005387	0.016435	0.050000	0.144589	0.366389	0.717614	0.965695	0.999897
		0.100	0.000429	0.002608	0.006834	0.018411	0.050000	0.132142	0.319566	0.638746	0.929245	0.999204
	85	0.050	0.000041	0.000660	0.002824	0.012160	0.050000	0.179824	0.492924	0.871478	0.996567	1.000000
		0.075	0.000092	0.001055	0.003815	0.013990	0.050000	0.162813	0.432853	0.808183	0.988880	0.999996
		0.100	0.000188	0.001610	0.005010	0.015904	0.050000	0.147953	0.378314	0.734839	0.971206	0.999935
	100	0.050	0.000015	0.000370	0.001962	0.010301	0.050000	0.200501	0.560805	0.923123	0.999225	1.000000
		0.075	0.000037	0.000630	0.002751	0.012034	0.050000	0.180742	0.495435	0.873183	0.996656	1.000000
		0.100	0.000085	0.001016	0.003735	0.013872	0.050000	0.163488	0.434698	0.809694	0.989019	0.999996

TABLE 4. The values of $h(d_1)$ at $\gamma = 0.1$ for $d_1 = 0.7, 0.75(0.025)0.95$, $m = 7(1)10$, $n = 70(15)100$ and $p = 0.05(0.025)0.1$ under $L = 0.05$, $T = 0.1$ and $d_0 = 0.825$

m	n	p	d_1									
			0.700	0.750	0.775	0.800	0.825	0.850	0.875	0.900	0.925	0.950
7	70	0.050	0.001378	0.007636	0.018218	0.043212	0.100000	0.219152	0.433919	0.724006	0.946232	0.999118
		0.075	0.002019	0.009499	0.020936	0.046113	0.100000	0.208483	0.402119	0.675847	0.918137	0.997369
		0.100	0.002878	0.011644	0.023853	0.049036	0.100000	0.198565	0.372280	0.626820	0.882057	0.993214
	85	0.050	0.000676	0.005123	0.014167	0.038487	0.100000	0.238431	0.489931	0.797929	0.976278	0.999894
		0.075	0.001057	0.006590	0.016607	0.041411	0.100000	0.226098	0.454192	0.752168	0.959362	0.999571
		0.100	0.001598	0.008331	0.019271	0.044376	0.100000	0.214639	0.420330	0.703678	0.935075	0.998551
	100	0.050	0.000340	0.003499	0.011166	0.034542	0.100000	0.256856	0.541385	0.853912	0.989926	0.999988
		0.075	0.000566	0.004649	0.013340	0.037456	0.100000	0.242928	0.502564	0.812568	0.980501	0.999936
		0.100	0.000906	0.006056	0.015756	0.040431	0.100000	0.229989	0.465383	0.766875	0.965320	0.999714
8	70	0.050	0.000714	0.005133	0.014053	0.038176	0.100000	0.241352	0.500532	0.813091	0.981426	0.999949
		0.075	0.001159	0.006770	0.016754	0.041419	0.100000	0.227412	0.460068	0.762829	0.964914	0.999731
		0.100	0.001803	0.008733	0.019716	0.044697	0.100000	0.214627	0.422093	0.709070	0.939658	0.998880
	85	0.050	0.000312	0.003227	0.010506	0.033438	0.100000	0.264257	0.563868	0.877154	0.993941	0.999997
		0.075	0.000550	0.004445	0.012850	0.036661	0.100000	0.248126	0.519427	0.833737	0.986350	0.999977
		0.100	0.000922	0.005965	0.015480	0.039950	0.100000	0.233333	0.477074	0.784441	0.972672	0.999859
	100	0.050	0.000140	0.002069	0.007971	0.029536	0.100000	0.286145	0.620458	0.920632	0.998118	1.000000
		0.075	0.000267	0.002973	0.009994	0.032709	0.100000	0.267928	0.573397	0.885143	0.994914	0.999998
		0.100	0.000482	0.004145	0.012314	0.035973	0.100000	0.251214	0.527795	0.842257	0.988086	0.999984
9	70	0.050	0.000436	0.003776	0.011475	0.034642	0.100000	0.259796	0.554284	0.870850	0.993408	0.999997
		0.075	0.000772	0.005248	0.014145	0.038156	0.100000	0.242732	0.506396	0.821999	0.984254	0.999970
		0.100	0.001294	0.007078	0.017128	0.041719	0.100000	0.227274	0.461341	0.766809	0.967351	0.999793
	85	0.050	0.000174	0.002253	0.008310	0.029934	0.100000	0.285745	0.622062	0.923436	0.998395	1.000000
		0.075	0.000340	0.003301	0.010566	0.033391	0.100000	0.265997	0.570632	0.884875	0.995131	0.999999
		0.100	0.000623	0.004668	0.013158	0.036934	0.100000	0.248098	0.521209	0.837673	0.987633	0.999984
	100	0.050	0.000071	0.001373	0.006114	0.026104	0.100000	0.310518	0.680995	0.955564	0.999631	1.000000
		0.075	0.000154	0.002118	0.008011	0.029471	0.100000	0.288232	0.627853	0.926875	0.998569	1.000000
		0.100	0.000307	0.003135	0.010249	0.032958	0.100000	0.268010	0.575605	0.888709	0.995521	0.999999
10	70	0.050	0.000303	0.002991	0.009821	0.032128	0.100000	0.274866	0.596804	0.907799	0.997495	1.000000
		0.075	0.000578	0.004350	0.012464	0.035864	0.100000	0.254924	0.54273	0.861860	0.992384	0.999996
		0.100	0.001030	0.006091	0.015466	0.039660	0.100000	0.237063	0.491678	0.806989	0.980966	0.999955
	85	0.050	0.000113	0.001714	0.006935	0.027464	0.100000	0.303319	0.667017	0.950275	0.999535	1.000000
		0.075	0.000241	0.002649	0.009124	0.031108	0.100000	0.280246	0.610120	0.916846	0.998086	1.000000
		0.100	0.000475	0.003915	0.011688	0.034858	0.100000	0.259557	0.554934	0.872703	0.993838	0.999998
	100	0.050	0.000043	0.001004	0.004980	0.023705	0.100000	0.330448	0.726560	0.973842	0.999919	1.000000
		0.075	0.000103	0.001646	0.006782	0.027226	0.100000	0.304434	0.669055	0.950988	0.999546	1.000000
		0.100	0.000224	0.002564	0.008961	0.030893	0.100000	0.281064	0.611651	0.917510	0.998104	1.000000

Example 2: In this example we discuss the algorithmic procedure based on a simulation data of size 30 from an inverted exponentiated exponential distribution with $\alpha = 0.75$ and $\beta = 1.66$. The generated data are:

```
0.399935,0.468751,0.495583,0.786440,0.952255,1.04402,1.125270,
1.222100,1.330590,1.501210,1.662980,1.870190,2.369760,2.433940,
2.442240,2.514060,2.712780,3.073840,3.171930,3.638930,5.028020,
7.375200,8.541830,13.927100,14.141900,38.331000,53.068300,
85.170200,111.517000,167.449000.
```

Next we obtain a progressive type I interval censored sample from this data. We consider $m = 10$ with equal length of intervals $T = 6.5$ and the pre-specified

removal percentages are $(p_1, p_2, \ldots, p_{17}) = (\underbrace{0.6, \ldots, 0.6}_{9 \text{ times}}, 1)$. The testing procedure about C_L is now given below:

Step 1: For a given lower lifetime limit $L_Y = 0.9312$ we observe that $L = -\ln(1-e^{-\beta/L_Y}) = -\ln(1-e^{-1.66/0.9312}) = 0.184154$. Next we obtain a progressive type I interval censored sample $(X_1, X_2, \ldots, X_{10}) = (3, 6, 1, 0, 0, 0, 0, 0, 0, 0)$ at the pre-set times $(T_1, T_2, \ldots, T_{10}) = (0.5, 7.0, 13.5, 20.0, 26.5, 33.0, 39.5, 46.0, 52.5, 59.0)$ with censoring scheme $(r_1, r_2 \ldots, r_{10}) = (16, 3, 0, 0, 0, 0, 0, 0, 0, 1)$.

Step 2: If required conforming rate P_r has to exceed 0.9048, then lifetime performance index value d_0 should be taken as 0.9 (see Table 1). Thus null hypothesis $H_0 : C_L \leq 0.9$ and the alternative hypothesis $H_1 : C_L > 0.9$ are constructed.

Step 3: Compute value of the test statistic $\hat{C}_L = 1 - \hat{\alpha}L = 0.962502$.

Step 4: At $\gamma = 0.05$ level of significance, the critical value will be $C_L^0 = 1 - L\left(\alpha_0 + Z_{1-\gamma}\sqrt{J^{-1}(\alpha_0)}\right) = 0.93912$.

Step 5: Since $\hat{C}_L = 0.962502 > C_L^0 = 0.93912$, therefore we reject the null hypothesis $H_0 : C_L \leq 0.9$ and conclude that the lifetime performance index of the product attains the required level.

6. Conclusion

Lifetime performance index have found wide applications in manufacturing industries. It is widely used to assess the capability performance of different processes. Many industrial experiments are conducted under certain restrictions and so observed data are often censored in nature. In this paper, we have considered estimation of lifetime performance index C_L on the basis of progressive type I interval censored data when product lifetimes follow a general class of inverse exponentiated distributions. A hypothesis testing procedure is proposed using the maximum likelihood estimator of C_L to test whether C_L attains the target value. We provided power curves for different values of n, m, p, γ and studied their effect on lifetime performance. We used a mechanical component lifetimes data to illustrate the proposed procedure for determining whether the lifetime performance index achieves the target estimate under the given censoring scheme. In sequel, we also presented a simulated data analysis.

FIGURE 1. Power function test at $\gamma = 0.1$ under $m = 7,\ p = 0.05$ for $n = 70, 85, 100$.

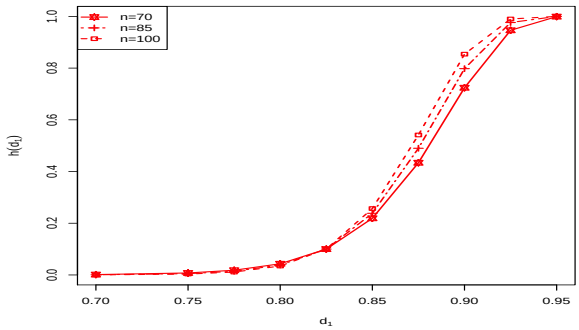


FIGURE 2. Power function test at $\gamma = 0.1$ under $n = 70,\ p = 0.05$ for $m = 7, 8, 9, 10$.

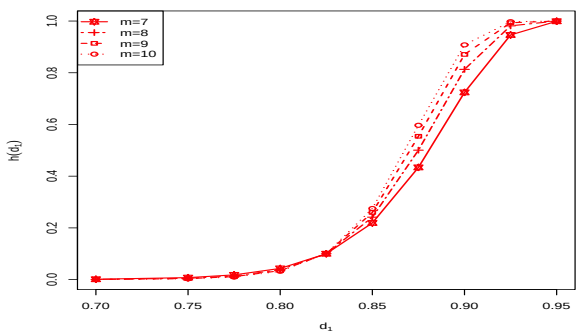


FIGURE 3. Power function test at $\gamma = 0.1$ under $n = 70,\ m = 7$ for $p = 0.05, 0.075, 0.1$.

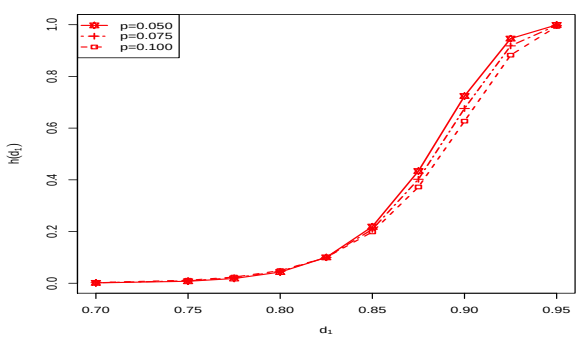


FIGURE 4. Power function test at $\gamma = 0.1$ under $n = 70,\ m = 7$ for $p = 0.05, 0.075, 0.1$.

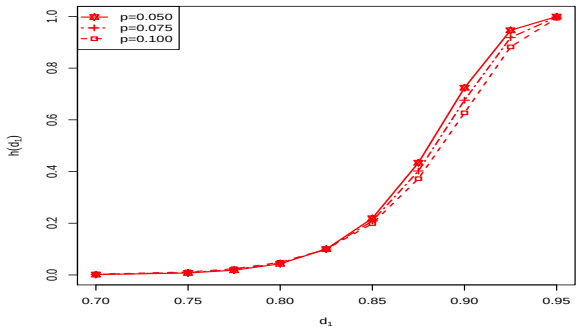


FIGURE 5. Power function test under $n = 70,\ m = 7,\ p = 0.05$ for $\gamma = 0.01, 0.05, 0.1$.

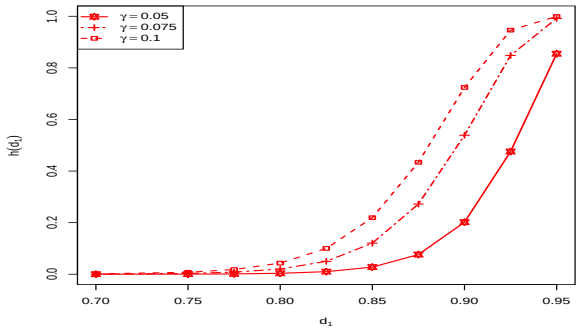
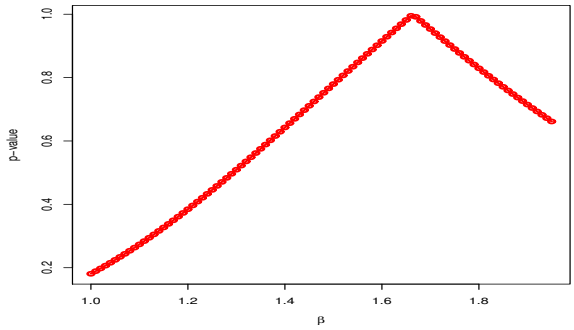


FIGURE 6. β vs $p-$value for the real data set.



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