J. Korean Math. Soc. **56** (2019), No. 1, pp. 285–287
https://doi.org/10.4134/JKMS.j180717
pISSN: 0304-9914 / eISSN: 2234-3008

CORRIGENDUM TO "FREE ACTIONS OF FINITE ABELIAN GROUPS ON 3-DIMENSIONAL NILMANIFOLDS" [J. KOREAN MATH. SOC. 42 (2005), NO. 4, PP. 795–826]

Dongsoon Choi and Joonkook Shin

In [1], Table 2 of Theorem 3.4 and Table 3 of Theorem 3.5 are incorrect in part, and so we here correct them with proofs.

Theorem 3.4. *Table 2 gives a complete list of all free actions* (*up to topological conjugacy*) *of finite abelian groups* G *on* $\mathcal{N}_p$ *which yield an orbit manifold homeomorphic to* $\mathcal{H}/\pi_3$.

Table 2

Groups G	Generators	AC classes of normal nilpotent subgroups	
$\mathbb{Z}_{\frac{p}{n}}$	${}^{\xi}\langle\alpha\rangle$	$\frac{p}{2n}\in\mathbb{N}$	$N=\langle t_1^{\frac{p}{2n}},\ t_2,\ t_3\rangle$
	${}^{\eta_4}\langle\alpha\rangle$	$\frac{p}{4n}\in\mathbb{N}$	$L_2=\langle t_1^{\frac{p}{4n}}t_2,\ t_2^2,\ t_3\rangle$
$\mathbb{Z}_{\frac{4p}{n}}$	${}^{\eta_2}\langle\alpha\rangle$	$\frac{p}{n}\in\mathbb{N},\ n\in 2\mathbb{N}$	$N_2=\langle t_1^{\frac{p}{n}}t_3,\ t_2,\ t_3^2\rangle$
	${}^{\zeta_5}\langle\alpha\rangle$	$\frac{p}{2n}\in\mathbb{N},\ n\in 2\mathbb{N}-1$	$K_5=\langle t_1^{\frac{p}{2n}}t_2,\ t_2^2t_3,\ t_3^2\rangle$
$\mathbb{Z}_{\frac{p}{n}}\times\mathbb{Z}_4$	${}^{\zeta_4}\langle\alpha,t_2\rangle$	$\frac{p}{2n}\in\mathbb{N},\ n\in 2\mathbb{N}-1$	$K_4=\langle t_1^{\frac{p}{2n}},\ t_2^2t_3,\ t_3^2\rangle$
$\mathbb{Z}_{\frac{2p}{n}}\times\mathbb{Z}_2$	${}^{\eta_1}\langle\alpha,t_3\rangle$	$\frac{p}{n}\in\mathbb{N},\ n\in 2\mathbb{N}$	$N_1=\langle t_1^{\frac{p}{n}},\ t_2,\ t_3^2\rangle$
	${}^{\zeta_2}\langle\alpha,t_3\rangle$	$\frac{p}{2n}\in\mathbb{N},\ n\in 2\mathbb{N}$	$K_2=\langle t_1^{\frac{p}{2n}}t_2,\ t_2^2,\ t_3^2\rangle$
	${}^{\zeta_3}\langle\alpha,t_2\rangle$	$\frac{p}{2n}\in\mathbb{N},\ n\in 2\mathbb{N}$	$K_3=\langle t_1^{\frac{p}{2n}}t_3,\ t_2^2,\ t_3^2\rangle$
	${}^{\zeta_6}\langle\alpha,t_2\rangle$	$\frac{p}{2n}\in\mathbb{N},\ n\in 2\mathbb{N}-1,$ $p\in 2\mathbb{N}+2$	$K_6=\langle t_1^{\frac{p}{2n}}t_3,\ t_2^2t_3,\ t_3^2\rangle$
$\mathbb{Z}_{\frac{p}{2n}}\times\mathbb{Z}_2$	${}^{\eta_3}\langle\alpha,t_2\rangle$	$\frac{p}{4n}\in\mathbb{N}$	$L_1=\langle t_1^{\frac{p}{4n}},\ t_2^2,\ t_3\rangle$
$\mathbb{Z}_{\frac{p}{n}}\times\mathbb{Z}_2\times\mathbb{Z}_2$	${}^{\zeta_1}\langle\alpha,t_2,t_3\rangle$	$\frac{p}{2n}\in\mathbb{N},\ n\in 2\mathbb{N}$	$K_1=\langle t_1^{\frac{p}{2n}},\ t_2^2,\ t_3^2\rangle$

Proof. First we deal with the case $K_5=\langle t_1^{\frac{p}{2n}}t_2, t_2^2t_3, t_3^2\rangle$, where $\frac{p}{2n}\in\mathbb{N}, n\in 2\mathbb{N}-1$. We will show that $\langle\alpha\rangle$ genetates the other elements in the following quotient group

$$\pi_3/K_5=\langle t_1,t_2,t_3,\alpha\rangle/\langle t_1^{\frac{p}{2n}}t_2,t_2^2t_3,t_3^2\rangle.$$

Received October 24, 2018; Accepted November 7, 2018.
2010 *Mathematics Subject Classification.* Primary 57S25; Secondary 57M05, 57S17.
Key words and phrases. affine conjugacy, almost Bieberbach group, group action, Heisenberg group.

From the following relations

$$\alpha^{\frac{p}{n}}K_5=(\alpha^2)^{\frac{p}{2n}}K_5=t_1^{\frac{p}{2n}}K_5=t_1^{\frac{p}{2n}}t_2^2t_3K_5=t_2t_3K_5=t_2t_3t_2^2t_3K_5=t_2^3K_5,$$
$$t_2^2K_5=t_2^2t_3^2K_5=t_3K_5,\quad t_2^4K_5=t_3^2K_5=K_5,$$

we have

$$\alpha^{\frac{2p}{n}}K_5=t_2^6K_5=t_2^2K_5=t_3K_5,\quad \alpha^{\frac{4p}{n}}K_5=t_3^2K_5=K_5,$$
$$(\alpha^{\frac{p}{n}}K_5)(t_2K_5)=\alpha^{\frac{p}{n}}t_2K_5=t_2^3t_2K_5=t_2^4K_5=K_5.$$

Therefore we can obtain that $t_2K_5=(\alpha K_5)^{-\frac{p}{n}}$ and

$$\pi_3/K_5=\langle t_1,t_2,t_3,\alpha\rangle/\langle t_1^{\frac{p}{2n}}t_2,t_2^2t_3,t_3^2\rangle=\langle \alpha K_5\rangle\cong\mathbb{Z}_{\frac{4p}{n}}.$$

Next we shall deal with the case $K_6=\langle t_1^{\frac{p}{2n}}t_3,t_2^2t_3,t_3^2\rangle$, where $\frac{p}{2n}\in\mathbb{N}, n\in 2\mathbb{N}-1, p\in 2\mathbb{N}+2$. In this case, since n is an odd number, we have

$$\alpha t_2\alpha^{-1}K_6=t_2^{-1}\alpha t_3^{-n}\alpha^{-1}K_6=t_2^{-1}t_3^nK_6=t_2t_3^{n+1}K_6=t_2K_6.$$

The following relations

$$\alpha^{\frac{p}{n}}K_6=t_1^{\frac{p}{2n}}K_6=t_1^{\frac{p}{2n}}t_3^2K_6=t_3K_6,\quad \alpha^{\frac{2p}{n}}K_6=t_3^2K_6=K_6,$$
$$t_2^2K_6=t_2^2t_3^2K_6=t_3K_6=\alpha^{\frac{p}{n}}K_6,\quad t_2^4K_6=t_3^2K_6=K_6$$

show that

$$\begin{aligned}\pi_3/K_6&=\langle t_1,t_2,t_3,\alpha\rangle/\langle t_1^{\frac{p}{2n}}t_3,t_2^2t_3,t_3^2\rangle\\&=\langle \alpha K_6,t_2K_6\mid (\alpha K_6)^{\frac{p}{n}}=(t_2K_6)^2,\ (t_2K_6)^4=1,\ (\alpha t_2)K_6=(t_2\alpha)K_6\rangle\\&\cong\mathbb{Z}_{\frac{2p}{n}}\times\mathbb{Z}_2.\end{aligned}$$

The other cases can be done similarly. □

Theorem 3.5. *Table 3 gives a complete list of all free actions (up to topological conjugacy) of finite abelian groups G on $\mathcal{N}_p$ which yield an orbit manifold homeomorphic to $\mathcal{H}/\pi_4$.*

Table 3

Group G	Generators	AC classes of normal nilpotent subgroups	
$\mathbb{Z}_2\times\mathbb{Z}_2$	$\langle\beta,\alpha\rangle$	$p=4n$	$N_1=\langle t_1,\ t_2,\ t_3\rangle$
$\mathbb{Z}_2\times\mathbb{Z}_4$	$^{\eta}\langle\beta,\alpha\rangle$	$p=2n$	$N_2=\langle t_1,\ t_2t_3,\ t_3^2\rangle$
	$^{\zeta_1}\langle\alpha,\beta\rangle$	$p=8n$	$L_1=\langle t_1t_2,\ t_2^2,\ t_3\rangle$
$\mathbb{Z}_4\times\mathbb{Z}_4$	$^{\zeta_2}\langle\beta,\alpha\rangle$	$p=4n$	$L_2=\langle t_1t_2t_3,\ t_2^2,\ t_3^2\rangle$

Proof. First we deal with the case $L_1=\langle t_1t_2,t_2^2,t_3\rangle$. From the following relations,

$$\alpha^2L_1=t_3L_1=L_1,\quad \beta^2L_1=t_1L_1=t_1t_2^2L_1=t_2L_1,\quad \beta^4L_1=t_2^2L_1=L_1,$$

we obtain that

$$\pi_4/L_1 = \langle t_1, t_2, t_3, \alpha, \beta\rangle / \langle t_1 t_2, t_2^2, t_3\rangle = \langle \alpha L_1, \beta L_1\rangle \cong \mathbb{Z}_2 \times \mathbb{Z}_4.$$

For the case of $L_2 = \langle t_1 t_2 t_3, t_2^2, t_3^2\rangle$, we know that $\alpha^4 L_2 = t_3^2 L_2 = L_2$. Since $[t_2, t_3] = 1$, we have

$$\beta^2 L_2 = t_1 L_2 = t_1 t_2^2 t_3^2 L_2 = t_2 t_3 L_2, \quad \beta^4 L_2 = t_2^2 t_3^2 L_2 = L_2,$$
$$t_2 L_2 = t_1 t_2 t_3 t_2 L_2 = t_1 t_3 L_2 = (\beta L_2)^2 (\alpha L_2)^2.$$

It is easy to show that

$$\pi_4/L_2 = \langle t_1, t_2, t_3, \alpha, \beta\rangle / \langle t_1 t_2 t_3, t_2^2, t_3^2\rangle = \langle \beta L_2, \alpha L_2\rangle \cong \mathbb{Z}_4 \times \mathbb{Z}_4. \qquad \square$$

Acknowledgement. The authors wish to thank Dr. Daehwan Koo for pointing out flaws and for valuable comments.

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[1] D. Choi and J. Shin, *Free actions of finite abelian groups on 3-dimensional nilmanifolds*, J. Korean Math. Soc. **42** (2005), no. 4, 795–826.

Dongsoon Choi
Daejeon Bongmyeong Middle School
Daejeon 34189, Korea
Email address: stsk23@hanmail.net

Joonkook Shin
Department of Mathematics Education
Chungnam National University
Daejeon 34134, Korea
Email address: jkshin@cnu.ac.kr