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Static output feedback pole assignment of 2-input, 2-output, 4^{th} order systems in Grassmann space

Su-Woon Kim*, Seong-Ho Song**, Min-Jae Kang***, Ho-Chan Kim*★

Abstract

It is presented in this paper that the static output feedback (SOF) pole-assignment problem of some linear time-invariant systems can be completely resolved by parametrization in real Grassmann space. For the real Grassmannian parametrization, the so-called Plücker matrix is utilized as a linear matrix formula formulated from the SOF variable's coefficients of a characteristic polynomial constrained in Grassmann space. It is found that the exact SOF pole assignability is determined by the linear independency of columns of Plücker sub-matrix and by full-rank of that sub-matrix. It is also presented that previous diverse pole-assignment methods and various computation algorithms of the real SOF gains for 2-input, 2-output, 4^{th} order systems are unified in a deterministic way within this real Grassmannian parametrization method.

Key words : Exact pole assignment, Grassmann space, Plücker matrix, static output feedback, real SOF gains

Ⅰ. Introduction

The 2-input, 2-output systems are a small class of general multivariable systems that are commonly encountered in practical applications such as gas-turbine fuel oil control in airplane systems, reel-to-reel tape drivers, artificial ventilation systems for respiratory failure patients, etc. However, the necessary and sufficient condition of exact pole-assignment by static output feedback (SOF) in 2-input, 2-output, n^{th} order strictly proper linear time-invariant systems, has been not known, and also stable computation algorithms covering whole solutions of the exact real SOF gains have been not developed [1]-[6]. The study of SOF pole-assignment for the exact pole locations was pioneered by Kimura [3], where it is proved that if $m+p>n$ in m-input, p-output, n^{th} order system, the pole-assignment by SOF can always be available in controllable and observable systems over distinct poles, provided that slight modification of the poles to be assigned is tolerable. A decade later, within the same state space frame of Kimura, a sufficient condition of exact pole-assignment (EPA) was derived by Fletcher and Magni without furnishing stable computation algorithms [7]-[9].

We propose in this paper a new EPA method based on the real Grassmannian space frame for

* Dept. of Electrical Engineering, Jeju National University
** School of Software, Hallym University
*** Dept. of Electronic Engineering, Jeju National University
★ Corresponding author
E-mail : hckim@jejunu.ac.kr, Tel : +82-64-754-3676
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resolving the SOF pole-assignment together with its related computation problems in frequency domain. In mathematical viewpoint, the basic coordinate setting for solving the nonlinear SOF equations for pole-assignment is naturally carried out in the specific Grassmann space without introducing the general affine space [10],[11]. For the real parametrization in Grassmann space, the Plücker matrix formula $Lk=a$ is introduced [12]-[15]. In this paper, the exact pole assignablitiy of 2-input, 2-output, 4^{th} order strictly proper linear systems (simply, (2,2,4) systems) is examined. Through the real Grassmannian parametrization approach, a necessary and sufficient condition is explicitly derived for the exact pole assignablitiy and the real SOF gains of EPA for the desired pole positions are always algebraically computable in a deterministic way, which can generalize and unify previous diverse methods for SOF pole-assignment and various computation algorithms of the real SOF gains.

II. Preliminaries

Regarding the pole assignabilities by SOF, the following two classifications are widely used: exact pole-assignment (EPA) and generic pole-assignment (GPA) [15].

Definition 1. An n^{th} order linear system with rational transfer function matrix $G(s)=N_R(s)D_R(s)^{-1}$ is exactly pole assignable by real SOF if any n^{th} order monic polynomial $p_c(s)=s^n+a_1s^{n-1}+\cdots+a_n$ with coefficients $\{a_1,\cdots,a_n\}$ in R^n can be achieved by the closed-loop characteristic equation using some real SOF K.

Definition 2. An n^{th} order linear system with rational transfer function matrix $G(s)$ is generically pole assignable by real SOF if the coefficient set $\{a_1,\cdots,a_n\}$ of the achievable closed-loop characteristic polynomial $p_c(s)$ by real SOF K, is open and dense in R^n.

In Definitions 1 and 2, $N_R(s)$ and $D_R(s)$ are right coprime polynomial matrices. In this section, a specific construction algorithm of $Lk=a$ for the (2,2,4) systems is introduced [5]. Consider a (2,2,4) system described by

$$y(s)=G(s)u(s) \tag{1}$$

where $G(s)$ is a strictly proper rational transfer function matrix.

A feedback control input $u(s)=-Ky(s)$ is applied to the systems. It is easily shown that the desired 4^{th} order closed-loop characteristic polynomial $p_c(s)$ is written by

$$p_c(s)=p(s)\det[I+KG(s)] \tag{2}$$

where $p(s)=s^4+b_1s^3+\cdots+b_4$ is an open-loop characteristic polynomial.

Equation (2) can be expressed as follows through matrix theory

$$\begin{aligned} p(s)&\det[I+KG(s)]\\ &=p(s)\det[\lambda I+KG(s)]_{\lambda=1}\\ &=p(s)[\lambda^2+tr(KG(s))\lambda+\det(KG(s))]_{\lambda=1}\\ &=p(s)[1+tr(KG(s))+\det(KG(s))]\\ &=p(s)[1+tr(KG(s))+\det(K)\det(G(s))]\end{aligned} \tag{3}$$

From the signal flow graph viewpoint, the elements of K and $G(s)$ can be written as below [1],[6],

$$K=\begin{bmatrix}k_1\, k_2\\ k_3\, k_4\end{bmatrix},\ \ G(s)=\frac{1}{p(s)}\begin{bmatrix}f_1(s)\, f_3(s)\\ f_2(s)\, f_4(s)\end{bmatrix},$$

where $f_i(s),(i=1,\cdots,4)$ is a numerator polynomial of $G_i(s)$.

And over the desired closed-loop characteristic polynomial $p_c(s)$, the Equation (3) becomes

$$p_c(s)=\prod_{i=1}^{4}(s-s_i)=p(s)+\sum_{i=1}^{5}f_i(s)k_i \tag{4}$$

where $s_1,\cdots,s_4$ are closed-loop poles and $f_i(s)=f_{i1}s^3+f_{i2}s^2+f_{i3}s+f_{i4}$ for all $i=1,\cdots,4$, $f_5(s)(:=p(s)\det(G(s)))=f_{52}s^2+f_{53}s+f_{54}$, and $k_5(:=\det(K))=k_1k_4-k_2k_3$.

Equating the coefficients of the same orders on both sides of Equation (4) yields

$$Lk = a \tag{5}$$

where

$$L=\begin{bmatrix}1 & 0 & \cdots & 0 & 0\\ b_1 & f_{11} & \cdots & f_{41} & 0\\ b_2 & f_{12} & \cdots & f_{42} & f_{52}\\ b_3 & f_{13} & \cdots & f_{43} & f_{53}\\ b_4 & f_{14} & \cdots & f_{44} & f_{54}\end{bmatrix}, k=\begin{bmatrix}1\\ k_1\\ \vdots\\ k_4\\ k_5\end{bmatrix}, a=\begin{bmatrix}1\\ a_1\\ a_2\\ a_3\\ a_4\end{bmatrix}$$

In Equations (4) and (5), it is known that the numerical construction algorithm of the Plücker matrix formula $Lk = a$ in (2,2,4) systems is implemented by the following procedure:

1st column of L: Filled by the coefficients of an open-loop characteristic polynomial $p(s)$.

2nd column of L: Filled by the coefficients of $p(s)G_1(s)$, which is matched with the gain k_1.

⋮

5th column of L: Filled by the coefficients of $p(s)G_4(s)$, which is matched with the gain k_4.

6th column of L: Filled by the coefficients of the interacting factor $p(s)(G_1(s)G_4(s)-G_2(s)G_3(s))$, which is matched with the gain k_5.

For the reduced formula, Equation (4) is rearranged into

$$\sum_{i=1}^{5} f_i(s)k_i = p_c(s) - p(s) \tag{6}$$

And equating the coefficient of the same orders on both sides of Equation (6) yields

$$L_{\text{sub}}k_{\text{sub}} = a_{\text{sub}} \tag{7}$$

where

$$L_{\text{sub}}=\begin{bmatrix}f_{11} & \cdots & f_{41} & 0\\ f_{12} & \cdots & f_{42} & f_{52}\\ f_{13} & \cdots & f_{43} & f_{53}\\ f_{14} & \cdots & f_{44} & f_{54}\end{bmatrix}, k_{\text{sub}}=\begin{bmatrix}k_1\\ \vdots\\ k_4\\ k_5\end{bmatrix}, a_{\text{sub}}=\begin{bmatrix}a_1-b_1\\ a_2-b_2\\ a_3-b_3\\ a_4-b_4\end{bmatrix}$$

Remark 1. In algebraic geometry, the elements $[1\,k_1 \cdots k_5]^t$ of k in Equation (5) are named inhomogeneous Plücker coordinates of Grassmann space Grass(2,4), constrained in a quadratic equation $k_5 - k_1k_4 + k_2k_3 = 0$, whose solution set is called Grassmann variety.

Remark 2. The general Plücker matrix formula $Lk = a$ for general (m,p,n) systems can be numerically constructed through the application of Binet-Cauchy theorem to determinantal matrix formula $\det[I+KG(s)]$ and its comparisons with the signal flow graph analysis of the closed-loop determinant Δ in Mason's gain formula [14].

III. Main Result

In Theorem 1, the necessary and sufficient condition for the exact pole assignability of (2,2,4) systems is explicitly provided for the Plucker matrix formula in Equation (7).

Theorem 1. The 2-input, 2-output, 4th order strictly proper linear systems are exactly pole assignable if and only if the last column ℓ_5 in L is zero under $rank(L_{\text{sub}}) = 4$. ■

Proof: The sufficient condition is obviously obtained by setting l_5 to be zero in L_{sub} of Equation (7). If l_5 is zero, then K_{sub} can be obtained under the condition $rank(L_{\text{sub}}) = 4$ for any given a_{sub} regardless of the relation for k_5. So EPA is satisfied.

Next, we will prove the necessary condition by contradiction. Assume that l_5 is not zero and the (2,2,4) system is exactly pole assignable. Because $rank(L_{\text{sub}}) = 4$, there happen three cases for the matrix L_{sub} when l_5 is not zero.

Case 1. One of 4 columns, $\ell_1,\cdots,\ell_4$ in L_{sub} is zero.

Case 2. Some 2 columns of L_{sub} are linearly dependent.

Case 3. Every two columns of L_{sub} are linearly independent

Case 1: If one of the 1st 4 columns of L_{sub},

$\ell_i(i=1,\cdots,4)$, is zero under $rank(L_{\text{sub}})=4$, then the four real SOF variables except k_i variable are always determined from $L_{\text{sub}}k_{\text{sub}}=a_{\text{sub}}$. Putting the four values in the 5 variable quadratic equation (QE), the QE $k_5-k_1k_4+k_2k_3=0$ is reduced into a 1 variable 1st order linear equation. Thus, the real solution-set of the SOF equtions, $Lk=a$ and QE, is complete on the real field R except a singular zero-value point where the gain multiplied to k_i in the QE is zero.

Case 2: In Case 2, there are three situations.

i) Two columns, $\{\ell_1,\ell_4\}$ or $\{\ell_2,\ell_3\}$ in L_{sub}, are linearly dependent.

If two columns of L_{sub}, $\{\ell_1,\ell_4\}$ or $\{\ell_2,\ell_3\}$ are linearly dependent, then from $L'_{\text{sub}}k'_{\text{sub}}=a'_{\text{sub}}$, four SOF variables where one combined variable X is represented by $X=k_1+\gamma k_4$ or $X=k_2+\delta k_3$ for real constants γ and δ, are always determined by real values. Putting the four real values into $k_5-k_1k_4+k_2k_3=0$, the QE is reduced into a 1 variable 2nd order equation. Thus by the algebraic character of the 1 variable 2nd order equation constructed for some real vector a, these (2,2,4) systems are NPA with some real-disconnected interval on the SOF gain variables in R.

ii) Two columns, ℓ_5 and one column of $\{\ell_1,\ell_2,\ell_3,\ell_4\}$ in L_{sub}, are linearly dependent.

If two columns in L_{sub}, $\{\ell_i,\ell_5\}\,(i=1,\cdots,4)$ are linearly dependent, then from $L'_{\text{sub}}k'_{\text{sub}}=a'_{\text{sub}}$, four variables where a combined variable X is represented by $X:=k_5+\lambda k_i$ for real constant λ, are always determined by real values. Putting the four real values into $k_5-k_1k_4+k_2k_3=0$, the QE is reduced into a 1 variable 1st order equation with a singular point. For example, let $k_i=k_2$, then from $X:=k_5+\lambda k_i=\alpha_1\alpha_4-\alpha_3k_2+\lambda k_2=\alpha_X$, the k_2 is obtained by

$$k_2=\frac{\alpha_1\alpha_4-\alpha_X}{\alpha_3-\lambda}$$

In this case, k_i is one of $\{k_1,k_2,k_3,k_4\}$ and $\alpha_X,\alpha_1,\alpha_3,\alpha_4$ indicate the real values of X,k_1,k_3,k_4, respectively, in $L'_{\text{sub}}k'_{\text{sub}}=a'_{\text{sub}}$. In this way, the SOF k_2 has a singular point at $\alpha_3=\lambda$ for a special real vector a. Thus, these (2,2,4) systems are exactly pole assignable except a singular point at k_i.

iii) Two columns, $\{\ell_1,\ell_2$ (or $\ell_3)\}$ or $\{\ell_4,\ell_2$ (or $\ell_3)\}$ in L_{sub}, are linearly dependent.

If two columns in L_{sub}, $\{\ell_1,\ell_2$ (or $\ell_3)\}$ or $\{\ell_4,\ell_2$ (or $\ell_3)\}$, are linearly independent, then from $L'_{\text{sub}}k'_{\text{sub}}=a'_{\text{sub}}$, four SOF variables where one combined variable X is represented by $X:=k_i+\rho k_j$ for real constant ρ, are always determined by real values. Putting the four real values in k_5, the QE is reduced into a 1 variable 1st order equation with a singular point. For example, let $k_i=k_1$ and $k_j=k_2$ then from $X:=k_1+\rho k_2=\alpha_X$, k_5 is obtained by $\alpha_5=k_1\alpha_4-\alpha_3(\alpha_X-k_1)/\rho$. Therefore, k_1 is given by

$$k_1=\frac{\alpha_5+\alpha_3\alpha_X}{\alpha_4-\alpha_3/\rho}$$

In this case, the SOF k_1 has a singular point at $\alpha_4+\alpha_3/\rho=0$ for a special real vector a. Thus, these (2,2,4) systems are exactly pole assignable except a singular point at k_i and k_j.

Case 3: If every 2 columns of L'_{sub} are linearly independent under $rank(L_{\text{sub}})=4$, then from the 1st 4 diagonalized matrix L'_{sub} in $L'_{\text{sub}}k'_{\text{sub}}=a'_{\text{sub}}$, 2∼4 variables among $k'_1,\cdots,k'_4$ in k'_{sub} become linear functions over the last remaining variable, k'_5.

i) 4 variable linear function case:

In L'_{sub} by rank-nullity theorem, rank (L'_{sub}) + null (L'_{sub}) = number of columns of L'_{sub}. Thus, the 1st 4 variables in $L'_{\text{sub}}k'_{\text{sub}}=a'_{\text{sub}}$ depend upon the last 1 variable k'_5.

$$\begin{bmatrix}1&0&0&0&\beta_1\\0&1&0&0&\beta_2\\0&0&1&0&\beta_3\\0&0&0&1&\beta_4\end{bmatrix}\begin{bmatrix}k'_1\\k'_2\\k'_3\\k'_4\\k'_5\end{bmatrix}=\begin{bmatrix}\alpha'_1\\\alpha'_2\\\alpha'_3\\\alpha'_4\end{bmatrix}$$

In this case, let $\beta_1\ell_1' + \beta_2\ell_2' + \beta_3\ell_3' + \beta_4\ell_4' = \ell_5'$ where ℓ_i' indicates the i^{th} column of L_{sub}', then all 4 variables $k_1',\cdots,k_4'$ are linear functions on the variable k_5'. Thus the QE, $k_5 - k_1k_4 + k_2k_3 = 0$ is always reduced into a 1 variable 2nd order equation of k_5' constructed through arbitrary selection of 4 variables among 5 variables in the QE for some real vector a.

ii) 3 variable linear function case:

In the same way as i), 3 variables among 4 variables $k_1',\cdots,k_4'$ in $L_{sub}'k_{sub}' = a_{sub}'$ depend upon the last 1 variable k_5'. For example, let $\beta_2\ell_2' + \beta_3\ell_3' + \beta_4\ell_4' = \ell_5'$, then 3 variables k_2', k_3', k_4' have linear functions with the variable k_5'. Thus the QE, $k_5 - k_1k_4 + k_2k_3 = 0$ is always reduced into a 1 variable 2nd order equation of k_5' constructed through arbitrary selection of 3 variables among 5 variables in the QE for some real vector a.

iii) 2 variable linear function case:

In the same way as ii), 2 variables among 4 variables $k_1',\cdots,k_4'$ in $L_{sub}'k_{sub}' = a_{sub}'$ depend upon the last 1 variable k_5'. For example, let $\beta_3\ell_3' + \beta_4\ell_4' = \ell_5'$, then 2 variables k_3', k_4' have linear functions with the variable k_5'. Thus the QE, $k_5 - k_1k_4 + k_2k_3 = 0$ is always reduced into a 1 variable 1st or 2nd order equation of k_5' constructed through arbitrary selection of 2 variables among 5 variables in the QE for some real vector a.

For the above three cases, 2-input, 2-output, 4th order strictly proper linear systems are not exactly pole assignable if l_5 is not zero. This is contradictory to the assumption. ■

Remark 3. The necessary and sufficient condition for the EPA of (2,2,4) systems given in Theorem 1 is equivalent to the condition, $\det(G(s)) = 0$ (called, rank-one systems) and no linear combination of the set of $\{G_i(s)\}$ vanishes [11],[15].

IV. Numerical Example

Consider a 2-input, 2-output, 4th order strictly proper system [16] with the system matrices given by

$$A = \begin{bmatrix} 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 1&0&1&0 \end{bmatrix}, B = \begin{bmatrix} 0&0 \\ 1&0 \\ 0&0 \\ 0&1 \end{bmatrix}, C = \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \end{bmatrix}$$

First, exact pole assignability is checked using the condition in Theorem 1.

Step 1: The system transfer function $G(s)$ $(:= C(sI-A)^{-1}B)$ is obtained by

$$G(s) = \begin{bmatrix} \frac{s^2-1}{s^4-s^2-1} & \frac{1}{s^4-s^2-1} \\ \frac{s^3-s}{s^4-s^2-1} & \frac{s}{s^4-s^2-1} \end{bmatrix}$$

From Equation (5), $Lk = a$ is constructed by

$$\begin{bmatrix} 1&0&0&0&0&0 \\ 0&0&1&0&0&0 \\ -1&1&0&0&0&0 \\ 0&0&-1&0&1&0 \\ -1&-1&0&1&0&0 \end{bmatrix} \begin{bmatrix} 1 \\ k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \end{bmatrix} = \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \tag{8}$$

without constraint of k_5. In the rank test, $rank(L_{sub}) = 4$, and the last column, l_5 of L_{sub} is zero. Thus, the condition for EPA in Theorem 1 is satisfied. So, this SOF system has EPA feature by real SOF. Next, the control gain K is obtained since the system satisfies pole assignability.

Step 2: From arbitrary desired pole positions of $(s+1)^2(s+2)^2 = 0$, the real coefficients of the closed-loop characteristic polynomial $p_c(s)$ are obtained by $a_1 = 6, a_2 = 13, a_3 = 12, a_4 = 4$. From $rank(L_{sub}) = 4$, the reduced row echelon form $L_{sub}'k_{sub}' = a_{sub}'$ is obtained by

$$\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} 14 \\ 6 \\ 19 \\ 8 \end{bmatrix} \tag{9}$$

From Equation (9), k_5 is calculated with

$k_5 = 14 \times 18 - 6 \times 19$ and the real solution K is directly obtained by

$$K = \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} = \begin{bmatrix} 14 & 6 \\ 19 & 18 \end{bmatrix}$$

Ⅴ. Conclusion

It is presented that the static output feedback pole-assignment problem and its related stabilization problem of 2-input, 2-output, 4th order strictly proper linear systems can be completely resolved by the real Grassmannian paramerization method within Plücker matrix formula $Lk = a$, as a self-contained algorithm in Grassmann space. In this paper, the necessary and sufficient condition was provided for the exact pole assignablitiy of static output feedback problem in 2-input, 2-output, 4th order linear systems. Futhermore, it can be shown that previous diverse pole-assignment methods and various computation algorithms of real gains are unified using this real Grassmannian parametrization method.

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BIOGRAPHY

Su-Woon Kim (Member)

1974 : BS degree in Electrical Engineering, Seoul National University.
1996 : PhD degree in Electrical Engineering, Univ. of Minnesota, U.S.A
2003~2010 : Lecturer, Dept. of Electrical Eng., Jeju National University

Min-Jae Kang (Member)

1982 : BS degree in Electrical Engineering, Seoul National University.
1991 : PhD degree in Electrical Engineering, Univ. of Louisvill, U.S.A
1992~Current : Professor, Dept. of Electronic Eng., Jeju National University

Seong-Ho Song (Member)

1987 : BS degree in Control & Instrument Eng., Seoul National University.
1991 : MS degree in Control & Instrument Eng., Seoul National University.
1995 : PhD degree in Control & Instrument Eng., Seoul National University
1996~Current : Professor, School of Software, Hallym University

Ho-Chan Kim (Member)

1987 : BS degree in Control & Instrument Eng., Seoul National University.
1989 : MS degree in Control & Instrument Eng., Seoul National University.
1994 : PhD degree in Control & Instrument Eng., Seoul National University
1995~Current : Professor, Dept. of Electrical Eng., Jeju National University