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A Fast Calculation of Apparent Soil Resistivity Using Exponential Sampling Method

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Abstract

The apparent soil resistivity is used for estimating multilayer soil parameters, such as, layer's depth and soil resistivity. The soil parameters are estimated by continuously revising those parameters until the error between the measured and calculated apparent soil resistivity reaches to allowable level. The equation for calculating the apparent soil resistivity is complicated and time consumed, because it is composed of an infinite integral which includes a zero order Bessel's function of the first kind. In this paper, a fast algorithm for calculating the apparent soil resistivity of horizontal multilayer earth structure is proposed using exponential sampling method.

Keywords: *apparent soil resistivity, estimated, soil parameters, Bessel function, multilayer earth structure.*

1. INTRODUCTION

It is important to know the earth structure in the given area when the grounding system is designed. Because badly designed grounding system cannot ensure the safety of equipment and personnel [1]. The soil is modeled as a uniform medium in early researches and the simplified formula is used to estimate the resistance of grounding system. For simplifying the problem, in a host of engineering application, multilayer soils are modeled by n horizontal layers with distinct resistivity and depths. A Wenner configuration method is commonly used to measure the apparent soil resistivity for this simplified earth model[2].

The inversion of soil parameters is an unconstrained nonlinear minimization problem [3],[4]. Supposing there are the n different layers below the ground surface then 2n-1 parameters need to be determined, because there exists different n-1 thicknesses and n resistivities in a Wenner configuration model[5]. Therefore, many different optimization methods have been carried out to invert soil parameters in the hope of improving the performance. However, there exist two difficulties in inverting the soil parameters using optimization methods. The first one, it is hard to obtain the derivatives of the optimized expression. The second one, the computing is hugely time consumed[6]. Among these difficulties, the computing time problem can be solved efficiently by using the proposed method in this paper.

The two and three-layer earth structures have been used in the numerical examples for simplicity to examine the proposed method, however this method also can be applied to a multi-layer structure more than three. The rest of this paper is organized as follows: Section 2 gives a brief overview of the apparent soil resistivity for a multi-layer earth model. The apparent soil resistivity can be measured by using a Wenner method, and can be

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calculated theoretically if the soil parameters are given. Section 3 presents a fast algorithm to calculate the apparent soil resistivity using exponential sampling method and the numerical example results in Section 4, and the conclusion is presented in Section 5.

2. APPARENT RESISTIVITY

2. 1. Measurement of apparent resistivity using a Wenner method

The most commonly used method for measuring apparent resistivity is a Wenner 4-point method which was published in 1915 by Frank Wenner. In this method, for simplifying the problem, multilayer soils are modeled by n horizontal layers with distinct resistivity and depths [2].

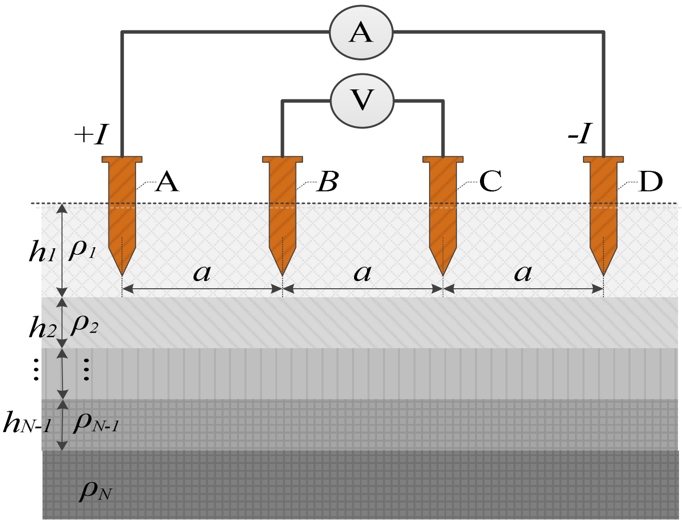

Figure 1. Wenner configuration for measuring apparent soil resistivity of N-layer earth structure

A current I is injected into the soil by applying the power between electrodes A and D and the potential difference ΔV_{BC} between electrodes B and C is measured. Then the resistance $R[\Omega]$ is obtained by $\Delta V_{BC}/I$. And the apparent resistivity $\rho[\Omega.m]$ can be expressed by definition as follows

$$\rho_a^m = 2\pi a \frac{\Delta V_{BC}}{I} = 2\pi a R \quad (1)$$

The apparent soil resistivity is named because the soil resistivity is measured on the earth surface. And in this paper its superscript, m means the measured apparent resistivity to distinguish from the calculated one (ρ_a^c) and its subscript, a is for meaning the apparent resistivity. The span between electrodes is established equally and by changing the test electrode span a, a set of apparent resistivity curves varying with electrode span can be obtained as shown in fig. 2. The data measured with the wide span a are used for analyzing the deep earth structure. Therefore, the range of span a is determined with the depth of investigated area. As shown in fig. 1 of the Wenner configuration, h_i $(i = 1,2,\cdots,N-1)$ and ρ_i $(i = 1,2,\cdots,N)$ are the thickness and the resistivity of the ith layer respectively for an n layer soil structure.

2. 2. Calculation of theoretical apparent resistivity

If the soil parameters, $h_i (i = 1,2,\cdots,N-1)$ and $\rho_i (i = 1,2,\cdots,N)$ are known, the apparent resistivity can be calculated theoretically using the following equation[1]

$$\rho_a^c = \rho_1\left[1 + 2a\int_0^\infty f(\lambda)[J_0(\lambda a) - J_0(2\lambda a)]d\lambda\right] \quad (2)$$

where ρ_1 is the soil resistivity of the first layer, a is electrode span, $J_0(\lambda r)$ is the zero order Bessel's function of the first kind and the kernel function $f(\lambda)$ is defined as follows [1]

$$f(\lambda) = \alpha_1(\lambda) - 1 \quad (3)$$

$$\alpha_1(\lambda) = 1 + \frac{2K_1 e^{-2\lambda h_1}}{1 - K_1 e^{-2\lambda h_1}} \qquad K_1(\lambda) = \frac{\rho_2\alpha_2 - \rho_1}{\rho_2\alpha_2 + \rho_1}$$

$$\alpha_2(\lambda) = 1 + \frac{2K_2 e^{-2\lambda h_2}}{1 - K_2 e^{-2\lambda h_2}} \qquad K_2(\lambda) = \frac{\rho_3\alpha_3 - \rho_2}{\rho_3\alpha_3 + \rho_2}$$

$$\vdots \qquad \vdots$$

$$\alpha_{n-1}(\lambda) = 1 + \frac{2K_{n-1} e^{-2\lambda h_{n-1}}}{1 - K_{n-1} e^{-2\lambda h_{n-1}}} \qquad K_{n-1}(\lambda) = \frac{\rho_n - \rho_{n-1}}{\rho_n + \rho_{n-1}}.$$

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The superscript of ρ_a^c, c is used for meaning the calculation apparent resistivity to distinguish from the measured one in the equation(1).

3. FAST CALCULATION OF THE APPARENT SOIL RESISTIVITY USING EXPONENTIAL SAMPLING METHOD

To estimate the soil parameters, $h_i\,(i = 1,2,\cdots,N-1)$ and $\rho_i\,(i = 1,2,\cdots,N)$, the theoretical apparent resistivity is calculated every time when new soil parameters are updated. In other words, the soil parameters are continuously revised until the calculated apparent resistivity using the given parameters reaches to close to the measured one. Nevertheless, it is hugely time consumed work to calculate the apparent resistivity as shown in equation (2). That is because of the infinite integral including Bessel's function in equation (2). In this paper, the fast and simple method has been suggested to calculate the apparent resistivity by using exponential sampling method.

The kernel function $f(\lambda)$ is assumed to be a continuous and smooth function with respect to λ, and the property of its asymptotic function is much similar to the damping exponential function [2]. So the kernel function $f(\lambda)$ can be approximated by a series of exponential function using Prony's method as follows[8]

$$f(\lambda) \simeq \sum_{k=1}^{n} b_k e^{-c_k\lambda} \quad \textbf{(4)}$$

where b_k, c_k are complex numbers, n is samples number. With the given soil parameters, the function $f(\lambda)$ is determined according to equation (3) and the *n* samples which are equally spaced are obtained from this function $f(\lambda)$. And then, b_k, c_k in (4) are also approximated using this *n* samples and Prony's method. The first step is to make the assumed kernel function $f(\lambda)$ with the updated soil parameters, and the next step is to obtain n samples from this $f(\lambda)$. With those n samples, b_k, c_k in (4) can be approximated using Prony's method. The samples number n can be set to any enough big number, but according to author's experience, it is suggested to set bigger than the twice of the layer's number (*N*).

Using the following Lipschitz integral

$$\int_0^\infty e^{-\lambda|c|} J_0(\lambda l)\,d\lambda = \frac{1}{\sqrt{c^2+l^2}} \tag{5}$$

The apparent resistivity ρ_a^c in (2) can be approximated as follows

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$$\rho_a^c(a) \simeq \rho_1 \left\{ 1 + 2a \sum_{k=1}^{N} b_k \left[\frac{1}{\sqrt{c_k^2 + a^2}} - \frac{1}{\sqrt{c_k^2 + 4a^2}} \right] \right\} \tag{6}$$

4. NUMERICAL EXAMPLES

The apparent resistivity is calculated by using equation (2) and (6). As shown fig. 2, it can be seen that the results by the method proposed in this paper are almost the same as those by conventional method. The two and three-layer earth structures have been tested in numerical examples to examine the proposed method, however this method can be applied to a multi-layer structure more than three. Fig.2 shows the effectiveness of the Prony's method and exponential sampling method. The corresponding parameters are shown in Table 1, 2.

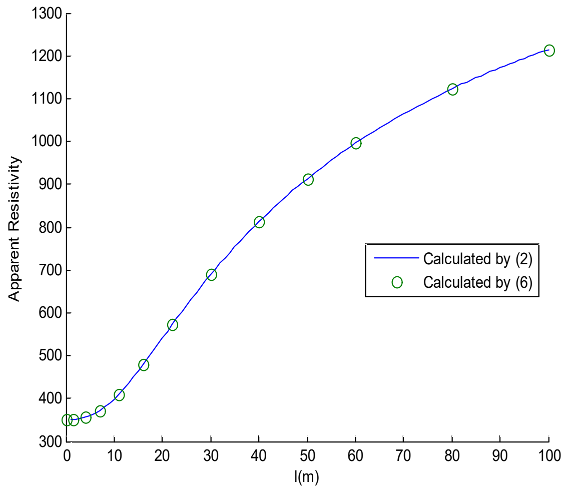
The simulation is performed in Matlab and numbers of cases have been simulated to compare the calculation time of two methods using Matlab function, tic, toc[7]. It has been seen that the execution time with the new method of equation (6) is several tens time faster than that with the conventional method of equation (2).

Table 1. Parameters of a two-layer earth structure

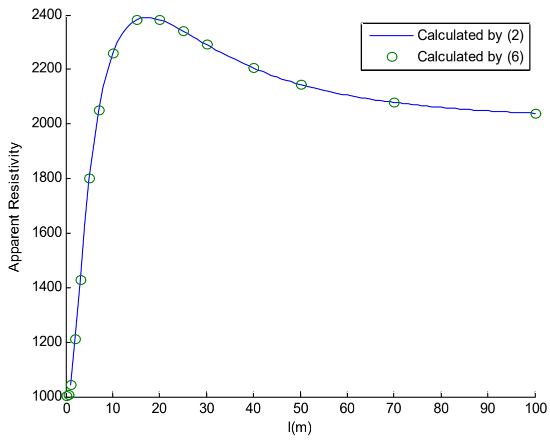
Layer No.	ρ ($\Omega \cdot m$)	h(m)
1	352	14
2	1600	∞

Table 2. Parameters of a three-layer earth structure

Layer No.	ρ (Ω·m)	h(m)
1	1,000	2
2	2,000	10
3	3,000	∞



(a)



(b)

Figure 2. the apparent resistivities(ρ_a^c) using eqn.(2) and (6)

(a) With the parameters shown in Table 1 (b) With the parameters shown in Table 2

5. CONCLUSION

In this paper, it has been introduced the fast method to calculate the theoretical apparent resistivity. The first step is to express the kernel function $f(\lambda)$ by a series of complex exponential function using Prony's method and the second step is to use Lipschitz formula to avoid the infinite integral which includes Bessel's function. The apparent resistivities with the suggested method has been agreed almost with those of the conventional method. However, in the view of calculating time, the new method had been shown several tens time faster than the previous method.

It is a kind of inverse problem to estimate soil parameters. The inverse problem is a hugely time consumed work because it needs to calculate every time with the new updated parameters until reaching to the allowable err between the measured and calculated values. Therefore, the suggested method in this paper is considered very useful to solve these time consumed problems.

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