

RULED SURFACES IN E^3 WITH DENSITY

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Abstract. In the present paper, we study curves in \mathbb{E}^3 with density $e^{ax^2+by^2}$, where $a, b \in \mathbb{R}$ not all zero constants and give the parametric expressions of the curves with vanishing weighted curvature. Also, we create ruled surfaces whose base curves are the curve with vanishing weighted curvature and the ruling curves are Smarandache curves of this curve. Then, we give some characterizations about these ruled surfaces by obtaining the mean curvatures, Gaussian curvatures, distribution parameters and striction curves of them.

1. Introduction

In different spaces, the geometry of curves and surfaces is an interesting area for differential geometers for a long time. The curvature of a curve $\alpha(u) = (x(u), y(u), 0)$ in a plane is an important invariant for a curve and it is defined as ([13], [17])

$$(1) \quad \kappa = \frac{x'(u)y''(u) - x''(u)y'(u)}{\left((x'(u))^2 + (y'(u))^2\right)^{3/2}}.$$

Also, the Smarandache curves which play an important role in Smarandache geometry have been obtained with the aid of the Frenet frame of a curve. If we denote TN –Smarandache curve as β_{TN} , TB –Smarandache curve as β_{TB} , NB –Smarandache curve as β_{NB} and TNB –Smarandache

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curve as β_{TNB} of a curve, then they are given by

$$\begin{aligned}\beta_{TN}(u) &= \frac{T(u) + N(u)}{\|T(u) + N(u)\|}, \quad \beta_{TB}(u) = \frac{T(u) + B(u)}{\|T(u) + B(u)\|}, \\ \beta_{NB}(u) &= \frac{N(u) + B(u)}{\|N(u) + B(u)\|} \text{ and } \beta_{TNB}(u) = \frac{T(u) + N(u) + B(u)}{\|T(u) + N(u) + B(u)\|},\end{aligned}$$

respectively. More information about Smarandache curves can be found in [1], [2], [18], [19] and etc.

On the other hand, ruled surfaces are one-parameter set of lines and they have been studied widely in differential geometry. Also, they have been applied on different areas such as architectural, CAD, electric discharge machining and etc ([4], [15]). A ruled surface is defined as

$$(2) \quad R := \varphi(u, v) = \alpha(u) + vX(u), \quad u, v \in I \subset \mathbb{R},$$

where the curve $\alpha(u)$ is called base curve and $X(u)$ is called the ruling of the ruled surface. The striction curve and distribution parameter of the ruled surface (2) are

$$(3) \quad \gamma(u) = \alpha(u) - \frac{\langle \alpha'(u), X'(u) \rangle}{\|X'(u)\|^2} X(u)$$

and

$$(4) \quad \delta = \frac{\det[\alpha'(u), X(u), X'(u)]}{\|X'(u)\|^2},$$

respectively [9], [14].

In recent years, weighted manifolds with density has started to be a popular topic in different areas. Density is the ratio of mass to volume; it is a measure of a material's or object's compactness and it is a way to describe mass in a continuous system $\varrho = \frac{m}{V}$. Manifolds with density arise naturally in mathematics, physics and economics. Here, we'll give some examples for them.

Manifolds with density arise in physics when considering surfaces or regions with differing physical density. An object may have differing internal densities so in order to determine the object's mass it is necessary to integrate volume weighted with density. As an example of an important two-dimensional surface with density is the Gauss plane, an Euclidean plane with volume and length weighted by $(2\pi)^{-1}e^{-r^2/2}$, where r is the distance from the origin. Also, Perelman's 2003 proof of the 1904 Poincare conjecture considers a manifold with a density (as in freshman physics or calculus). In government and economics, it is often

necessary to consider aggregate properties of groups and subgroups of people. For large groups, these aggregate properties can be determined by integrating over the members of the group, the differing individual properties (much like different densities).

Actually, manifold with density is a Riemannian manifold with positive density function e^φ used to weight volume and area. In terms of the underlying Riemannian volume dV_0 and area dA_0 , the new weighted volume and area are given by

$$\begin{aligned} dV &= \varphi dV_0, \\ dA &= \varphi dA_0. \end{aligned}$$

From the first variation of weighted area, Gromow [6] has introduced φ -curvature (or weighted curvature) κ_φ of a curve and φ -mean curvature (or weighted mean curvature) H_φ of an n -dimensional hypersurface on a manifold with density e^φ and they are defined by

$$\begin{aligned} \kappa_\varphi &= \kappa - \frac{d\varphi}{dN}, \\ H_\varphi &= H - \frac{1}{n-1} \frac{d\varphi}{d\eta}, \end{aligned}$$

where κ is Riemannian curvature and N is the normal vector of the curve; H is Riemannian mean curvature and η is the normal vector field of the hypersurface.

Following, φ -Gaussian curvature (or weighted Gaussian curvature) G_φ of a Riemannian manifold with density e^φ has been defined by [5]

$$G_\varphi = G - \Delta\varphi,$$

where G is Gaussian curvature of the hypersurface and Δ is the Laplacian operator and the generalized Gauss-Bonnet formula for a smooth topological disc R is obtained as

$$\int_R G_\varphi + \int_{\partial R} \kappa_\varphi = 2\pi.$$

Also, Bakry and Émery [3] have defined a generalization of the Ricci tensor of Riemannian manifold M^n with density e^φ by

$$Ric_\varphi^\infty = Ric - Hess\varphi,$$

where, $Hess\varphi$ is Hessian of φ and Ric is Ricci curvature of M^n . For more details about manifolds with density, one can see [6], [7], [8], [10], [11], [12], [16], [20], [21] and etc.

2. Curves With Vanishing Weighted Curvature in E^3 With Density $e^{ax^2+by^2}$

In this section, firstly we obtain the weighted curvature of a curve in \mathbb{E}^3 with positive density $e^{ax^2+by^2}$. Also, we create the curves with vanishing weighted curvature according to cases of $b = 0$ and $a = 0$ and obtain Smarandache curves of one of these curves.

Let $\alpha(u) = (x(u), y(u), 0)$ be a curve in \mathbb{E}^3 . The weighted curvature of the curve $\alpha(u)$ in \mathbb{E}^3 with density $e^{ax^2+by^2}$ is obtained as

$$(5) \quad \kappa_\varphi = \kappa - \frac{d\varphi}{dN} \\ = \frac{x'(u)y''(u) - x''(u)y'(u) + 2(ax(u)y'(u) - bx'(u)y(u))(x'(u)^2 + y'(u)^2)}{(x'(u)^2 + y'(u)^2)^{3/2}}.$$

So,

Proposition 2.1. *Weighted curvature κ_φ of the curve $\alpha(u) = (x(u), y(u), 0)$ in \mathbb{E}^3 with density $e^{ax^2+by^2}$ is zero if and only if*
 $x'(u)y''(u) - x''(u)y'(u) + 2(ax(u)y'(u) - bx'(u)y(u))(x'(u)^2 + y'(u)^2) = 0$
is satisfied.

Corollary 2.2. *If the curve $\alpha(u) = (x(u), y(u), 0)$ is a unit speed curve, then the weighted curvature κ_φ of the curve $\alpha(u)$ in \mathbb{E}^3 with density $e^{ax^2+by^2}$ is*

$$\kappa_\varphi = x'(u)y''(u) - x''(u)y'(u) + 2(ax(u)y'(u) - bx'(u)y(u)).$$

Now, we will examine the weighted curvature of a curve in \mathbb{E}^3 with density $e^{ax^2+by^2}$ for different values of constants a and b .

If $b = 0$, $a \neq 0$, then from (5) the weighted curvature κ_φ of the curve $\alpha(u)$ in \mathbb{E}^3 with density e^{ax^2} is obtained as follows

$$\kappa_\varphi = \frac{x'(u)y''(u) - x''(u)y'(u) + 2ax(u)y'(u)(x'(u)^2 + y'(u)^2)}{(x'(u)^2 + y'(u)^2)^{3/2}}.$$

Hence, we have

Proposition 2.3. *Weighted curvature κ_φ of the curve $\alpha(u)$ in \mathbb{E}^3 with density e^{ax^2} vanishes if and only if*

$$(6) \quad x'(u)y''(u) + 2ax(u)y'(u)(x'(u)^2 + y'(u)^2) = x''(u)y'(u)$$

is satisfied.

From (6), we have

$$y(u) = c_2 \mp \int_1^u \frac{x'(k)}{\sqrt{-1 + c_1 e^{2ax(k)^2}}} dk.$$

Thus, taking the sign of "±" which has been stated in the last equation as "+", we have

Theorem 2.4. *The curve $\alpha_1(u)$ with vanishing weighted curvature in \mathbb{E}^3 with density e^{ax^2} can be parametrized by*

$$(7) \quad \alpha_1(u) = \left(x(u), c_2 + \int_1^u \frac{x'(k)}{\sqrt{-1 + c_1 e^{2ax(k)^2}}} dk, 0 \right).$$

One can see the graph of the curve $\alpha_1(u)$ for $x(u) = \sqrt[4]{u}$, $c_1 = 1$, $c_2 = 0$ and different values of a in Figure 1.

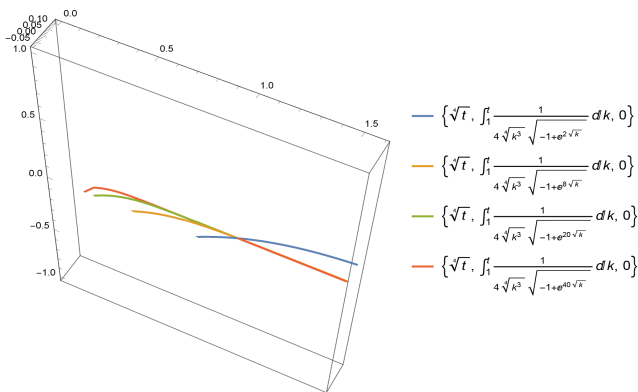


FIGURE 1

If $a = 0$, $b \neq 0$, then from (5) the weighted curvature κ_φ of the curve $\alpha(u)$ in \mathbb{E}^3 with density e^{by^2} is obtained as follows

$$\kappa_\varphi = \frac{x'(u)y''(u) - x''(u)y'(u) - 2bx'(u)y(u)(x'(u)^2 + y'(u)^2)}{(x'(u)^2 + y'(u)^2)^{3/2}}.$$

So, we get

Proposition 2.5. *Weighted curvature κ_φ of the curve $\alpha(u) = (x(u), y(u), 0)$ in \mathbb{E}^3 with density e^{by^2} is zero if and only if*

$$(8) \quad x'(u)y''(u) = x''(u)y'(u) + 2bx'(u)y(u)(x'(u)^2 + y'(u)^2)$$

is satisfied.

From (8), we have

$$x(u) = c_2 \pm \int_1^u \frac{y'(k)}{\sqrt{-1 + c_1 e^{2by(k)^2}}} dk.$$

So,

Theorem 2.6. *The curve $\alpha_2(u)$ with vanishing weighted curvature in \mathbb{E}^3 with density e^{by^2} can be parametrized by*

$$(9) \quad \alpha_2(u) = \left(c_2 \pm \int_1^u \frac{y'(k)}{\sqrt{-1 + c_1 e^{2by(k)^2}}} dk, y(u), 0 \right).$$

Now, let we construct the Smarandache curves of the curve (7).

The tangent, normal and binormal vectors of the curve $\alpha_1(u)$ are obtained as

$$\begin{aligned} T &= \frac{1}{\sqrt{x'(u)^2 + y'(u)^2}} (x'(u), y'(u), 0) \\ &= \frac{1}{\sqrt{c_1 e^{2ax(u)^2}}} \left(\sqrt{-1 + c_1 e^{2ax(u)^2}}, 1, 0 \right), \\ N &= \frac{1}{\sqrt{x'(u)^2 + y'(u)^2}} (-y'(u), x'(u), 0) \\ &= \frac{1}{\sqrt{c_1 e^{2ax(u)^2}}} \left(-1, \sqrt{-1 + c_1 e^{2ax(u)^2}}, 0 \right), \\ B &= (0, 0, 1), \end{aligned}$$

respectively. So, the TN -Smarandache curve β_{TN} , TB -Smarandache curve β_{TB} , NB -Smarandache curve β_{NB} and TNB -Smarandache

curve β_{TNB} of the curve $\alpha_1(u)$ are written as

$$\begin{aligned} \beta_{TN}(u) &= \left(\frac{\sqrt{-1 + c_1 e^{2ax(u)^2}} - 1}{\sqrt{2c_1 e^{2ax(u)^2}}}, \frac{\sqrt{-1 + c_1 e^{2ax(u)^2}} + 1}{\sqrt{2c_1 e^{2ax(u)^2}}}, 0 \right), \\ (10) \quad \beta_{TB}(u) &= \left(\frac{\sqrt{-1 + c_1 e^{2ax(u)^2}}}{\sqrt{2c_1 e^{2ax(u)^2}}}, \frac{1}{\sqrt{2c_1 e^{2ax(u)^2}}}, \frac{1}{\sqrt{2}} \right), \\ \beta_{NB}(u) &= \left(\frac{-1}{\sqrt{2c_1 e^{2ax(u)^2}}}, \frac{\sqrt{-1 + c_1 e^{2ax(u)^2}}}{\sqrt{2c_1 e^{2ax(u)^2}}}, \frac{1}{\sqrt{2}} \right), \\ \beta_{TNB}(u) &= \left(\frac{\sqrt{-1 + c_1 e^{2ax(u)^2}} - 1}{\sqrt{3c_1 e^{2ax(u)^2}}}, \frac{\sqrt{-1 + c_1 e^{2ax(u)^2}} + 1}{\sqrt{3c_1 e^{2ax(u)^2}}}, \frac{1}{\sqrt{3}} \right), \end{aligned}$$

respectively. Hence, Figure 2 shows the graph of the Smarandache curve $\beta_{NB}(u)$ of the curve $\alpha_1(u)$ for $x(u) = \sqrt[4]{u}$, $c_1 = 1$ and $a = 1, 2000, 4000, 6000$.

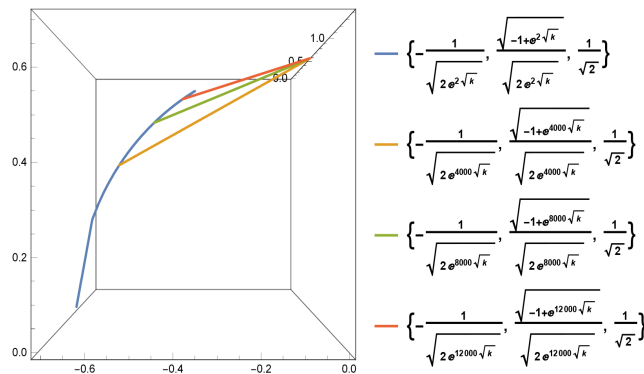


FIGURE 2

Similarly, the graphs of the Smarandache curves $\beta_{TN}(u)$, $\beta_{TB}(u)$ and $\beta_{TNB}(u)$ of the curve $\alpha_1(u)$ can be drawn for different choosings of $x(u)$, c_1 and a .

3. Ruled Surfaces Generated by the Curve (7) and its Smarandache Curves

In this section, firstly we construct the ruled surfaces with the help of the curve $\alpha_1(u)$ and its Smarandache curves. Also, we obtain the mean

curvatures, Gaussian curvatures, distribution parameters and striction curves for each of these ruled surfaces and give some characterizations for them.

Throughout this section, the base curves of the ruled surfaces will be taken as the curve (7).

Let the ruling curve of the ruled surface be the TN -Smarandache curve $\beta_{TN}(u)$ of the curve $\alpha_1(u)$. Thus from (2), (7) and (10), the ruled surface R_{TN} can be parametrized by

$$(11) \quad R_{TN} := \varphi_{TN}(u, v) = \alpha_1(u) + v\beta_{TN}(u) \\ = (x(u) + v \left(\frac{A_1 - 1}{\sqrt{2}A_2} \right), \\ c_2 + \int_1^u \frac{x'(k)}{\sqrt{-1 + c_1 e^{2ax(k)^2}}} dk + v \left(\frac{A_1 + 1}{\sqrt{2}A_2} \right), 0),$$

where $A_1 = \sqrt{-1 + c_1 e^{2ax(u)^2}}$ and $A_2 = \sqrt{c_1 e^{2ax(u)^2}}$.

Since the ruled surface R_{TN} is a parametrization of a plane, it is clear that the Gaussian curvature, mean curvature and from (4), the distribution parameter δ_{TN} of it are zero and it is developable.

From (3), the parametrization of the striction curve $\gamma_{TN}(u)$ on the ruled surface R_{TN} is

$$\gamma_{TN}(u) = \alpha_1(u) - \frac{A_2}{2\sqrt{2}ax(u)}\beta_{TN}(u).$$

So, we can state the following Theorem:

Theorem 3.1. *The base curve and the striction curve of the ruled surface R_{TN} never intersect.*

Let the ruling curve of the ruled surface be the TB -Smarandache curve $\beta_{TB}(u)$ of the curve $\alpha_1(u)$. Thus from (2), (7) and (10), the ruled surface R_{TB} can be parametrized by

$$(12) \quad R_{TB} := \varphi_{TB}(u, v) = \alpha_1(u) + v\beta_{TB}(u) \\ = (x(u) + \frac{A_1 v}{\sqrt{2}A_2}, c_2 + \int_1^u \frac{x'(k)}{\sqrt{-1 + c_1 e^{2ax(k)^2}}} dk + \frac{v}{\sqrt{2}A_2}, \frac{v}{\sqrt{2}}).$$

The Gaussian and mean curvatures of the ruled surface R_{TB} are

$$G = -\frac{4a^2(A_2)^2x(u)^2}{((A_2)^2 + 4a^2v^2x(u)^2)^2},$$

$$H = \frac{av(\sqrt{2}A_2A_1 - 2\sqrt{2}aA_2A_1x(u)^2 + 4a^2vx(u)^3)}{((A_2)^2 + 4a^2v^2x(u)^2)^{3/2}},$$

respectively.

Also from (4), the distribution parameter of the ruled surface R_{TB} is

$$\delta_{TB} = \frac{A_2}{2ax(u)}.$$

It is known that, a ruled surface is developable, if its distribution parameter vanishes. So, we have

Theorem 3.2. *The ruled surface R_{TB} is not developable.*

From (3), the parametrization of the striction curve $\gamma_{TB}(u)$ on the ruled surface R_{TB} is

$$\gamma_{TB}(u) = \alpha_1(u).$$

So,

Theorem 3.3. *The base curve and the striction curve of the ruled surface R_{TB} coincide.*

Let the ruling curve of the ruled surface be the NB -Smarandache curve $\beta_{NB}(u)$ of the curve $\alpha_1(u)$. Thus from (2), (7) and (10), the ruled surface R_{NB} can be parametrized by

(13)

$$R_{NB} := \varphi_{NB}(u, v) = \alpha_1(u) + v\beta_{NB}(u)$$

$$= (x(u) - \frac{v}{\sqrt{2}A_2}, c_2 + \int_1^u \frac{x'(k)}{\sqrt{-1 + c_1e^{2ax(k)^2}}}dk + \frac{vA_1}{\sqrt{2}A_2}, \frac{v}{\sqrt{2}}).$$

The Gaussian and mean curvatures of the ruled surface R_{NB} are

$$G = 0,$$

$$H = \frac{ax(u)}{\sqrt{2((A_2)^2 + 2\sqrt{2}aA_2vx(u) + 2a^2v^2x(u)^2)}},$$

respectively.

Also from (4), the distribution parameter of the ruled surface R_{NB} is

$$\delta_{NB} = 0.$$

So, we have

Theorem 3.4. *The ruled surface R_{NB} is developable.*

From (3), the parametrization of the striction curve $\gamma_{NB}(u)$ on the ruled surface R_{NB} is

$$\gamma_{NB}(u) = \alpha_1(u) - \frac{A_2}{2\sqrt{2}ax(u)}\beta_{NB}(u).$$

So, the following Theorem can be given:

Theorem 3.5. *The base curve and the striction curve of the ruled surface R_{NB} never intersect.*

Finally, let the ruling curve of the ruled surface be the TNB -Smarandache curve $\beta_{TNB}(u)$ of the curve $\alpha_1(u)$. Thus from (2), (7) and (10), the ruled surface R_{TNB} can be parametrized by

$$\begin{aligned} (14) \quad R_{TNB} &:= \varphi_{TNB}(u, v) = \alpha_1(u) + v\beta_{TNB}(u) \\ &= (x(u) + \left(\frac{A_1 - 1}{\sqrt{3}A_2}\right)v, \\ &\quad c_2 + \int_1^u \frac{x'(k)}{\sqrt{-1 + c_1 e^{2ax(k)^2}}} dk + \frac{v(A_1 + 1)}{\sqrt{3}A_2}, \frac{v}{\sqrt{3}}). \end{aligned}$$

The Gaussian and mean curvatures of the ruled surface R_{TNB} are

$$\begin{aligned} G &= -\frac{a^2(A_2)^2x(u)^2}{((A_2)^2 + 2\sqrt{3}aA_2vx(u) + 4a^2v^2x(u)^2)^2}, \\ H &= \frac{a(\sqrt{3}A_2vA_1 + (A_2)^2x(u) - 2\sqrt{3}aA_2v(A_1 - 2)x(u)^2 + 8a^2v^2x(u)^3)}{2\sqrt{2}((A_2)^2 + 2\sqrt{3}aA_2vx(u) + 4a^2v^2x(u)^2)^{3/2}}, \end{aligned}$$

respectively.

Also from (4), the distribution parameter of the ruled surface R_{TNB} is

$$\delta_{TNB} = \frac{A_2}{4ax(u)}.$$

So, we have

Theorem 3.6. *The ruled surface R_{TNB} is not developable.*

From (3), the parametrization of the striction curve $\gamma_{TNB}(u)$ on the ruled surface R_{TNB} is

$$\gamma_{TNB}(u) = \alpha_1(u) - \frac{\sqrt{3}A_2}{4ax(u)}\beta_{TNB}(u).$$

So, the following Theorem can be given:

Theorem 3.7. *The base curve and the striction curve of the ruled surface R_{TNB} never intersect.*

In figure 3, one can see the ruled surfaces R_{TN} , R_{TB} , R_{NB} and R_{TNB} for $x(u) = \sqrt[4]{u}$, $c_1 = 1$, $c_2 = 0$ and $a = 1$.

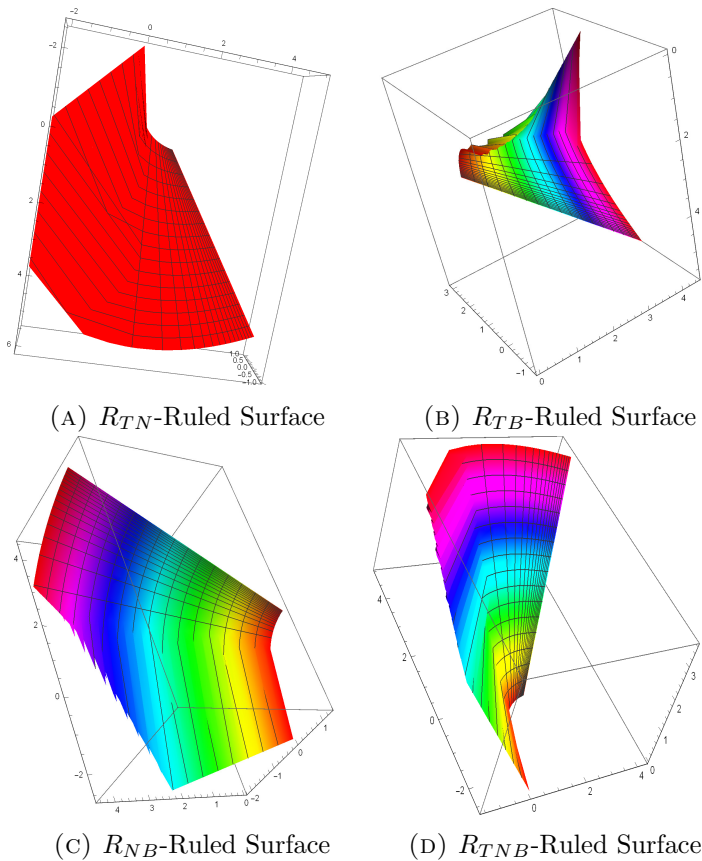


FIGURE 3. Ruled Surfaces

4. Conclusion and Future Work

In this paper, we obtain the weighted curvature of a planar curve in \mathbb{E}^3 with density $e^{ax^2+by^2}$, $a, b \in \mathbb{R}$ not all zero constants. Then, we get the curves with vanishing weighted curvature according to the cases of constants a and b and create the Smarandache curves of one of these curves. Following, we construct the ruled surfaces whose base curves are the curve with vanishing weighted curvature and ruling curves are the Smarandache curves of this curve. By giving the distribution parameters, striction curves, mean curvatures and Gaussian curvatures for each of these ruled surfaces, we state some results for them. Also, we draw the obtaining curves and ruled surfaces with the aid of Mathematica.

In addition, one can study these curves and surfaces in different spaces with different densities and we think that, these studies can be useful for geometers and physicists.

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