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INDIVIDUAL AND SOCIAL INCENTIVES VERSUS R&D NETWORK RESTRICTION

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ABSTRACT. This paper examines individual and social strategies to form profitable cooperation networks. These two types of strategies measure network stability and efficiency that may not meet in a single network. We apply restrictions on knowledge flows (R&D spillovers) and links formation to integrate these benefits into structures that ensure high outcomes for both strategies. The results suggest that linking the spillovers to the firms' positions and restricting cooperation contribute to reducing the conflict between the individual and social strategies in the development of cooperative networks.

1. INTRODUCTION

The investment and cooperation of companies in R&D are associated with issues such as the amount and duration of the investment and the type of products of other firms in the market. When the concept of the network was introduced into R&D literature, there are other issues related to cooperative behavior in the social and economic perspective. For example, the effect of network size (population in the network) and collaborative links between collaborators. Also, the extent to which individual and social perspectives are close to identifying the most beneficial network.

The concept of the network is a new visualization of the R&D cooperation that represents firms in the form of nodes and R&D partnerships in the form of links. When any two firms in the market decided to participate in R&D together, they will be represented by nodes connected by a link. The bilateral links together with firms will ultimately form a network called an R&D cooperative network. The type of the network varies with the partnership decisions between firms. Also, the number of potential networks increases with the number of collaborators in R&D.

Goyal and Moraga-Gonzalez (2001) is one of those who studied the R&D cooperation under the network framework [1]. Their network game begins by giving firms freedom to choose the R&D partnerships. At this stage, the cooperation network is set where R&D spillovers are imposed between non-linked firms to ensure knowledge flow between non-cooperators. After

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that firms together choose their investments in R&D in order to reduce the production cost. Then, they compete in the market by setting their quantities of products to maximize their profits. One of the main conclusion of that paper concerned the strategies of firms in forming and developing R&D networks and determining the strength of these strategies in the light of social desires. They found that when firms sell homogeneous goods, maximum benefits are obtained from an individual perspective in a structure that ensures the cooperation of all existing firms; whereas very low-cooperative structures are socially preferable.

The present paper contributes to the R&D literature in terms of the theoretical framework. The focus is on equilibrium outcomes where individual strategies are a major source of network growth. We develop the network game of Goyal and Moraga-Gonzalez by applying measures from social network theory to restrict the R&D spillovers between non-cooperators and limit the formation of links. The aim of this paper is to create opportunities to attract individual and social incentives in networks with almost similar characteristics.

Firstly, we assume that the R&D spillovers between non-cooperating firms are not free from the network structure. To do this, we apply two different types of constraints. In the first type, we assume that the spillovers are managed by the density of the network (a ratio of actual links out of potential links that can be set with the same number of firms). In the second type, we assume that the spillovers to firms are managed through their activities in R&D (number of R&D agreements). These enforcements on the spillover reflect the role played by the R&D network development in controlling transmission of knowledge between firms. There are some authors who have tried to manage the benefits among agents in the network by applying measures from social network theory (e.g., [2, 3, 4]). Secondly, we assume that firms are not free to choose their R&D partners. In particular, firms are divided into groups such that firms in one group cannot cooperate. Also, firms cannot establish more than cooperative link. In the network formation stage, these restrictions limit inter-firm linkages. In the empirical literature, similar patterns of collaborative networks were observed. Under different data sets, some authors found that complex systems of the R&D collaboration had a low level of interaction compared to a fully collaborative structure with the same number of firms [5, 6, 7].

The outcomes of this paper show that linking the R&D spillover to the density of the network does not present an intrinsic change in the relationship between the individual and social benefits. However, linking the spillover to the firms' positions in the network reduces the conflict between the individual and social benefits. In terms of the links formation, the outcomes show that restricting the R&D agreements has a positive effect on pooling the individual and social incentives. It creates favorable organizations where individual and social benefits reach maximum results, especially when the number of R&D agreements for each firm is reduced to a minimum.

The paper proceeds as follows. In the next section, we review some issues related to economics and networks. In the third section, we review related literature. In the fourth and fifth sections, we present our results. In the sixth section, we conclude the paper.

2. Background

2.1. **The Model.** The emphasis in this paper is on the linear-quadratic function of consumers given by Hackner (2000) [8]:

$$U = \alpha \sum_{i=1}^{n} q_i - \frac{1}{2}\left(\lambda \sum_{i=1}^{n} q_i^2 + 2\beta \sum_{j \neq i} q_i q_j\right) + I\ . \tag{2.1}$$

Here the demand parameters $\alpha > 0$ denotes the willingness of consumers to pay and $\lambda > 0$ is the diminishing marginal rate of consumption. To simplify the analysis, we assumed that $\lambda = 1$. The parameter q_i is the quantity consumed of good i and I measures the consumer's consumption of all other products. The parameter β such that $-1 \leq \beta \leq 1$ captures the marginal rate of substitution between different goods. In this paper, we consider the case when firms sell homogeneous goods ($\beta = 1$).

■ **Payoffs.** Let p_i be the price to produce a unit of good i and m be a consumer's income. If the consumer buys q_i of good i, then the money spent is $p_i q_i$ and the balance is $I = m - p_i q_i$. By substituting into equation (2.1), we determine the optimal consumption of good i by calculating

$$\frac{\partial U}{\partial q_i} = \alpha - q_i - \beta \sum_{j \neq i} q_j - p_i = 0\ .$$

This implies the inverse demand function for each good i

$$D_i^{-1} = p_i = \alpha - q_i - \beta \sum_{j \neq i} q_j, \quad i = 1, \ldots, n\ .$$

If c_i is the cost to produce a unit of good i, then the cost to produce q_i of good i is $c_i q_i$. Thus, the profit for firm i is

$$\pi_i = (p_i - c_i) q_i = \left(\alpha - q_i - \beta \sum_{j \neq i} q_j - c_i\right) q_i\ . \tag{2.2}$$

The total welfare is the sum of consumer surplus (CS) and industry surplus (Π):

$$TW = \underbrace{\frac{(1-\beta)}{2} \sum_{i=1}^{n} q_i^2 + \frac{\beta}{2}\Big(\sum_{i=1}^{n} q_i\Big)^2}_{CS} + \underbrace{\sum_{i=1}^{n} \pi_i}_{\Pi}\ . \tag{2.3}$$

■ **Cost Reduction.** The effective amount of investment of each firm is sum of its expenditure on R&D and part of investments of other firms in an industry [9]. The part benefit is defined by an external parameter ϕ called an R&D spillover that captures knowledge flow acquired from firms not cooperated in R&D. In the case of two firms in the industry, the effective investment of firm i is defined as follows:

$$S_i = s_i + \phi s_j\ ,$$

where s_i is the amount of investment of firm i in R&D and $\phi \in [0, 1)$ is the R&D spillover. The effective investment reduces firm i's marginal cost (c_0) of production:

$$c_i = c_0 - S_i = c_0 - s_i - \phi s_j \ .$$

2.2. **Network.** A **network** is a set of objects (called nodes or vertices) that are connected together by edges or links [20]. Let N be a set of all vertices labeled by numbers or letters $N = \{i, j, k, ...\}$ and $E = \{ij, jk, ...\}$ be a set of all edges in the network. Let $\mathcal{G}^n$ be a set of all distinct networks generated from n firms. Then, $G(N, E) \in \mathcal{G}^n$ denotes a network with nodes N and links E, and for simplicity the network is denoted by G. For the purpose of this article, we assume that each link joins two different vertices and serve both sides (i.e., undirected networks).

A set of **neighbors** of node i consists of all nodes that are linked to it: $N_i = \{j \in N : ij \in E\}$. The length of the neighbors' set of node i is used to refer to the **degree** of that node i.e., for each node $i \in N$, $deg(i) = |N_i|$ where $0 \leq deg(i) \leq n - 1$. In this paper, the degree of node i sometimes indicates the size of the neighbors of that node. Thus, if $|N| = n$ is the number of nodes, the degree centrality of node i is $C_d(i) = deg(i)/(n - 1)$. If $|E| = m$ is the number of links, the density of the network G is $D = 2m/n(n - 1)$ where $0 \leq D \leq 1$.

A **bipartite network** is a graph whose nodes can be divided into two disjoint sets V_1 and V_2 where nodes that belong to the same set cannot be linked. An example of this type of networks is **star networks** in which central nods linked to others that are located at the periphery where the latter nodes are not linked to each other. A **k-partite network** is a graph that may be partitioned into k sets such that no node in any of the k sets connects to another node of that same set. A **complete k-partite** graph is a k-partite graph in which there is a link between every pair of nodes of different independent sets. A **matching** M in G is a set of links such that no two links share a common vertex. A **maximal matching** in the network G is a matching in which contains the largest possible number of links. A **complete network** is a graph in which each two nodes are linked and an **empty network** is a graph consists of nodes without links between them.

■ **R&D Network Model.** The R&D network can be understood as the quest to a better capture of R&D partnerships between firms. Assume n firms in the industry, the R&D cooperation can be represented by a network G where the partnerships of firms are represented by links. We assume that the R&D agreement between any two firms requires the consent and full participation of both firms. This means in a network framework, each link between any two firms serve both sides. This means in a network framework, each link between any two firms serve both sides. The network game of Goyal and Moraga-Gonzalez consists of three stages as follows:

The first stage: Firms choose their partners in R&D. At the end of this stage, the cooperation networks G is constructed and firms know their locations in that network. In practice, the network $G \in \mathcal{G}^n$ is captured by a symmetric $n \times n$ adjacency matrix $A = (a_{ij})$ where $a_{ij} \in \{0, 1\}$. If $a_{ij} = 1$, firms i and j are linked (i.e., they cooperate in R&D), and $a_{ij} = 0$ otherwise.

The second stage: Firms choose the amounts of investment (effort) in R&D simultaneously and independently in order to reduce the cost of production. According to the model of Goyal and Moraga-Gonzalez, the effective investment in R&D for each firm i in the network G is

$$S_i = s_i + \sum_{j \in N_i} s_j + \phi \sum_{k \notin N_i} s_k, \quad i = 1, \ldots, n \,, \tag{2.4}$$

where N_i is the set of neighbors of firm i in the cooperation network and $\phi \in [0, 1)$ captures knowledge spillovers acquired from firms not cooperated in R&D with firm i [1]. It can be observed that from equation 2.4, the effective investment for firm i varies with the network structure. According to this, each firm i has a set of the effective investments corresponding to the network structures: $\mathcal{S}_i = \{S_i(G_t) : G_t \in \mathcal{G}^n\}$.

If the adjacency matrix A represents the cooperation network G, the effective R&D investment (2.4) can be rewritten as follows:

$$\begin{aligned} \mathbf{S}' &= A\mathbf{s}' + \phi A^c \mathbf{s}' \\ &= \big(A + \phi A^c\big)\mathbf{s}' \\ &= \mathbf{A}\,\mathbf{s}' \,, \end{aligned}$$

where $\mathbf{S} = [S_1, S_2, ..., S_n]$ is a matrix of the effective investments and $\mathbf{s} = [s_1, s_2, ..., s_n]$ is a matrix of the investments. The matrix A^c represents the complement network of the cooperation network G. Now, for each $a_{ij} \in \mathbf{A}$, we have

$$a_{ij} = \begin{cases} 1 & : \ ij \in E; \\ \phi & : \ otherwise. \end{cases}$$

As in the original model, the spillover is free from the network structure and the firms' positions in the network. In the following, we develop the model of Goyal and Moraga-Gonzalez by justifying the definition of the spillover.

Model A: Linking the spillover to the network density.

$$S_i = s_i + \sum_{j \in N_i} s_j + \sum_{k \notin N_i} \phi^{1-D(G)} s_k, \quad i = 1, \ldots, n \,, \tag{2.5}$$

where $D(G)$ is the density of the network G. Since $\phi \in [0, 1)$, then the fraction $\phi^{1-D(G)}$ increases with the density and this indicates that in a dense network, the spillover reaches to the maximum values. In the case of a complete network (each two firms are linked), the fraction equals one. In the case of an empty network (firms without links), the fraction equals $\phi^{1-D(G)} = \phi$ and firms obtain the lowest R&D spillover. In these two cases, Model A corresponds to the Goyal and Moraga-Gonzalez model.

Model B: Linking the spillover to the neighbors size.

$$S_i = s_i + \sum_{j \in N_i} s_j + \sum_{k \notin N_i} \phi^{1-C_d(i)} s_k, \quad i = 1, \ldots, n \,, \tag{2.6}$$

where $C_d(i)$ is the degree centrality of firm i. For each firm i, the fraction $\phi^{1-C_d(i)}$ increases with the degree centrality. This means that firms in highly central positions obtain high spillovers; whereas isolated firms obtain very low spillovers. In the case of the complete (empty) network, the fraction for each firm equals one (ϕ). This means Model B is compatible with the Goyal and Moraga-Gonzalez model in these two networks.

We assume that the marginal cost c_0 is constant and equal for all firms. The effective R&D investment is cost reducing in the sense that it reduces firm i's marginal cost of production:

$$c_i = c_0 - S_i \ ,$$

where the effective investment S_i depends on the model used.

The third stage: Firms compete in the product market by setting quantities (Cournot competition). At this stage, firms choose their levels of production in order to maximize their profits.

The investment of firms is assumed to be costly and the function of the cost is quadratic, so that given the investment $s_i \in [0, c_0]$, the cost of R&D is $C(s_i) = \mu s_i^2$, where $\mu > 0$ refers to the effectiveness of R&D expenditure [9]. From this, the profit function (2.2) becomes

$$\pi_i = (p_i - c_i)q_i - C(s_i) = \left(\alpha - c_0 - q_i - \beta \sum_{j \neq i}^{n} q_j + S_i \right) q_i - C(s_i) \ , \qquad (2.7)$$

where the marginal cost satisfies $\alpha > c_0$ and S_i depends on the network structure and model used.

■ **Stability and efficiency of R&D networks.** The study of R&D cooperation under the network game involves the concepts of pairwise stability and efficiency [4]. The pairwise stability depends on firms' profit functions and it examines the individual incentives in forming and developing the cooperation network. Meaning that when firms form a stable network, they do not have incentives to form or delete links.

Definition 1 (Pairwise Stability). *A network $G \in \mathcal{G}^n$ is stable if the following two conditions are satisfied for any two firms i, $j \in G$:*

(1) *If $ij \in G$, $\pi_i(G) \geq \pi_i(G - ij)$ and $\pi_j(G) \geq \pi_j(G - ij)$,*
(2) *If $ij \notin G$ and if $\pi_i(G) < \pi_i(G + ij)$, then $\pi_j(G) > \pi_j(G + ij)$,*

The network $G - ij$ is resulting from deleting the link ij from the network G and the network $G + ij$ is resulting from adding the link ij to the network G.

The efficiency of a network is determined by the total welfare and it examines the social benefit in forming and developing the cooperation network.

Definition 2 (Network Efficiency). *A network $G \in \mathcal{G}^n$ is efficient if $TW(G) > TW(G')$ for all $G' \in \mathcal{G}^n$.*

2.3. **Nash Equilibria.** We identify the sub-game perfect Nash equilibrium by using backwards induction. If we assume that firms sell homogeneous goods ($\beta = 1$), then from the profit function (2.7), we calculate $\frac{\partial \pi_i}{\partial q_i} = 0$ to have the best response function of quantity for good i:

$$q_i = \frac{(\alpha - c_0) + S_i - \sum_{j \neq i} q_j}{2} .$$

By substituting the best response functions into each other, we have the symmetric equilibrium output for each good i

$$q_i^* = \frac{(\alpha - c_0) + nS_i + \sum_{j \neq i} S_j}{n+1} . \tag{2.8}$$

To find the symmetric equilibrium profit, we substitute the equilibrium output (2.8) into the profit function (2.7) which gives

$$\pi_i^* = \left[\frac{(\alpha - c_0) + nS_i + \sum_{j \neq i} S_j}{n+1} \right]^2 - C(s_i) . \tag{2.9}$$

Thus the profit function can be expressed in the following form

$$\pi_i^* = q_i^{*2} - C(s_i) . \tag{2.10}$$

Now to have the final list of the equilibria, we need to know the network structure. By knowing the structure, we have the effective investment of each firm i. By substituting into the profit function (2.9) and calculating $\frac{\partial \pi_i^*}{\partial s_i} = 0$, we have the best response function of the R&D investment for each firm i. To find the symmetric equilibrium investment s_i^*, we plug the best response functions into each other. Then, we use the backwards induction to have the final equilibria. In Appendix A, we provide the final equilibrium equations for the investment and output. To find the final equilibrium equations of the profit and the total welfare, substitute the equilibria given in the Appendix A into equations (2.10) and (2.3), respectively.

3. Review of Related Literature

There are extensive studies focusing on the cooperation of firms in R&D under a network concept. The role of the network is reflected in the issue of cooperation through the use of statistical measures and tools to suit the impact of individual and social characteristics in the development of R&D partnerships (e.g., [6, 10, 11, 12]). In theoretical literature, there have been major developments in the theory of cooperation in R&D, which dealt with the evolution of relations versus economic returns.

Among the theoretical models used in this literature, we consider an important model presented by Goyal and Moraga-Gonzalez [1]. They defined the cooperation of firms in R&D as a network established by bilateral links. They extended the work by D'Aspremont and Jacquemin who studied R&D cooperation within a homogeneous Cournot duopoly (a market with two firms) [9]. The extension was based on the R&D model where they represented the cooperation of firms in R&D as a network game. They assumed that in the cooperation case, bilateral collaborative links are formed to link the cooperators. However, in the case of non-cooperation, firms are not linked and face an identical spillover ($\phi \in [0, 1)$). The work

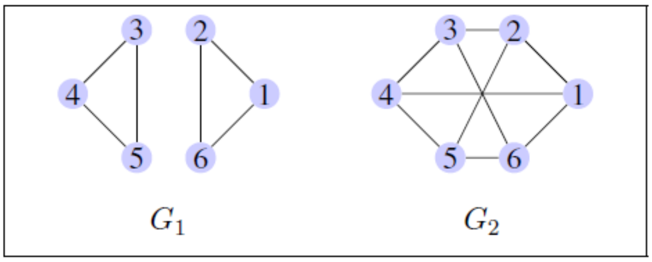

Figure 1. An example of symmetric networks with six firms. In each network, firms have the same number of links.

by Goyal and Moraga-Gonzalez have been developed and extended by many authors (e.g., [10, 12, 13, 14, 15, 16]).

The authors studied Cournot oligopoly (a market with more than two firms) for two main cases of the cooperation networks. The first case is symmetric (regular) R&D networks, which mean that each firm has the same number of cooperative links or neighbors (see Figure 1). In this case, they assumed that the cooperation network does not involve the spillover term. They did the study for an arbitrary number of firms and provided results for firms sell in two different market structures: independent and homogeneous goods. The second case is asymmetric (irregular) R&D networks, which mean the collaborative activity is asymmetrically distributed. They studied this case for three firms produce homogeneous goods with the spillover term. Figure 2 displays different cooperation networks that can be generated from three firms in the market.

The contribution of Goyal and Moraga-Gonzalez mainly concerns the relationship between the cooperative links and both R&D investments and on the incentives of firms to cooperate. In addition to this, they studied stability and efficiency of the R&D networks. They concluded that there exist situations in which the two terminologies never meet. They found that firms mostly prefer the complete network since the cooperative links always improve their own profit. Whereas, the social benefit is maximized in a network that is characterized by low cooperation activity (number of links). The difference between the individual and social incentives reflects the conflict between the stability and efficiency of the R&D networks (Propositions 6 and 8 for symmetric networks and Propositions 9 and 10 for asymmetric networks [1]).

In the current paper, we develop the Goyal and Moraga-Gonzalez network game by applying measures from social network theory to restrict the R&D spillovers and limit the links formation. The main objective of this work is to create networks that ensure high results in both individual and social returns. The restriction of the R&D network has been considered in many studies for other purposes. For example, Meagher and Rogers modeled the innovation as a function of the network structure [17]. One of their conclusions is that network density has a significant effect on the innovators. In addition, Narola and Santangelo used a data set and found that the new R&D collaborations are sensitive to their positions in the network [18].

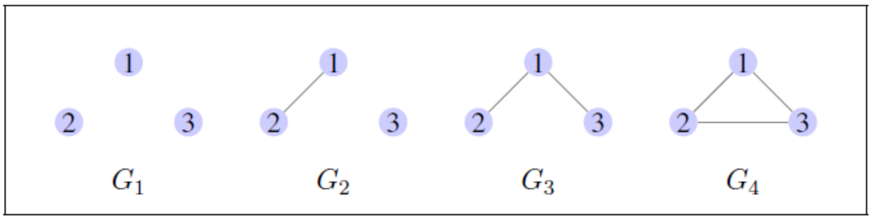

Figure 2. The distinct networks with three firms. The distribution of links is heterogeneous. In each of G_1 and G_4, there is one group of firms, but the other networks, there are two groups. In the network G_3, there is a hub (firm 1) and peripheries (firms 2 and 3) and in the network G_2, there are linked firms (firms 1 and 2) and an isolated firm (firm 3).

Zhang and Yang introduced a graphical algorithm to rebuild the R&D network to reduce the risks associated with R&D activities [19].

4. Restriction of the R&D spillover

In this section, we restrict the knowledge spillover to the network structure and firms' positions in that network. We examine changes in the outcomes and decide whether these constraints have a role in decreasing the gap between individual and social desires in developing the cooperation network. Firstly, we assume that for each market size, the spillover is a function of cooperative links (density of the network). Secondly, we assume that the spillover to a firm is corresponding to its neighbors size.

4.1. **Restriction of the Spillover to the Network Density.** We assume that the R&D spillover in the cooperation network is fixed for all firms and controlled by the density of that network. As defined in Model A (equation 2.5), the relationship between the spillover and the density is positive in a sense that the spillover between non-cooperating firms increase with the density of the network. Based on this relationship, we generate the equilibrium outcomes for all economic variables (R&D investment, production quantity, profit and total welfare) for three firms in a market with homogeneous goods. For this number of firms, there are four possible cooperation networks given in Figure 2.

Figure 3 shows the equilibrium outcomes for different values of the spillover under Model A. The outcomes suggest that linking the spillover to the network density does not provide significant changes from those generated from the original model by Goyal and Moraga-Gonzalez. To verify this point, we provide the outcomes under the original model in Appendix C. It can be observed that the behavior of the economic variables under the two models is almost identical. The main difference between the two models is in the value of the spillover which determines the stability and efficiency of the network. Table 1 compares between the original model and Model A in terms of the two terminologies. From the table, when considering a network with low density (i.e., network G_2) as a stable and efficient network, then the range of the spillover that makes that network stable or efficient is slightly large under Model A. This means that

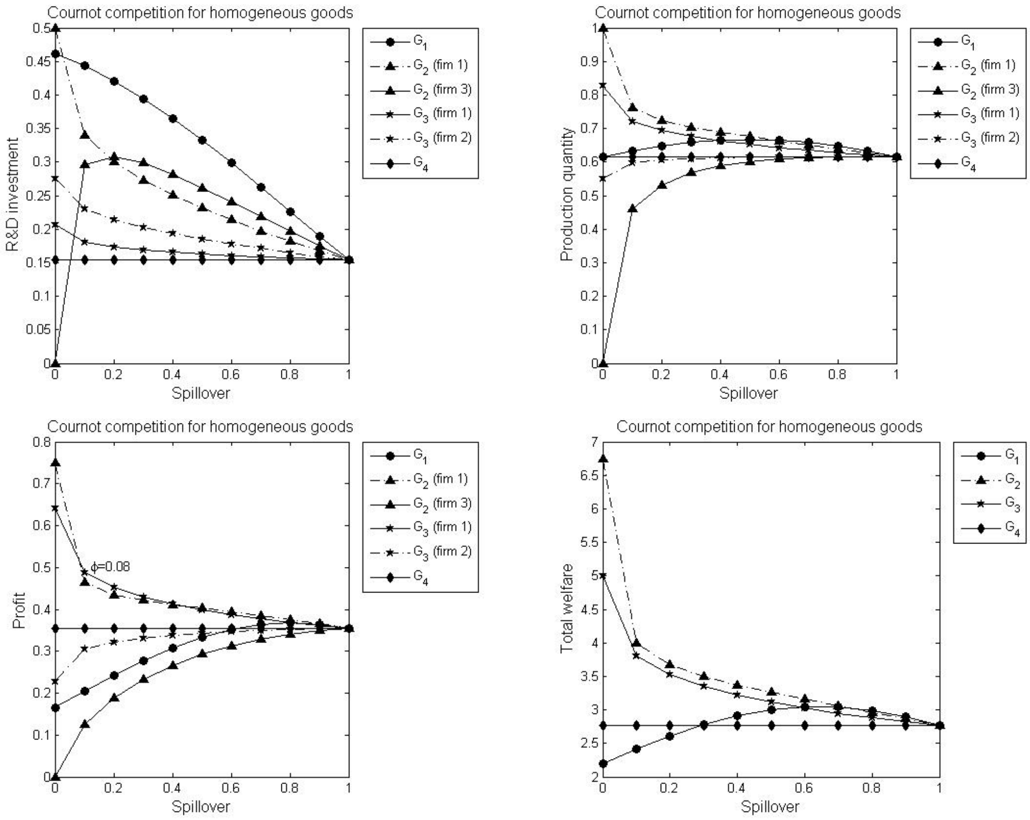

Figure 3. The equilibrium outcomes for the networks given in Figure 2 under Model A. The parameters used to plot the results are $a = 12, c_0 = 10$ and $\mu = 1$.

Model A enlarges the conflict between the stability and efficiency of the network if the cooperation between firms in R&D is low. However, if the R&D cooperation is intense; meaning that firms form a complete network (G_4), then Model A reduces the conflict between the stability and efficiency of the cooperative network. Now, since the complete network is stable for all values of the spillover, we can say that linking the spillover to the network density generally decrease the gap between the individual and social desires in developing the cooperation network.

4.2. **Restriction of the Spillover to the Neighbors Size.** We examine the impact of the position of firms in the network on the individual and social outcomes. We assume that the spillover is not identical to all firms but depends on their neighbors size (degree centralities). According to Model B (equation 2.6), the spillover to a firm increases with its degree centrality. We apply this model for three firms in the market with homogeneous goods and generate the equilibrium outcomes for the networks given in Figure 2.

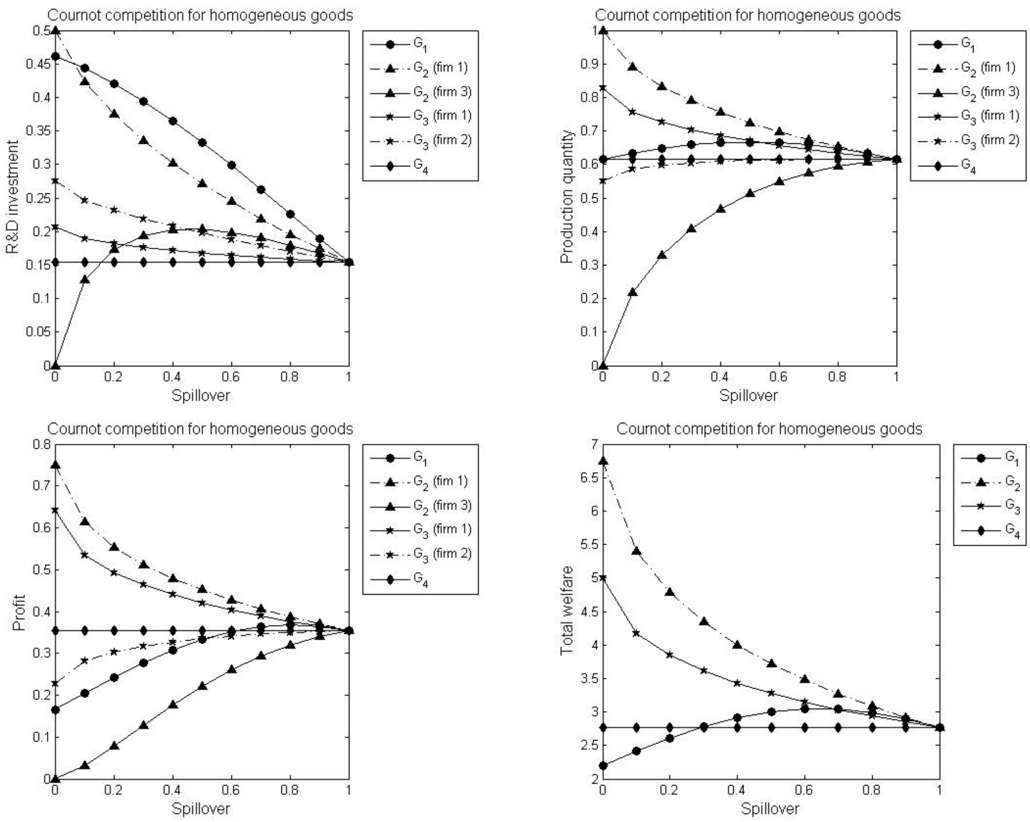

Figure 4. The equilibrium outcomes for the networks given in Figure 2 under Model B. The parameters used to plot the results are $a = 12, c_0 = 10$ and $\mu = 1$.

Figure 4 shows the outcomes for different values of the spillover. From the figure, we have the following observations. The first observation is that the behavior of the equilibrium outcomes with respect to the spillover and the links is consistent with those under the original model (given in Appendix C). The second observation is that the investment of low active (or isolated) firms under Model B is lower than under the original model. We can verify this result by comparing the investment of firm 3 in network G_2 under the two models. The third observation is that the complete network (G_4) is a unique stable network under Model B. This is one of the main differences between Model B and the original model by Goyal and Moraga-Gonzalez where under the latter model, there exist another stable network for small values of the spillover, that is the network G_2. The fourth observation is that the network G_2 is efficient for all values of the spillover. However, under the original model, the network G_1 is also efficient for high values of the spillover.

This indicates that the main contrast between the two models is in the individual and social incentives in developing the R&D networks. For very small values of the spillover, the Goyal

Table 1. The stability and efficiency of the networks given in Figure 2 under the three models.

	Network	G_1	G_2	G_3	G_4
	D	0	$1/3$	$2/3$	1
Stability	Original model	-	$0 \leq \phi \leq 0.06$	-	$0 \leq \phi \leq 1$
	Model A	-	$0 \leq \phi \leq 0.08$	-	$0 \leq \phi \leq 1$
	Model B	-	-	-	$0 \leq \phi \leq 1$
Efficiency	Original model	$0.5 < \phi \leq 1$	$0 \leq \phi \leq 0.5$	-	-
	Model A	$0.7 < \phi \leq 1$	$0 \leq \phi \leq 0.7$	-	-
	Model B	-	$0 \leq \phi \leq 1$	-	-

and Moraga-Gonzalez model is individually and socially preferable since there is no gap between their benefits in choosing the cooperation network. From Table 1, the network G_2 is stable and efficient for $0 \leq \phi \leq 0.06$. However, with increasing the spillover, Model B appears to be better in terms of reducing the gap between the stability and efficiency of the R&D network.

5. Restriction of the R&D Cooperation

In this section, we adopt the network game by Goyal and Moraga-Gonzalez with constraining the first stage of the game, that is the link formation. Instead of choosing free R&D partners, we will impose some limitations that make network density small. We will assume that firms are divided into groups and firms in one group cannot cooperate. Also, we will assume that each firm is able to establish only one agreement, i.e., $deg(i) \leq 1$ for any $i \in N$. In addition to this, we assume that at least one R&D agreement will be formed. In the theory of graphs, these conditions can be interpreted on the assumption that firms can only form a multilateral network containing only matches. Figure 5 displays bipartite and tripartite graphs. Therefore, very knowing networks in R&D cooperation literature will not be considered in this paper; for example, the complete network, the star network and the empty network.

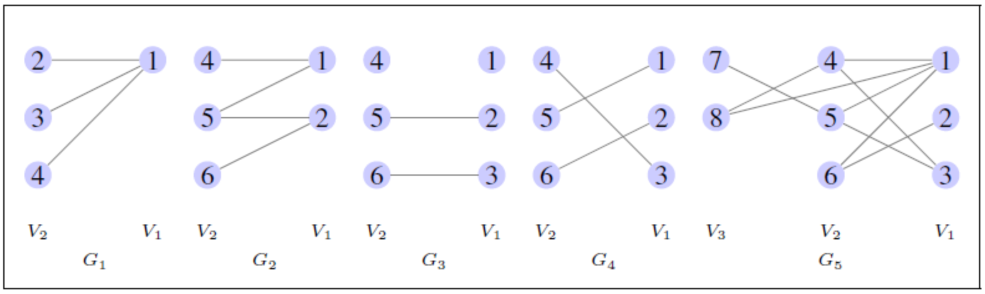

Figure 5. Examples of bipartite networks with different number of firms. The networks G_1, G_2, G_3 and G_4 are bipartite networks. The network G_3 is a tripartite network. The networks G_3 and G_4 contain only matchings.

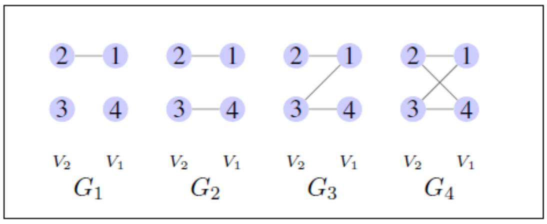

Figure 6. Some bipartite networks generated from four firms. Each set contains two firms, i.e., $|V_1| = |V_2| = 2$. The networks G_1 and G_2 only contain matchings.

5.1. **Forming Bipartite R&D Networks.** In this section, we assume that firms are divided into two groups such that firms in one group cannot cooperate in R&D. Also, we assume that firms cannot form more than one link as networks G_3 and G_4 in Figure 5.

When considering four firms in the market, there are 11 potential networks representing R&D cooperation(see Figure 10 in Appendix B). However, when restricting the R&D relationships in a way that the bipartite networks only exist, then the number of possible networks decreases. For the purpose of this paper, we focus on the four distinct networks given in Figure 6. In a market with homogeneous goods, Figure 7 displays the equilibrium outcomes for those networks.

Since the profits of firms increase with their new R&D agreements, then network G_4 is individually preferable (a stable network). This is one of the main results of Goyal and Moraga-Gonzalez (Proposition 6). In the social perspective, the optimal network for most values of the spillover is low density. When comparing the individually and socially optimal structures, we find that the difference between the densities is small compared to the case when considering all distinct networks generated from cooperating four firms. Table 2 displays the density of the four networks and Table 4 in the Appendix B displays the density of all generated networks from four firms.

A further decline in the gap between individual and social desires can be obtained if more restrictions are applied to the formation of linkages. If we assume that firms can only create one link, we will limit our interest to networks G_1 and G_2. Therefore, assuming that cooperation takes one of these two networks, then for most of the values of the spillover, the individual and social benefits are maximized in a single network. As shown in Figure 7 and Diagram 1, the network G_2 is stable and it is efficient if the spillover is not large ($\phi \leq 0.5$).

The restrictions on the formation of links include other consequences. While increasing R&D agreements reduces firms spending in R&D, reducing cooperation requires more investment. As shown in Figure 7, R&D expenditures by firms in networks G_1 and G_2 are higher

Table 2. The density of the networks given in Figure 6.

Network	G_1	G_2	G_3	G_4
D	$1/6$	$1/3$	$1/2$	$2/3$

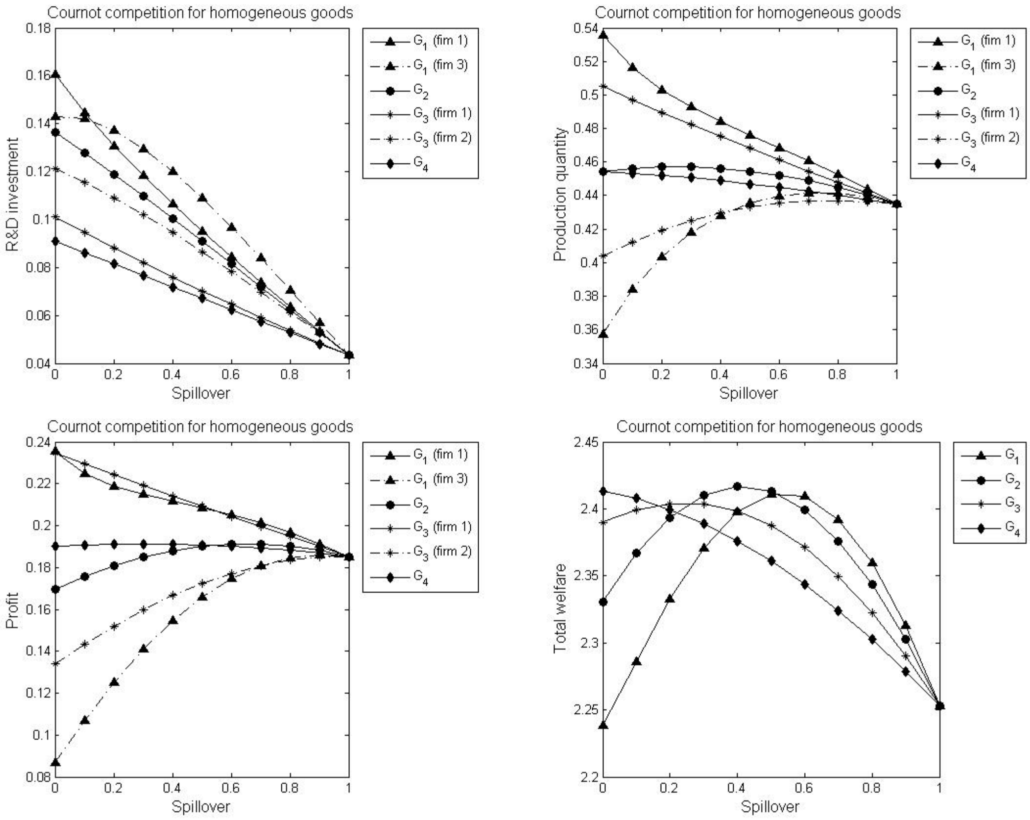

Figure 7. The equilibrium outcomes for the networks given in Figure 6. The parameters used to plot the results are $a = 12, c_0 = 10$ and $\mu = 2$.

than in other networks. However, in terms of the production quantity and the profit, the restriction of the cooperation has a negative role. This can be seen by comparing the amount of production and profit of the firm 1 in the network G_3 with those in networks G_1 and G_2. In addition, if the star network exists, then the firm that is at the center gains a higher profit than other firms in all given networks. This is because the profits of firms are increasing with their own links, especially when the cooperation between the neighbors is low.

5.2. **Forming Tripartite R&D Networks.** In this section, we assume that firms are divided into three groups such that firms in one group cannot cooperate in R&D. However, we assume that firms are allowed to connect to more one group. For example, if firm 1 is in group V_1, this firm has a choice to cooperate with firms in group V_2 or in group V_3, or in both groups.

We do this study for six firms in a market with homogeneous goods such that each two firms form one group. For the purpose of this paper, we focus on the networks given in Figure 8. As

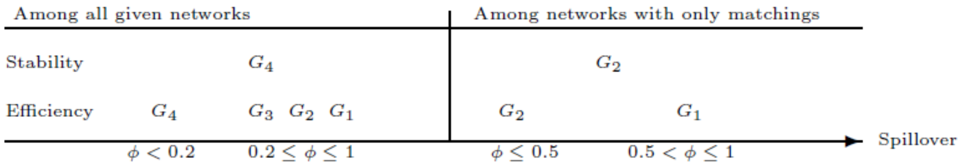

Diagram 1. The stability and efficiency of the R&D networks given in Figure 7. When restricting the R&D cooperation, the gap between network stability and efficiency is reduced.

shown in this figure, the three groups are placed next to each other where firms are linked in adjacent groups only.

Since the results of individuals differ with the cooperative links, it seems better to discuss the overall results instead of the individual outcomes. Figure 9 displays the aggregate quantities, the industry profit and the total welfare. The results show that for most values of the spillover, the total quantities in the low-density network are higher than in the other networks. Also, when considering the gap between individual and social incentives, we note that this gap decreases with the reduction of the cooperation in R&D. To verify this point, we compare the total welfare of the network G_7 (a stable network) with the total welfare of the networks G_1, G_2 and G_5 (bipartite networks that contain only matchings).

When considering all seven networks, the stable network (G_7) is efficient for small values of the spillover (see Diagram 2). However, when the cooperation is reduced, the network G_5 becomes stable for all values of the spillover and is efficient for a large range of the spillover. In addition, with increasing the spillover, the difference between the stable and efficient networks in terms of the density is small (see Diagram 2 and Table 3). This indicates that with increasing knowledge flow between firms, the individual and social returns are maximized in a network characterized by low levels of R&D cooperation activity.

On the other hand, an R&D organization with a small number of agreements has a negative impact on industry profits. As shown in Figure 9, for most values of the R&D spillover, the industry profit is maximized in high density networks i.e., G_7 and G_6. However, with increasing the spillover, the industry profit increases with decreasing the density. Moreover, firms in the middle group in the networks G_6 and G_7 dominate other firms in terms of high returns because of the positive relationship between corporate profits and their cooperative relationships.

Table 3. The density of the networks given in Figure 8.

Network	G_1	G_2	G_3	G_4	G_5	G_6	G_7
D	$2/21$	$1/7$	$1/7$	$4/21$	$4/21$	$2/7$	$8/21$

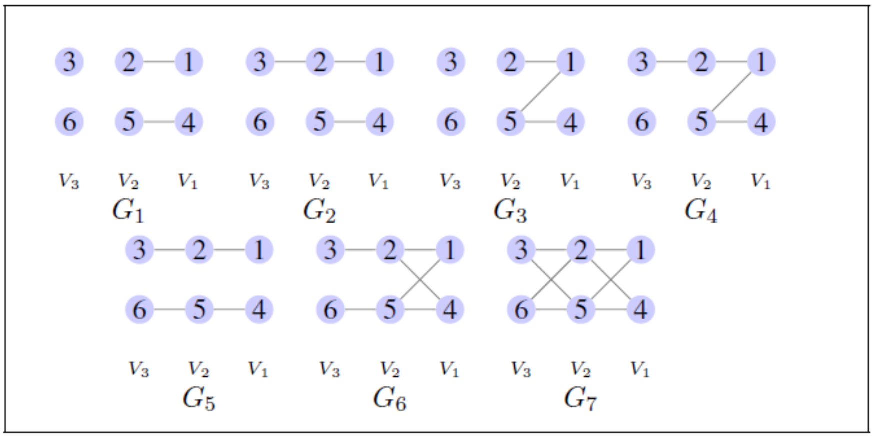

Figure 8. Some tripartite networks generated from six firms. Each group contains two firms. The networks G_1, G_2 and G_5 only contain matchings.

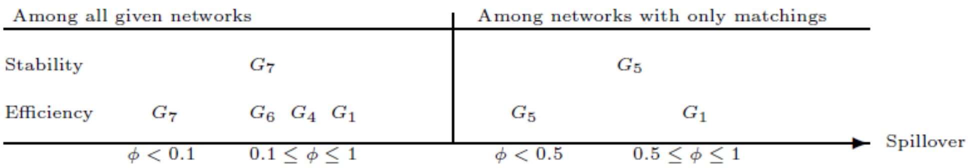

Diagram 2. The stability and efficiency of the R&D networks given in Figure 9. The gap between network stability and efficiency is small when we focus on a small number of R&D relationships.

6. Conclusion

In this paper, we have developed the theory of network formation for inter-firm cooperation in R&D. The results addressed the relationship between individual and social incentives to develop a network of R&D cooperation. Firstly, we restrict the R&D spillover to the network density. This restriction does not create a significant change in the relationship between the individual and social preferences in choosing the network architecture. Secondly, we restrict the R&D spillover to the firms' positions in the network. This restriction reduces the gap between the two perspectives in terms of the preferred structure of the R&D network. Finally, we restrict the R&D agreements between firms. This restriction reduces the density of networks which in turn has a positive impact in pooling the individual and social incentives in structures that guarantee high results.

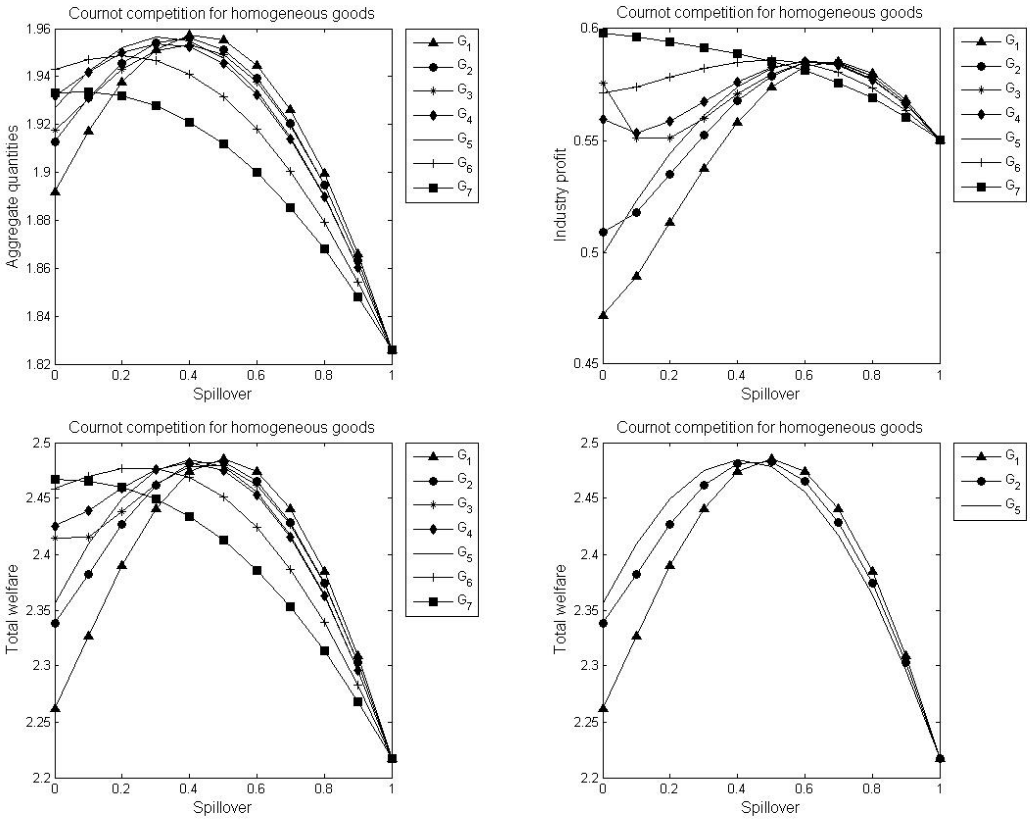

Figure 9. The equilibrium outcomes for the networks given in Figure 8. The parameters used to plot the results are $a = 12, c_0 = 10$ and $\mu = 2$.

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APPENDIX

Appendix A: For the networks given in Figure 2.

A.1 Equilibria Under the Goyal and Moraga-Gonzalez model.

$$s_{G_1} = \frac{((2\phi - 3)(\alpha - c_0))}{(-4\phi^2 + 4\phi - 16\mu + 3)}$$
$$q_{G_1} = \frac{-(4\mu(\alpha - c_0))}{(-4\phi^2 + 4\phi - 16\mu + 3)}$$

$$s_{G_2}(firm1) = \frac{((\alpha - c_0)(\phi - 2)(-2\phi^2 + 5\phi + 4\mu - 3))}{(2(2\phi^4 - 7\phi^3 + 12\phi^2\mu + 4\phi^2 - 40\phi\mu + 7\phi - 32\mu^2 + 34\mu - 6))}$$
$$q_{G_2}(firm1) = \frac{-(2\mu(\alpha - c_0)(-2\phi^2 + 5\phi + 4\mu - 3))}{(2\phi^4 - 7\phi^3 + 12\phi^2\mu + 4\phi^2 - 40\phi\mu + 7\phi - 32\mu^2 + 34\mu - 6)}$$
$$s_{G_2}(firm3) = \frac{((2\phi - 3)(\alpha - c_0)(-\phi^2 + 3\phi + 2\mu - 2))}{(2\phi^4 - 7\phi^3 + 12\phi^2\mu + 4\phi^2 - 40\phi\mu + 7\phi - 32\mu^2 + 34\mu - 6)}$$
$$q_{G_2}(firm3) = \frac{-(4\mu(\alpha - c_0)(-\phi^2 + 3\phi + 2\mu - 2))}{(2\phi^4 - 7\phi^3 + 12\phi^2\mu + 4\phi^2 - 40\phi\mu + 7\phi - 32\mu^2 + 34\mu - 6)}$$

$$s_{G_3}(firm1) = \frac{-((\alpha - c_0)(\phi^2 - 3\phi + 4\mu + 2))}{(-8\phi^2\mu + \phi^2 + 16\phi\mu - 3\phi - 64\mu^2 + 4\mu + 2)}$$
$$q_{G_3}(firm1) = \frac{-(4\mu(\alpha - c_0)(\phi^2 - 3\phi + 4\mu + 2))}{(-8\phi^2\mu + \phi^2 + 16\phi\mu - 3\phi - 64\mu^2 + 4\mu + 2)}$$
$$s_{G_3}(firm2) = \frac{(4\mu(\alpha - c_0)(\phi - 2))}{(-8\phi^2\mu + \phi^2 + 16\phi\mu - 3\phi - 64\mu^2 + 4\mu + 2)}$$
$$q_{G_3}(firm2) = \frac{-(16\mu^2(\alpha - c_0))}{(-8\phi^2\mu + \phi^2 + 16\phi\mu - 3\phi - 64\mu^2 + 4\mu + 2)}$$

$$s_{G_4} = \frac{(\alpha - c_0)}{(16\mu - 3)}$$
$$q_{G_4} = \frac{(4\mu(\alpha - c_0))}{(16\mu - 3)}$$

A.2 Equilibria Under Model A.

$$s_{G_1} = \frac{((2\phi - 3)(\alpha - c_0))}{(-4\phi^2 + 4\phi - 16\mu + 3)}$$
$$q_{G_1} = \frac{-(4\mu(\alpha - c_0))}{(-4\phi^2 + 4\phi - 16\mu + 3)}$$

$$s_{G_2}(firm1) = \frac{((\phi^{2/3}-2)(\alpha-c_0)(4\mu+5\phi^{2/3}-2\phi^{4/3}-3))}{(2(34\mu-40\phi^{2/3}\mu+12\phi^{4/3}\mu-7\phi^2+7\phi^{2/3}+4\phi^{4/3}+2\phi^{8/3}-32\mu^2-6))}$$
$$q_{G_2}(firm1) = \frac{-(2\mu(\alpha-c_0)(4\mu+5\phi(2/3)-2\phi(4/3)-3))}{(34\mu-40\phi^{2/3}\mu+12\phi^{4/3}\mu-7\phi^2+7\phi^{2/3}+4\phi^{4/3}+2\phi^{8/3}-32\mu^2-6)}$$
$$s_{G_2}(firm3) = \frac{((2\phi^{2/3}-3)(\alpha-c_0)(2\mu+3\phi^{2/3}-\phi^{4/3}-2))}{(34\mu-40\phi^{2/3}\mu+12\phi^{4/3}\mu-7\phi^2+7\phi^{2/3}+4\phi^{4/3}+2\phi^{8/3}-32\mu^2-6)}$$
$$q_{G_2}(firm3) = \frac{-(4\mu(\alpha-c_0)(2\mu+3\phi^{2/3}-\phi^{4/3}-2))}{(34\mu-40\phi^{2/3}\mu+12\phi^{4/3}\mu-7\phi^2+7\phi^{2/3}+4\phi^{4/3}+2\phi^{8/3}-32\mu^2-6)}$$

$$s_{G_3}(firm1) = \frac{-((\alpha-c_0)(4\mu-3\phi^{1/3}+\phi^{2/3}+2))}{(4\mu+16\phi^{1/3}\mu-8\phi^{2/3}\mu-3\phi^{1/3}+\phi^{2/3}-64\mu^2+2)}$$
$$q_{G_3}(firm1) = \frac{-(4\mu(\alpha-c_0)(4\mu-3\phi^{1/3}+\phi^{2/3}+2))}{(4\mu+16\phi^{1/3}\mu-8\phi^{2/3}\mu-3\phi^{1/3}+\phi^{2/3}-64\mu^2+2)}$$
$$s_{G_3}(firm2) = \frac{(4\mu(\phi^{1/3}-2)(\alpha-c_0))}{(4\mu+16\phi^{1/3}\mu-8\phi^{2/3}\mu-3\phi^{1/3}+\phi^{2/3}-64\mu^2+2)}$$
$$q_{G_3}(firm2) = \frac{-(16\mu^2(\alpha-c_0))}{(4\mu+16\phi^{1/3}\mu-8\phi^{2/3}\mu-3\phi^{1/3}+\phi^{2/3}-64\mu^2+2)}$$

$$s_{G_4} = \frac{(\alpha-c_0)}{(16\mu-3)}$$
$$q_{G_4} = \frac{(4\mu(\alpha-c_0))}{(16\mu-3)}$$

A.3 Equilibria Under Model B.

$$s_{G_1} = \frac{((2\phi-3)(\alpha-c_0))}{(-4\phi^2+4\phi-16\mu+3)}$$
$$q_{G_1} = \frac{-(4\mu(\alpha-c_0))}{(-4\phi^2+4\phi-16\mu+3)}$$

$$s_{G_2}(firm1) = \frac{-((\alpha-c_0)(\phi-2)(2\phi-4\mu-5\phi^{1/2}+3))}{(2(3\phi+34\mu-8\phi\mu+4\phi^2\mu-24\phi^{1/2}\mu-4\phi^2+4\phi^{1/2}+2\phi^3+4\phi^{3/2}-3\phi^{5/2}-32\mu^2-6))}$$
$$q_{G_2}(firm1) = \frac{(2\mu(\alpha-c_0)(2\phi-4\mu-5\phi^{1/2}+3))}{(3\phi+34\mu-8\phi\mu+4\phi^2\mu-24\phi^{1/2}\mu-4\phi^2+4\phi^{1/2}+2\phi^3+4\phi^{3/2}-3\phi^{5/2}-32\mu^2-6)}$$
$$s_{G_2}(firm3) = \frac{((2\phi^{1/2}-3)(\alpha-c_0)(-\phi^2+3\phi+2\mu-2))}{(3\phi+34\mu-8\phi\mu+4\phi^2\mu-24\phi^{1/2}\mu-4\phi^2+4\phi^{1/2}+2\phi^3+4\phi^{3/2}-3\phi^{5/2}-32\mu^2-6)}$$
$$q_{G_2}(firm3) = \frac{-(4\mu(\alpha-c_0)(-\phi^2+3\phi+2\mu-2))}{(3\phi+34\mu-8\phi\mu+4\phi^2\mu-24\phi^{1/2}\mu-4\phi^2+4\phi^{1/2}+2\phi^3+4\phi^{3/2}-3\phi^{5/2}-32\mu^2-6)}$$

$$s_{G_3}(firm1) = \frac{-((\alpha - c_0)(\phi + 4\mu - 3\phi^{1/2} + 2))}{(\phi + 4\mu - 8\phi\mu + 16\phi^{1/2}\mu - 3\phi^{1/2} - 64\mu^2 + 2)}$$

$$q_{G_3}(firm1) = \frac{-(4\mu(\alpha - c_0)(\phi + 4\mu - 3\phi^{1/2} + 2))}{(\phi + 4\mu - 8\phi\mu + 16\phi^{1/2}\mu - 3\phi^{1/2} - 64\mu^2 + 2)}$$

$$s_{G_3}(firm2) = \frac{(4\mu(\phi^{1/2} - 2)(\alpha - c_0))}{(\phi + 4\mu - 8\phi\mu + 16\phi^{1/2}\mu - 3\phi^{1/2} - 64\mu^2 + 2)}$$

$$q_{G_3}(firm2) = \frac{-(16\mu^2(\alpha - c_0))}{(\phi + 4\mu - 8\phi\mu + 16\phi^{1/2}\mu - 3\phi^{1/2} - 64\mu^2 + 2)}$$

$$s_{G_4} = \frac{(\alpha - c_0)}{(16\mu - 3)}$$

$$q_{G_4} = \frac{(4\mu(\alpha - c_0))}{(16\mu - 3)}$$

Appendix B: The possible networks generated from four firms and their densities.

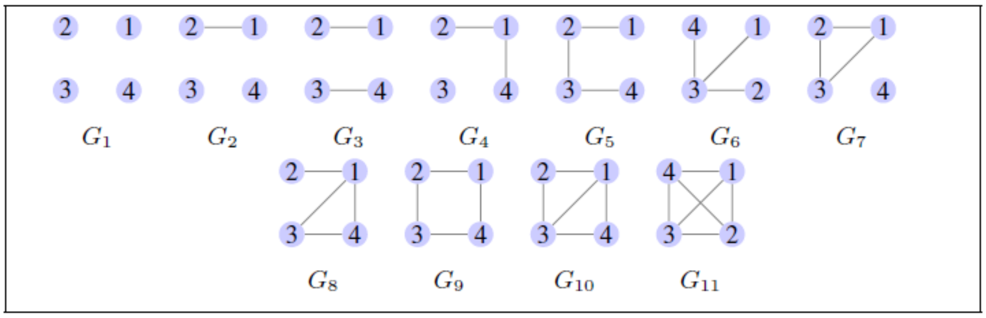

Figure 10. The distinct networks with size four firms. In each network, the individual outcomes depend on the own number of links.

Table 4. The density of the networks in Figure 10.

Network	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}
D	0	1/6	1/3	1/3	1/2	1/2	1/2	2/3	2/3	5/6	1

Appendix C: The outcomes under the model of Goyal and Moraga-Gonzalez.

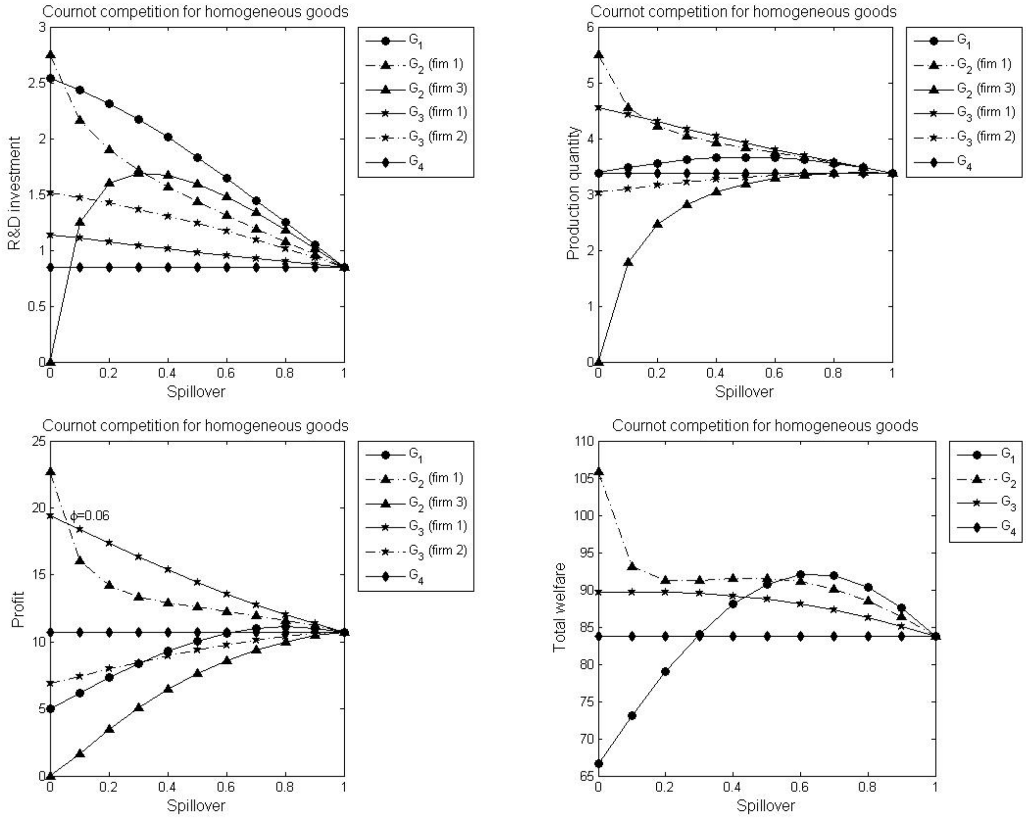

Figure 11. The equilibrium outcomes for the networks given in Figure 2 under the equation 2.4. The parameters used to plot the results are $a = 12, c_0 = 10$ and $\mu = 1$.