

Estimation of Antenna Correlation Coefficient of N -Port Lossy MIMO Array

Susilo Ady Saputro, Satya Nandiwardhana  and Jae-Young Chung

This paper proposes a simple yet accurate method for estimating the antenna correlation coefficient (ACC) of a high-order multiple-input multiple-output (MIMO) antenna. The conventional method employed to obtain the ACC from three-dimensional radiation patterns is costly and difficult to measure. An alternate method is to use the S -parameters, which can be easily measured using a network analyzer. However, this method assumes that the antennas are highly efficient, and it is therefore not suitable for lossy MIMO antenna arrays. To overcome this limitation, we define and utilize the non-coupled radiation efficiency in the S -parameter-based ACC formula. The accuracy of the proposed method is verified by the simulation results of a 4-port highly coupled lossy MIMO array. Further, the proposed method can be applied to N -port arrays by expanding the calculation matrix.

Keywords: Antenna correlation coefficient (ACC), Antenna impedance, Embedded radiation efficiency, MIMO antenna, Non-coupled radiation efficiency, Scattering parameter.

I. Introduction

Currently, flagship smartphones are equipped with 4-port multiple-input multiple-output (MIMO) antenna arrays in order to enhance the data throughput. The MIMO technique has attracted even more attention in the development of fifth-generation wireless communication networks because of the requirement for them to have higher channel capacity. The antenna array in a MIMO system needs to be optimized in order to fully utilize the theoretical channel capacity [1].

The antenna correlation coefficient (ACC) is an important antenna parameter, and it is a measure of the independence between adjacent antennas. The lower the ACC, the more independent the antennas are of each other. Subsequently, multipath signals can have a higher degree of independence for a high communication data rate.

The ACC was originally calculated from complex radiation patterns that contain phase and polarization information in all of the spherical directions [2]. An anechoic chamber is required to measure the radiation patterns for each antenna in a MIMO array, and it is a time-consuming and expensive process. To simplify the process, Blanch and others [3] proposed a method to estimate the ACC by using the scattering parameters (S -parameters). Then, it is used as a common method in numerous publications. However, Blanch's method assumes that the antennas are highly efficient, that is, with a radiation efficiency, $\eta \approx 100\%$, to fulfill the energy conservation principle. Therefore, it is not suitable for most closely packed small handset antennas whose radiation efficiency values are less than or equal to 50%.

To eliminate this limitation, antenna losses were incorporated into the calculation, as reported in [4] and [5]. Hallbjorner [5] included radiation efficiencies in their S -parameter-based formula. However, the radiation

Manuscript received Aug. 21, 2017; accepted Feb. 19, 2018.

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efficiency used here is an isolated one that did not consider the non-excited antennas that were adjacent to the excited ones. To estimate the ACC, Li and others [6] proposed an equivalent circuit model to split a MIMO array into lossy and lossless components. The values of lossy components were obtained from measurements of the radiation efficiency. This method provided an accurate estimation of the ACC for 2-port MIMO arrays. Later, it was also applied to a 4-port array [7]. However, it is complex to obtain the equivalent circuits, and ABCD matrices appear unappealing, particularly in the case of higher-order MIMO arrays.

In this letter, we propose an *S*-parameter-based ACC formula that is suitable for extending its dimension of application to *N*-port lossy MIMO arrays. The new formula considers the MIMO antenna's non-coupled radiation efficiency, that is, η_{nc} [8], which includes only the material losses of the tested antenna and not the coupling losses that are due to the adjacent antennas. The usage of η_{nc} simplifies the ACC formula in a matrix form, which can be readily implemented in a high-order *N*-port MIMO array. The validity of the proposed formula was demonstrated by using full-wave simulation data of a closely packed 4-port MIMO array.

II. ACC Formula Derivation

An equation for calculating ACC (ρ_c) from the three-dimensional (3D) radiation patterns in an *N*-port MIMO array given in [2] is rewritten below:

$$\rho_c(i, j, N) = \frac{\iint [\mathbf{F}_i(\theta, \phi) \cdot \mathbf{F}_j(\theta, \phi)] d(\theta, \phi)}{\sqrt{\iint |\mathbf{F}_i(\theta, \phi)|^2 d(\theta, \phi) \iint |\mathbf{F}_j(\theta, \phi)|^2 d(\theta, \phi)}}, \quad (1)$$

where $\mathbf{F}_i(\theta, \phi)$ and $\mathbf{F}_j(\theta, \phi)$ are the complex radiation patterns of the antennas *i* and *j*, respectively, when all other antennas are terminated by matched loads (usually 50 Ω). The operator \cdot is the Hermitian product. The conventional *S*-parameter method proposed by Blanch for an *N*-port MIMO array expressed in [9] is rewritten below

$$\rho_c(i, j, N) = \frac{-\left(\sum_{n=1}^N S_{i,n}^* S_{n,j}\right)}{\sqrt{\prod_{k=i,j} \left(1 - \sum_{n=1}^N S_{k,n}^* S_{n,k}\right)}}, \quad (2)$$

where S_{ij} represents the *S*-parameters between the ports *i* and *j*, and $*$ is the complex conjugate operator. This equation assumes that the power received by an antenna is equal to the radiated power, implying that there is no loss, which in turn implies that the *S*-parameters are lossless.

In this study, the proposed method extracts the lossless *S*-parameters from lossy *S*-parameters by considering the radiation efficiency. Then, (2) can be appropriately used to calculate the ACC of a lossy array. Steps for obtaining the lossless *S*-parameters are described as follows:

1. The non-coupled radiation efficiency, η_{nc} , is obtained from the measured impedance matrix (*Z*-parameters) and the embedded radiation efficiency, η_{emb} . The latter is obtained by exciting one antenna, with all of the other antennas remaining unexcited but properly terminated with their matched loads [8].
2. The lossless *Z*-parameters are obtained by multiplying η_{nc} with the measured antenna input impedances.
3. The lossless *S*-parameters are obtained from the lossless *Z*-parameters using *Z*-to-*S* matrix conversion.

1. Embedded and Non-coupled Radiation Efficiency

In a MIMO array, the antenna analysis should be done by considering the presence of adjacent antennas because the mutual coupling between them is not trivial. Figure 1 shows the equivalent circuit of an $N \times N$ MIMO array, including the mutual coupling effects. Here, all of the antennas are assumed to be identical and reciprocal, and thus, $Z_{ii} = Z_{jj} = Z_{nn}$ and $Z_{ij} = Z_{ji}$. This implies that the proposed method is limited to MIMO arrays with identical elements. The input impedance of an $N \times N$ antenna array, where the *i*th antenna is excited with all the other antennas terminated with matched-load Z_L , is:

$$Z_{in,i} = \frac{V_i}{I_i} = Z_{ii} + \sum_{j=1, j \neq i}^N Z_{ij} \frac{I_j}{I_i}, \quad (3)$$

the ratio between currents I_j and I_i ($i \neq j$) can be calculated from the *Z*-matrix as follows:

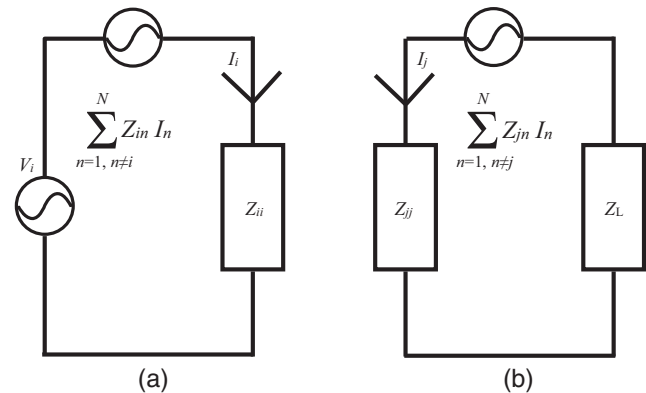


Fig. 1. Equivalent circuit of an $N \times N$ MIMO antenna array. (a) is the *i*-th excited element and (b) is the terminated element for $j = \{1, 2, N | j \neq i\}$.

$$\begin{bmatrix} \frac{I_i}{I_i} \\ \frac{I_{j+1}}{I_i} \\ \vdots \\ \frac{I_N}{I_i} \end{bmatrix} = - \begin{bmatrix} Z_{jj} + Z_L & Z_{j,j+1} & \cdots & Z_{jN} \\ Z_{j+1,j} & Z_{j+1,j+1} + Z_L & \cdots & Z_{j+1,N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{Nj} & Z_{N,j+1} & \cdots & Z_{NN} + Z_L \end{bmatrix}^{-1} \begin{bmatrix} Z_{j,i} \\ Z_{j+1,i} \\ \vdots \\ Z_{Ni} \end{bmatrix} \quad (4)$$

On the other hand, the total power consumed by an antenna (P_{tot}) is defined as

$$P_{\text{tot}} = P_{\text{rad}} + P_{\text{mm}} + P_{\text{ohmic}} + P_L, \quad (5)$$

where P_{rad} is the radiated power, P_{mm} is the mismatch loss, P_{ohmic} is the material loss (that is, conduction and dielectric losses), and P_L is the power absorbed by Z_L in all of the terminated antennas. Based on (5), the η_{emb} of the i th antenna with all the other antennas terminated is denoted as

$$\eta_{\text{emb},i} = \frac{P_{\text{rad},i}}{P_{\text{tot},i} - P_{\text{mm},i}} = \frac{P_{\text{tot},i} - P_{\text{mm},i} - P_{\text{ohmic}} - P_L}{P_{\text{tot},i} - P_{\text{mm},i}}, \quad (6)$$

where $P_{\text{tot},i} - P_{\text{mm},i}$ is the power received by the i th antenna, and is expressed by

$$P_{\text{acc},i} = \frac{1}{2} \text{Re}(Z_{in,i}) |I_i|^2, \quad (7)$$

where $\text{Re}(\cdot)$ denotes the real part of the resulting equation. By definition, we can obtain η_{nc} by eliminating the efficiency loss that is due to the power absorbed by Z_L from (6). η_{nc} can be expressed as

$$\eta_{\text{nc}} = \eta_{\text{emb}} + \frac{P_L}{P_{\text{acc}}}, \quad (8)$$

and by substituting (7) into (8), the value of η_{nc} of the excited i th antenna with others terminated by Z_L can be obtained as shown below

$$\eta_{\text{nc},i} = \eta_{\text{emb},i} + \sum_{j=1, j \neq i}^N \left| \frac{I_j}{I_i} \right|^2 \frac{\text{Re}(Z_L)}{\text{Re}(Z_{in,i})}, \quad (9)$$

where $Z_{in,i}$ and $|I_j/I_i|$ are calculated from (3) and (4), respectively. A network analyzer can be used to obtain the Z -parameters in (3) and (4). On the other hand, to obtain η_{emb} , it is necessary to use radiation efficiency measurement equipment. For example, a fast and inexpensive wheeler cap or reverberation chamber can be used instead of an anechoic chamber with full 3D scanning capability.

2. Antenna Correlation Coefficient

The radiation efficiencies $\eta_{\text{emb},i}$ and $\eta_{\text{nc},i}$ can also be expressed as

$$\eta_{\text{emb},i} = \frac{R_{\text{rad}} \left(\sum_{i=1}^N |I_i|^2 \right)}{(R_{\text{rad}} + R_{\text{ohmic}}) \sum_{i=1}^N |I_i|^2 + \text{Re}(Z_L) \sum_{j=1, j \neq i}^N |I_j|^2}, \quad (10)$$

$$\eta_{\text{nc},i} = \frac{R_{\text{rad}} \left(\sum_{i=1}^N |I_i|^2 \right) + \text{Re}(Z_L) \sum_{j=1, j \neq i}^N |I_j|^2}{(R_{\text{rad}} + R_{\text{ohmic}}) \sum_{i=1}^N |I_i|^2 + \text{Re}(Z_L) \sum_{j=1, j \neq i}^N |I_j|^2}, \quad (11)$$

where R_{rad} and R_{ohmic} refer to the antenna radiation resistance and loss resistance, respectively. From (10), the antenna input impedance can be written as

$$\begin{aligned} Z_{in,i} &= \text{Re}(Z_{in,i}) + j\text{Im}(Z_{in,i}) \\ &= (R_{\text{rad}} + R_{\text{ohmic}}) \left(1 + \sum_{j=1, j \neq i}^N \left| \frac{I_j}{I_i} \right|^2 \right) \\ &\quad + \text{Re}(Z_L) \sum_{j=1, j \neq i}^N \left| \frac{I_j}{I_i} \right|^2 + j\text{Im}(Z_{in,i}). \end{aligned} \quad (12)$$

By eliminating R_{ohmic} from the lossy input impedance, we can define the lossless input impedance as:

$$\begin{aligned} Z_{in,i,\text{lossless}} &= (R_{\text{rad}}) \left(1 + \sum_{j=1, j \neq i}^N \left| \frac{I_j}{I_i} \right|^2 \right) \\ &\quad + \text{Re}(Z_L) \sum_{j=1, j \neq i}^N \left| \frac{I_j}{I_i} \right|^2 + j\text{Im}(Z_{in,i}). \end{aligned} \quad (13)$$

The expression $Z_{in,i,\text{lossless}}$ can be re-written using $\eta_{\text{nc},i}$ and Z_{in} from (11) and (12) as follows

$$Z_{in,i,\text{lossless}} = (\eta_{\text{nc},i} \text{Re}(Z_{in,i})) + j\text{Im}(Z_{in,i}). \quad (14)$$

Having obtained $Z_{in,i,\text{lossless}}$, the lossless self-impedance can be calculated from (3). The mutual impedance is constant, and it is only affected by the antenna structures and their separation [10]. Finally, the lossless S -parameters that are compatible with (2) can be obtained from the Z - to S -matrix conversion.

III. Validity Verification Using Simulation Results

The validity of the proposed method was verified by performing full-wave simulations. Figure 2 depicts two different configurations of the 4-port microstrip dipole array

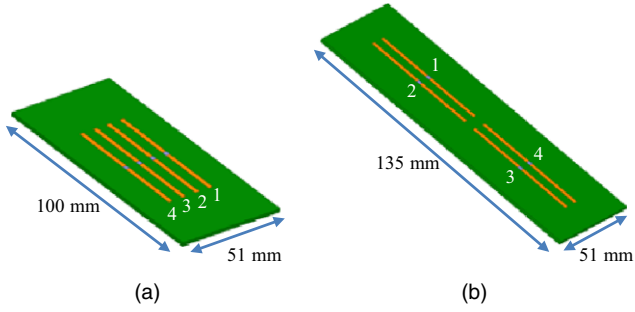


Fig. 2. Geometry of 4-port dipole array used for simulation. (a) 4 dipoles in a row (Array 1) and (b) 4 dipoles with 2×2 symmetry (Array 2).

used for the simulation. In Fig. 2(a), the dipoles are tightly packed in the row located above the FR-4 substrate ($\epsilon_r = 4.4$ and $\tan\delta = 0.02$). Their length and width were adjusted to resonate at 1.95 GHz (Array 1). The separation between the dipoles was only 4 mm ($< \lambda_g/35$ at 1.95 GHz), and consequently, their mutual couplings were very high (> -5 dB). Otherwise, in Fig. 2(b), the dipoles are in a 2×2 configuration (Array2). The radiation efficiency at a resonant frequency of 1.95 GHz is $\eta_{Arr1} = \{0.43, 0.25, 0.25, 0.43\}$ for each element in Array 1, and $\eta_{Arr2} = \{0.35, 0.35, 0.35, 0.35\}$ for each element in Array 2. The information about the mutual coupling between the MIMO antennas of Array 1 and Array 2 are given in Fig. 3(a) and Fig. 3(b), respectively. Figure 3(a) shows that with reference to the excited dipole, the closer dipole absorbs more power than the other dipoles; hence, it has stronger mutual coupling, leading to higher ACC, as can be seen in (2). However, unlike Array 1, the mutual coupling between the two dipoles that have the same distance but different position with symmetric properties, is the same as shown by S_{14} and S_{23} of Array 2 in Fig. 3(b).

Using the models of Array 1 and Array 2, the Z-parameters and η_{emb} were obtained and incorporated into the ACC calculation process described in Section II. In particular, for the 1st antenna in the 4-port array, (3) and (4) become

$$Z_{in,1} = Z_{11} + \sum_{j=2}^4 Z_{1j} \frac{I_j}{I_1}, \quad (15)$$

$$\begin{bmatrix} I_2/I_1 \\ I_3/I_1 \\ I_4/I_1 \end{bmatrix} = - \begin{bmatrix} Z_{22} + Z_L & Z_{23} & Z_{24} \\ Z_{32} & Z_{33} + Z_L & Z_{34} \\ Z_{42} & Z_{43} & Z_{44} + Z_L \end{bmatrix}^{-1} \begin{bmatrix} Z_{21} \\ Z_{31} \\ Z_{41} \end{bmatrix}, \quad (16)$$

respectively. $Z_{in,1}$ can be obtained by substituting (16) into (15). $Z_{in,2}$, $Z_{in,3}$, and $Z_{in,4}$ can also be calculated by the

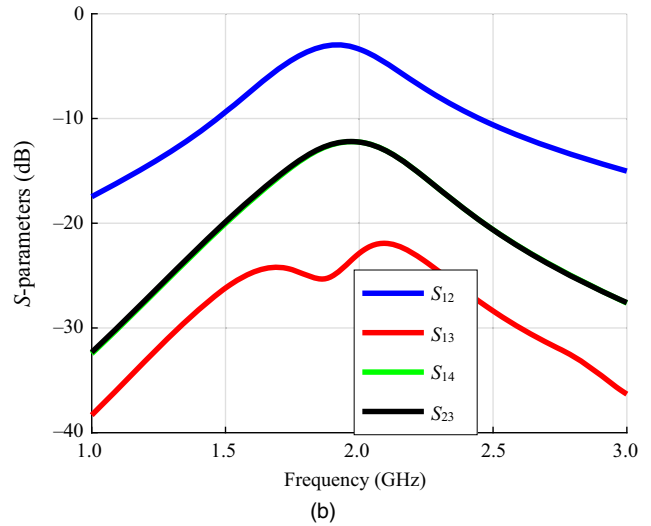
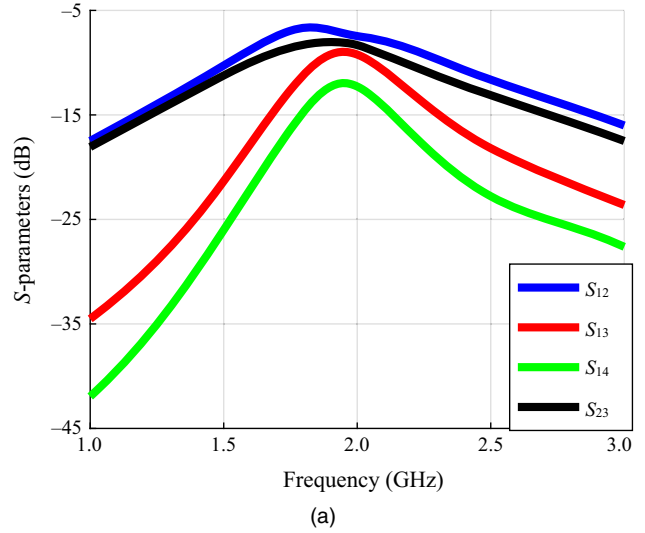


Fig. 3. Mutual Coupling (a) Array 1 and (b) Array 2.

same procedure. Then, $\eta_{nc,i}$ of each antenna is calculated by (9) together with $\eta_{emb,i}$ from the simulations and the known matched load $Z_L = 50 \Omega$. Finally, from $Z_{in,i,lossless}$ in (14), the lossless Z matrix is generated and converted into S-parameters to calculate ACC using (2).

Figures 4 and 5 show the calculated ACC of Array 1 and Array 2, respectively, for frequency values of 1.8 GHz–2 GHz. Here, the ACC from the proposed formula is compared with the ones obtained from radiation patterns [2] and from the conventional S-parameter method [3]. In Figs. 4 and 5, ρ_{ij} refers to ACC between the antennas i and j . It is observed from Fig. 4 that the ACC obtained from the proposed formula and from the radiation patterns have similar values and tendencies as the antenna separation increases ($\rho_{12} > \rho_{13} > \rho_{14}$). In spite of the same separation, ρ_{23} is lower than ρ_{12} because of the difference in the mutual coupling loss. However,

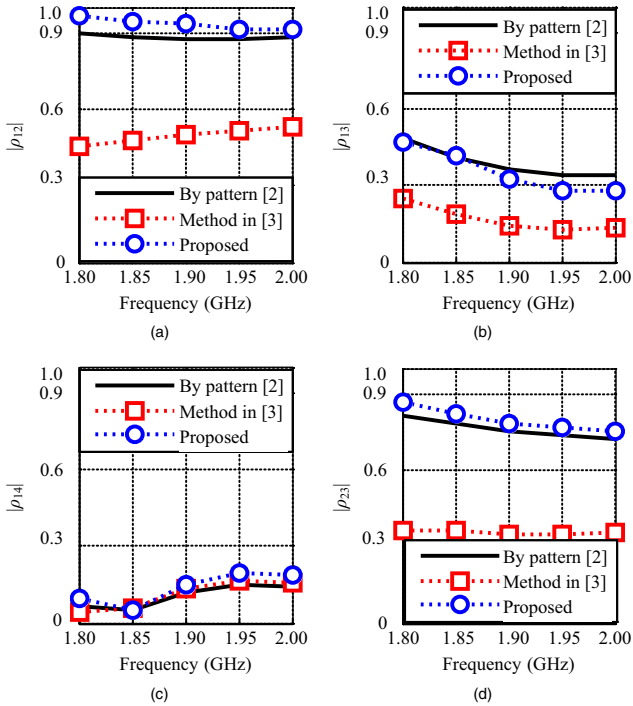


Fig. 4. ACC of Array 1 (a) ρ_{12} , (b) ρ_{13} , (c) ρ_{14} , and (d) ρ_{23} .

the value of ACC obtained from the conventional S -parameter method is underestimated except for ρ_{14} , whose mutual coupling is low. For example, in the case of ρ_{12} , the conventional S -parameter method shows an averaged error, while the proposed method has a closer approximation of ACC with reference to the value of ACC obtained from the radiation pattern. The latter is due to errors in the simulated Z -parameters and η_{emb} results.

Figure 5 also demonstrates the effectiveness of the proposed formula even when the antenna arrangement is changed. The conventional S -parameter method shows additional errors as the loss due to mutual coupling is higher, for example, ρ_{12} . Otherwise, ACC from the proposed method agrees well with the references, that is, ACC values obtained from the radiation patterns. ρ_{13} and ρ_{14} show a better match between the three methods as the dipole's centers (that is, phase center) have larger separations leading to lower ACC [11], [12].

IV. Conclusion

This paper proposed simple and useful formulas to estimate the ACC values of a high-order MIMO array. The formulas were derived by considering all possible antenna loss factors. In the meantime, η_{nc} was introduced to distinguish between the losses due to materials and mutual couplings. From the results obtained, the procedure to obtain lossless Z -parameters was simplified,

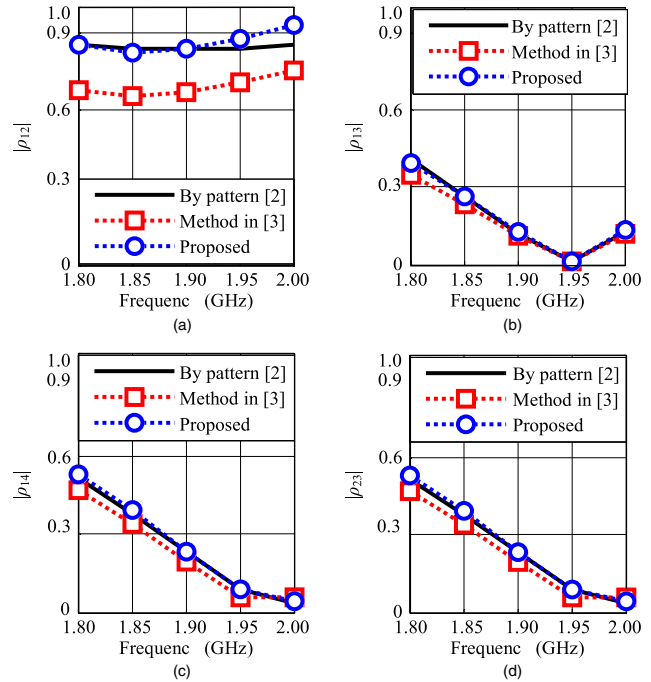


Fig. 5. ACC of Array 2 (a) ρ_{12} , (b) ρ_{13} , (c) ρ_{14} , and (d) ρ_{23} .

and may be applied to the lossless S -parameter-based method. The proposed formulae were validated by using the simulation results of two different 4-port dipole array configurations that are closely packed on a lossy substrate. The results clearly indicated that the proposed formula could capture the impact of the mutual coupling loss on ACC. The proposed method is expected to provide a fast and accurate evaluation of ACC in complicated MIMO array structures with various loss factors.

Acknowledgements

This work was supported in part by an Institute for Information & Communications Technology Promotion (IITP) grant funded by the Korea government (MSIT) (No. 2015-0-00855), and in part by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (No. 2015R1C1A1A01052381).

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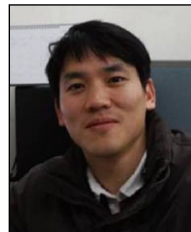
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