

Error Rate and Capacity Analysis for Incremental Hybrid DAF Relaying using Polar Codes

Natarajan Madhusudhanan  and Rajamanickam Venkateswari

The deployment of an incremental hybrid decode-amplify and forward relaying scheme is a promising and superior solution for cellular networks to meet ever-growing network traffic demands. However, the selection of a suitable relaying protocol based on the signal-to-noise ratio threshold is important in realizing an improved quality of service. In this paper, an incremental hybrid relaying protocol is proposed using polar codes. The proposed protocol achieves a better performance than existing turbo codes in terms of capacity. Simulation results show that the polar codes through an incremental hybrid decode-amplify-and-forward relay can provide a 38% gain when $\gamma_{th(1)}$ and $\gamma_{th(2)}$ are optimal. Further, the channel capacity is improved to 17.5 b/s/Hz and 23 b/s/Hz for 2×2 MIMO and 4×4 MIMO systems, respectively. Monte Carlo simulations are carried out to achieve the optimal solution.

Keywords: Capacity, Cooperative relaying, Error rate performance, IHDAF protocol, Polar codes.

I. Introduction

Cooperative relaying is a technique employed in wireless communications to guarantee the system performance in fifth-generation (5G) cellular networks. Compared to fourth-generation (4G) networks, 5G cellular networks incorporate the deployment of femtocell-based relay nodes to provide good quality of service to all users in a given area. Essentially, relaying is a type of protocol that is used to expand coverage in different locations, and it is an alternative to base stations. Because relay nodes are considered as low-power base stations, they can be deployed anywhere within the cellular coverage area. They also provide diversity gain, hence improving the quality of service in terms of capacity in fading environments [1]. The major classifications of relays are: amplify-and-forward (AF), decode-and-forward (DF), compress-and-forward (CF), demodulate-and-forward, and filter-and-forward protocols. In the AF relaying protocol, the received signal is amplified along with the noise and interference, and is forwarded to the destination. The DF relaying protocol will detect the signal first, and then it retransmits the received signals to the destination. However, the DF relaying protocol requires the use of signal processing methods and channel coding techniques to rectify errors [2]. In the case of the CF relaying protocol, the relay node quantizes and transmits the received signals to the destination [3]. In the above-mentioned schemes, AF and CF are analog schemes, whereas compression-based relaying schemes are digital cooperative relaying schemes. To overcome drawbacks such as the noise amplification, latency, and other factors, several cooperation schemes have been proposed based on selective cooperation using threshold values of the signal-to-noise ratio (SNR) [3].

With reference to the characteristics, DF relaying can be further divided into two categories, such as fixed DF

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(FDF) and adaptive DF (ADF). In FDF, the relay decodes and forwards the received signal to the destination. However, in the case of ADF, the relay will transfer information to the destination only if the received signals are correct. This scheme is also referred to as selective DF. The selective-DF relaying scheme determines whether it is essential to forward the received signals by evaluating the quality of the channel with respect to the SNR threshold. In order to improve the spectral efficiency, incremental relaying protocols have been proposed. In [4], the authors investigated the amplify-and-forward incremental relaying technique to provide feedback from the destination and achieved gain at a high SNR. In [5], the average frame error rate was analyzed using an improved SNR threshold-based approximation method for a hybrid decode-amplify-and-forward relaying (HDAF) protocol. The closed-form expression for conventional AF, adaptive DF, and HDAF relaying schemes was derived to improve the performance in terms of the gain [6]. In [7], to improve the performance in terms of the outage probability and bit error rate (BER), the authors proposed an HDAF cooperative communication for multi-relay networks with the selection of the relay and SNR threshold in the multi-relay scenario over independent and nonidentical flat fading channels. To do this, they used the maximal ratio combining (MRC) technique. The incremental relaying technique utilizes the feedback method from the destination to specify when the relay is permitted to convey the information from the source to the destination. If the destination decodes the data successfully, an acknowledgment (ACK) message will be generated to communicate the status, and the relay does nothing. If it fails, a negative acknowledgement (NACK) will be sent back to indicate that the source should retransmit the information [8]. In [9], an incremental best relaying scheme has been introduced to exploit spatial diversity over non-identical Rayleigh fading channels to estimate the BER, outage performance, and average capacity. In [10], the authors addressed “when to cooperate” and “whom to cooperate with” for multi-node relay networks to achieve bandwidth efficiency.

An incremental DF (IDF) along with selective DF, namely, the incremental-selective decode-and-forward (ISDF-) based relaying protocol over the Rayleigh fading channel has been presented in [11], and it shows that the ISDF relaying scheme outperforms IDF. The authors of [2] proposed an incremental HDAF relaying scheme, where the relay forwards the information to the destination either in DF mode or AF mode according to the channel quality and the SNR threshold for the estimation of the outage probability with reduced error and minimal power. In [12], the error-rate performance and power allocation

for cooperative diversity networks with distributed Alamouti code using Marcum-Q function have been derived and simulated. In [13], the authors investigated important techniques to realize cooperative communications with better improvement in the network capacity for LTE-Advanced systems. The opportunistic space-time coding (OSTC-) based relaying scheme has been proposed with an opportunistic selection of the best relay from k relays in order to update the channel state information (CSI), and analytical expressions have been derived for the estimation of the ergodic capacity and outage probability [14]. The relay placement and power allocation for DF relaying has been investigated over lognormal shadowing Rayleigh fading channels in order to validate the symbol error [15].

Channel coding plays an important role in achieving the performance improvement in terms of the redundancy and channel capacity. Various coding schemes, such as turbo codes, low-density parity-check (LDPC) codes, and convolutional codes have been proposed for wireless communication systems in existing literature. Owing to the high computational complexity during encoding and decoding processes, as well as the long latency in existing channel codes, a new type of channel coding, called “polar codes,” was introduced by Erdal Arıkan in 2009. Polar codes have been shown to be a better channel coding scheme because of the achievement of Shannon approximation. These codes are applicable for different channels such as binary-input discrete memoryless channels (B-DMCs), the Rayleigh channel, and other fading channels in cooperative communication systems. Polar codes with succession-cancellation (SC) decoders have been considered as an important candidate for future cellular networks that should achieve the channel capacity nearest to the Shannon limit with an acceptable complexity [16]–[19]. It provides a peak data rate of 27 Gbps. There are two important parameters in polar coding, that is the channel capacity and Bhattacharya parameter. These parameters are used to measure the rate of information and capacity-related parameters. The polar code utilizes the concept of channel polarization using the Bhattacharya parameter. A smart relaying technique with polar codes [18] has been proposed and designed to detect erroneous values using SC decoders. Moreover, polar codes were considered as an option to upgrade high spectrum efficiency when compared to present cellular standards, as well as to improve the link reliability compared with turbo codes using polar-coded spatial modulation (PCSM) [20]. Polar codes are verified and tested with different parameter settings for 5G cellular networks using multicarrier modulation schemes with

channel emulators [21]. These channel codes are suitable for DF and CF relaying protocols owing to the capacity performance in physically degraded channels [22]. In [23], the authors proposed a cyclic redundancy check (CRC)-aided decoding scheme, and they showed a performance gain improvement over turbo codes. Several analytical expressions and methodologies have been given in [24]–[27] for the estimation of capacity and coverage.

The main objective of this study is to increase the capacity and reduce the error performance of cooperative relaying-based cellular networks. An incremental-based HDFAF (IHDAF) relaying scheme is considered, which uses polar codes with higher-order quadrature amplitude modulation (QAM). To achieve the capacity in the closed form, an SC list (SCL-) based decoder is utilized at the relay to determine whether the relay will forward the information to the destination with cooperation. Various practical parameters are taken into account for evaluations, such as the BER, SNR, and capacity. The performance of the proposed relaying protocol is analyzed in the presence of the Rayleigh fading channel.

The rest of the paper is organized as follows: Section II describes the notation and construction of polar codes. The concept of the IHDAF relaying scheme with polar codes is discussed in Section III. Section IV shows the performance analysis of the proposed system. Simulation results are described in Section V, and Section VI concludes the paper.

II. Notation and Construction of Polar Codes

1. Notation of Polar Codes

For a given vector \mathbf{x} , the i th element can be represented by x_i and its sub vector, which are limited within the range x_i to x_j . The standard notation for mutual information and the conditional mutual information is represented by $I(X; Y)$ and $I(X|Y)$, respectively. In addition, if a binary discrete memoryless channel (B-DMC) has transition probability $W(y|x)$, then the conditional mutual information or symmetric capacity can be denoted by $I(W)$. In B-DMC, $W(y|x)$ is the transition probability, where x and y denote the transmitted and received signals [17], [20]. The computational order of the polar code is assumed to be $O(N \log N)$ [17].

2. Construction of Polar Codes

Polar codes are an emerging class of error-correcting codes that closely approximate to the Shannon capacity. Polar encoding schemes are classified into three different

scenarios, namely non-systematic coding, systematic coding, and concatenated coding. In [20], the authors proved that polar codes are constructed by discriminating two sets of distinguishable channels, that is, free and frozen polar sets. The input present in the frozen polar set arrangements are fixed, and it is known to the transmitter and receiver. For the above scenario, a B-DMC is considered as a major element with low encoding and decoding complexity. In B-DMC, $W(y|x)$ is the transition probability, where x and y denote the transmitted and received signals, respectively [17], [20].

A popular method of constructing polar codes is channel polarization, which causes these codes to achieve high capacity. With the aid of the above method, channels are classified into two different scenarios: noise and noiseless channels. It also includes combine and split operations to transform N independent copies of transmitting channel W into virtual channels W_N^j , where the value of j ranges between 1 and N . The probability mass function of transmit channel W^N is a part of the virtual channel, and is given by [17]

$$W^N\left(\frac{y}{x}\right) = \prod_{i=1}^n W\left(\frac{y_i}{x_i}\right). \quad (1)$$

In the polar code design, noiseless bit channels are considered to transmit information from the source to the destination. In this structure, the code block length can be denoted by $N = 2^n$, $n \geq 0$, where n is the level of the code. As a family of block codes, polar codes for the above code block length and binary sequence can generally be denoted by [22]

$$\mathbf{x} = \mathbf{u}\mathbf{G}_N. \quad (2)$$

The above expression denotes the encoding operation represented by \mathbf{x} , where the binary source signal \mathbf{u} includes both fixed (information) and frozen bits, and N represents the length of the block. \mathbf{G}_N is the generator matrix of size N , which comes from the n th Kronecker power of the kernel matrix \mathbf{G}_2 [3], [14]. Then, the standard $N \times N$ generator matrix \mathbf{G}_N is represented by [17]

$$\mathbf{G}_N = \mathbf{Q}_N \mathbf{G}_2^{\otimes n}. \quad (3)$$

In (3), \mathbf{Q}_N denotes the $N \times N$ permutation matrix. The matrix \mathbf{G}_N enables channel combination. The standard kernel matrix can be denoted as [3], [17]

$$\mathbf{G}_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}. \quad (4)$$

From [17], the relationship between the transmit channel W^N and raw channel W_N is given by

$$W_N\left(\frac{y}{u}\right) = \prod_{i=1}^N W\left(y_i | (\mathbf{u}\mathbf{G}_N)_i\right). \quad (5)$$

The probability distribution function of the j th virtual channel W_n^j can be obtained by splitting the operation of W_N as [17]

$$W_n^j(y, u_1^{i-1} | u_i) = \left(\frac{1}{2}\right)^{N-1} \sum_{u_{i+1}^N} \left\{ \prod_{i=1}^N W(y_i | (\mathbf{u}\mathbf{G}_N)_i) \right\}. \quad (6)$$

The structure of the proposed system consists of K information and frozen bits transmitted by the source. First, the source encodes the information (F_3) and frozen bits (F_1). The encoding operation is purely dependent on the generator matrix. A frozen set of bits F_1 is selected during the encoding operation at the source. Therefore, the indices are given by [3], [22]

$$F_1 = \{i \in 0, 1, \dots, N - 1\} : Z(W_1^{(i)}) \leq \delta_N, \quad (7)$$

where $Z(W_1^{(i)})$ indicates the construction procedure for i channels, and depends on combining and splitting operations, as expressed in [3], [22]. The common randomness procedure can be utilized for the generation of frozen bits, and it is shown in Fig. 1. Further, $Z(W_1^{(i)})$ denotes the Bhattacharya parameter of the i th channel with bound $\delta_N = \frac{1}{N} 2^{-N^\beta}$; because β is a constant, $\beta < 1/2$, the polar code is considered to be an extremely capacity-achieving for W_1 channel. The Bhattacharya parameter is used to measure the reliability of virtual channels. The polar encoder selects information indices with respect to the virtual channels [17]. Bhattacharya parameter $Z(W)$ and system capacity parameters determine whether the channel is noisy or noiseless, and can be expressed as [3].

$$Z(W) = \sum_y \sqrt{W\left(\frac{y}{0}\right) W\left(\frac{y}{1}\right)} dy. \quad (8)$$

The capacity of the symmetric channel is given by [1]

$$I(W) = \sum_y \sum_x \frac{1}{2} W(y|x) \log \frac{W(y|x)}{\frac{1}{2} W(y|0) + \frac{1}{2} W(y|1)}, \quad (9)$$

where $W(y|x)$ represents the transition probability of the continuous channel. On the other hand, F_3 represents the

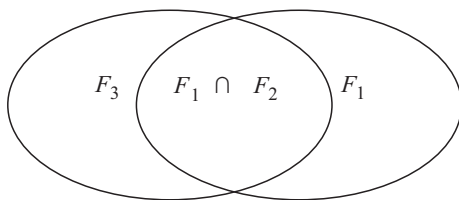


Fig. 1. Basic structure of polar codes.

set of information bits. A set of indices for channel W_3 is represented by [3], [22]

$$F_3 = \{i \in 0, 1, \dots, N - 1\} : Z(W_3^{(i)}) \leq \delta_N, \quad (10)$$

where $Z(W_3^{(i)})$ corresponds to the Bhattacharya parameter of the i th channel between the source and destination. In Fig. 1, the intersection between two sets of bits retrieves the remaining bits using the SC scenario. The SC decoder retrieves original information with reduced computational complexity of order $O(N \log N)$. From (6), the decision for \mathbf{u}_i would be decided based on the previous value of \mathbf{u}_1^{i-1} and the transition probability $W_n^j(\mathbf{y}, \mathbf{u}_1^{i-1} | u_i)$. Therefore, the decision of the system is given by [17]

$$\hat{u}_i = \begin{cases} \operatorname{argmax}_{u_i \in \{0,1\}} W_n^j(\mathbf{y}, \hat{\mathbf{u}}_1^{i-1} | u_i), & i \in F_1 \\ 0, & i \in F_3. \end{cases} \quad (11)$$

Following the above expression, to estimate the next decision, the SC decoder will receive the information based on a decision as a priori information. Further, the decoder would decode the received information when the rate is minimum compared to the capacity.

III. System Model in IHDAF Relaying Environment Based on Polar Codes

1. System Channel Model

The basic cooperative relay channel consists of three nodes, namely a source (S), relay (R), and a destination (D), as shown in Fig. 2. It illustrates the proposed system model of $M_t \times M_r$ multiple-input multiple-output (MIMO) systems, where M_t and M_r represent the number of transmit and receive antennae used in the system. The binary source bits of size $n = \log_2(mM_t)$ are polar encoded with outputs vector X_s , where M denotes the order of QAM. X_s is classified into two binary sets, namely free

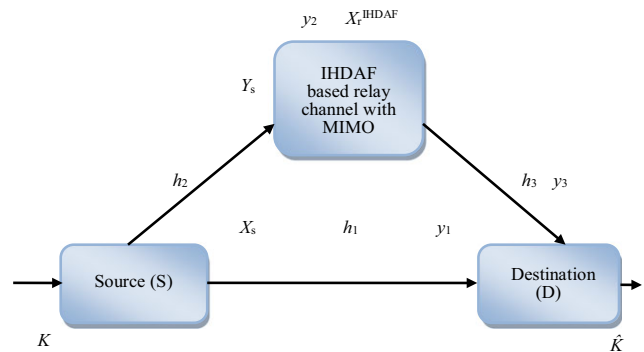


Fig. 2. Typical IHDAF cooperative relaying system.

polar sets (F_3) and frozen polar sets (F_1). The symbol mapper assigns the free polar sets to the M-QAM symbol. After assigning symbols in the code block, the polar-coded spatial mapper assigns the symbols to the corresponding antenna. All of the symbols in the block are transmitted over the MIMO Rayleigh channel.

Initially, the original information of K bits as an input is passed to the system, encoded to X_s , and transmitted by the source. Then, it is received by both the relay and destination. The relay generates an estimation denoted by Y_s , which is mapped into y_2 , and then the signal received from the source will be forwarded to the destination. Coefficients such as h_1 , h_2 , and h_3 represent the channel gain between the source-destination, source-relay, and relay-destination, respectively. y_3 and y_1 are the channel outputs at the relay and at the destination, respectively, and X_T^{IHDAF} is the vector of bits put into the channel by the relay. In this situation, the source communicates with the destination in two different forms, namely the direct and indirect modes. It is assumed that the relaying protocol operates in half-duplex mode, which is the simplest and most reasonable approach. The focus of this paper is the IHDAF relaying scheme with selective cooperation using a Rayleigh channel. Figure 3 illustrates the dataflow processing diagram of the IHDAF relaying protocol, which utilizes a polar encoder at the transmitter, and an SC decoder at the receiver. The proposed model will work either in DF or AF modes based on the SNR threshold. Initially, the source transfers information to the destination in the non-cooperative mode. The relay remains idle until the instantaneous SNR at the destination γ_1 is above the threshold SNR between the source and destination $\gamma_{\text{th}(1)}$. Therefore, the source again transmits new information to the destination during the next phase. If the above condition is not satisfied, the cooperative relay plays a vital role. The relaying protocol estimates the channel before transporting information to the destination based on the SNR threshold, and it would improve the system performance. In the IHDAF relay, the proposed system operates in DF mode when the instantaneous SNR between the source and relay exceeds the threshold SNR between the source and relay $\gamma_{\text{th}(2)}$, and then the polar code-based relay will decode and forward the information using the cooperative mode. In contrast, the above system will be switched over into AF relaying mode to improve the system performance in the source-to-relay link based on the SNR. In the non-cooperative mode, the source transfers information directly to the destination, and the combining technique, that is, the MRC, was employed to combine the signals received from two phases. Further, the MIMO configuration has been utilized to estimate the

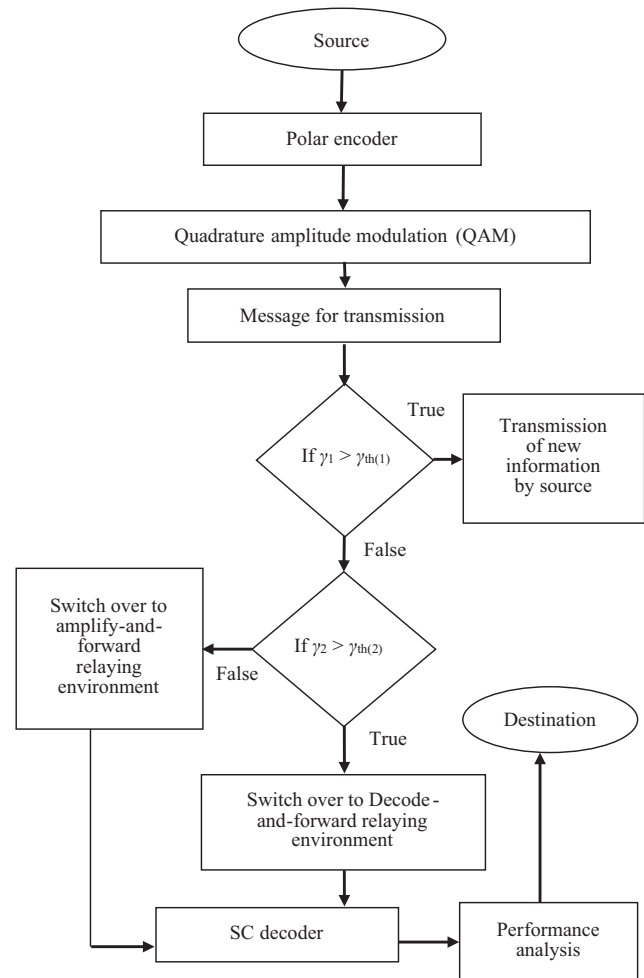


Fig. 3. Data processing diagram of polar coded IHDAF relaying.

error rate corresponding to the SNR. The destination extracts original information from both the direct and indirect modes.

If the source knows that the destination did not decode correctly, then the source retransmits the information to the destination or through a relay with the aid of a feedback mechanism. As is the case with the IHDAF relaying scheme, the relay either decodes or amplifies the incoming signal based on the incremental method with respect to the threshold values, and forwards it to the destination. In direct mode, the signal received at the destination is given by

$$y_1 = \sqrt{P_s}h_1s + N_1. \quad (12)$$

During the first phase, the signal received at the relay is represented by

$$y_2 = \sqrt{P_s}h_2s + N_2, \quad (13)$$

where h_2 and h_1 are the channel fading coefficients between the source and the relay and the source and

destination, respectively, and are distributed as Rayleigh channel models; s is the signal transmitted from the source with unit average power, and N_1 and N_2 are independent and identically distributed (i.i.d.) complex Gaussian random variables, which are additive noises with mean zero, variance N_0 , and transmitted power from the source P_s .

During the second phase, the relay works in DF mode. Therefore, the received signal at the destination is given by

$$y_3 = \sqrt{P_r}h_3s_r + N_3, \tag{14}$$

where h_3 denotes the channel gain of the relay-destination link. s_r represents the signal sent by the relay based on the threshold condition, and P_r is the relay's transmission power. N_3 indicates the i.i.d. complex Gaussian random variable. If the threshold between the source-relay is greater than the SNR of the source and relay, then the relay switches over to AF mode. In AF mode, the signal received at the destination can be expressed as

$$y_3 = \beta_r h_3 y_2 + N_3. \tag{15}$$

In (15), β_r denotes the scaling factor. In the AF protocol, the relay simply amplifies the desired signal from the source S, and forwards the received signal to the destination in order to balance out the channel fade between the source and the relay. The relay scales the received signal by a factor β_r , which is inversely proportional to the received power, and is represented by

$$\beta_r = \frac{\sqrt{P}}{\sqrt{P|h_1|^2 + N_0}}. \tag{16}$$

The signal transmitted from the relay is given by $\beta_r y_1$, and it has total power P , which equals the transmitted signal power received from the source. To estimate the mutual information, the instantaneous SNR is required. Therefore, the instantaneous SNR for the source-destination, source-relay, and relay-destination links are expressed as below:

$$\gamma_1 = \frac{|h_1|^2 P_s}{N_0}, \tag{17}$$

$$\gamma_2 = \frac{|h_2|^2 P_s}{N_0}, \tag{18}$$

$$\gamma_3 = \frac{|h_3|^2 P}{N_0}. \tag{19}$$

The channel coefficients mentioned above obey Rayleigh's density function. The probability density

function (pdf) of the instantaneous SNR $\gamma_j (j = 1, 2, 3)$ is denoted by

$$p_{\gamma_j}(\gamma) = \frac{1}{\bar{\gamma}_j} \exp\left(-\frac{\gamma}{\bar{\gamma}_j}\right), \gamma \geq 0, \tag{20}$$

where $\bar{\gamma}_j (j = 1, 2, 3)$ denotes the average SNR of the source, relay, and destination links, respectively. In the proposed system, the channel model uses an $M_t \times M_r$ antenna configuration with M_t antennas at the transmitter and M_r antennas at the receiver. It follows a Rayleigh distribution. Assume that $h_{M_r M_t}$ is the channel gain between the M_t th transmit antenna and M_r th receive antenna. While transmitting symbols via the transmitting antennas, the received signal is expressed in [24] as

$$y_i = \sum_{j=1}^{M_t} h_{ij} s_j + N_j. \tag{21}$$

In the above expression, h_{ij} represents the channel gain between the j th transmit antenna and i th receive antenna. $s_i (i = 1, 2, \dots, M_t)$ denotes symbols that are transmitted via M_t antennas. $N_j (j = 1, 2, \dots, M_r)$ is the additive noise at the receiver side, and (21) can be expressed in matrix form as

$$y_{M_r} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{M_r} \end{bmatrix} = \begin{bmatrix} h_{11}h_{12} & \cdots & h_{1M_t} \\ \vdots & \ddots & \vdots \\ h_{M_r1}h_{M_r2} & \cdots & h_{M_rM_t} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_{M_t} \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_{M_r} \end{bmatrix}, \tag{22}$$

where $y_1 y_2 \dots y_{M_r}$ denotes the received signal at the receiver. In (21) and (22), channel coefficients "h" and the additive noise are i.i.d.

IV. Analysis of Error Probability and Capacity

This section presents the error probability and channel capacity for the proposed IHDAF relaying scheme. A channel-polarization method is considered for the construction of polar codes, and it transforms multiple independent copies of a given binary channel into a set of successive usage of binary input channels. The polarization procedure consists of channel splitting and combining operations.

During the channel-splitting operation, N binary input channels $W_N^{(i)}$ are considered. For each "ith" symbol ($i \in 0, 1, 2, 3, \dots, N-1$), the transition probability can be denoted by

$$W_N^{(i)}(y, u_0^{i-1} | u_i) = \frac{1}{2^{N-1}} \sum_{u_{i+1}^{N-1}} W_N(y | u), \quad (23)$$

where $W_N(y | u)$ denotes the output channel vector. In (21), N tends to infinity, and the channels $W_N^{(i)}$ are classified as noisy and noiseless. The information is transferred through the noiseless channels effectively Polar codes are considered as G_N coset codes with simulation parameters (N, K, F_3, F_1) , where F_1 is the frozen vector. F_3 denotes the fixed information set, and is chosen as a K -element of the subset of $\{1, 2, 3, \dots, N\}$.

In addition, the channel polarization rate can be measured with the help of the Bhattacharya parameter $Z(W)$. It is essentially an upper bound on the probability of the maximum-likelihood decision error during the transmission of binary values. The Bhattacharya parameter $Z(W)$ should satisfy $Z(W_N^{(i)}) \leq Z(W_N^{(j)})$ for all $i \in F_3$ and $j \in F_1$. The indices in F_3 are good channel indices, whereas those in F_1 represent bad channel indices.

Therefore, the average Bhattacharya parameter for N binary channels can be represented using the transition probability, as mentioned in [20, Eq. (8)].

$$Z(W_N^{(i)}) = \sum_{y \in Y^N} \sum_{u_0^{i-1} \in X^{i-1}} \sqrt{\prod_{u_i \in 0,1} W_N^{(i)}(y, u_0^{i-1} | u_i)}. \quad (24)$$

At the destination node, the received information is decoded using the SC decoder. Initially, from the received vector y_2 , the SC decoder decodes the relay-destination link, and the estimate is represented by \hat{K} . Then, the estimated information is evaluated in the feedback process to extract the original information.

In the proposed system, the QAM scheme is used. The closed-form expression for the error probability of M-QAM over the Rayleigh fading channel is

$$P_e^{\text{QAM}}(\gamma | \bar{\gamma}) = Q_{N_i}(\gamma). \quad (25)$$

where Q_{N_i} denotes the closed-form expression for the error probability over Rayleigh channels, which is given by

$$Q_{N_i} = \left(1 - \frac{1}{\sqrt{M}}\right) \left(1 - \sqrt{\frac{g_{\text{QAM}} \gamma_{\text{th}}}{1 + g_{\text{QAM}} \gamma_{\text{th}}}}\right) + \left(1 - \frac{1}{\sqrt{M}}\right)^2 \left[\frac{4}{\pi} \sqrt{\frac{g_{\text{QAM}} \gamma_{\text{th}}}{1 + g_{\text{QAM}} \gamma_{\text{th}}}} \tan^{-1} \sqrt{\frac{1 + g_{\text{QAM}} \gamma_{\text{th}}}{g_{\text{QAM}} \gamma_{\text{th}}} - 1}\right]. \quad (26)$$

In (26), g_{QAM} is a function of QAM, and it is given by $3/2(M - 1)$. The value of Q_{N_i} achieves its optimal limit

for different values of order of modulation (that is M), and γ_{th} represents the threshold SNR of the system.

Let e, e_1 , and e_2 represent the overall error events in source-relay, and relay-destination links, respectively. Then, the expression for the BER polar-coded incremental hybrid relay is given by

$$P_e(\text{IHDAF}) = \sum_{i \in F_3} Z(W_N^{(i)}) \left[Q_{N_i}(\gamma) P(e, e_2) + \Pr(e, 1 - P(e_2)) (\Pr(\gamma_1) \leq \gamma_{\text{th}(1)}) P_e(nc) \Pr(\gamma_1 \leq \gamma_{\text{th}(1)}) P_e(D) + (\Pr(\gamma_1) \leq \gamma_{\text{th}(1)}) + (1 - \Pr(\gamma_2 > \gamma_{\text{th}(2)})) P_e(A) + y_{M_t} \right]. \quad (27)$$

The parameters in (27) are the sum of the Bhattacharya parameter, $Z(W_N^{(i)})$ for “ r ” binary input channels, the probability of error in DF mode, $P_e(D)$, the error probability when the relay works in AF mode, $P_e(A)$, and the probability of error in non-cooperative mode, $P_e(nc)$, with multiple antenna (y_{M_t}) systems. Here, the Bhattacharya parameter is used as a bound for the estimation of the BER at a reduced level based on the threshold (γ_{th}) value of the SNR between the source, relay, and destination.

When γ_1 exceeds $\gamma_{\text{th}(1)}$, the destination detects the desired signal from the source directly. Then, the error probability for the direct mode through polar codes can be approximated as

$$P_e(nc) = \sum_{i \in F_3} Z(W_N^{(i)}) Q_{N_i}(\gamma) p_\gamma(\gamma | \gamma_1 > \gamma_{\text{th}(1)}) P_1(e) + y_{M_t}, \quad (28)$$

where

$$P_1(e) = \frac{\bar{\gamma}_1}{\bar{\gamma}_1 + 1} \frac{1 - e^{-\gamma_{\text{th}} \left(\frac{1}{\bar{\gamma}_1} + 1\right)}}{1 - e^{-\gamma_1 / \bar{\gamma}_1}}. \quad (29)$$

This equation shows the probability of error between the source and destination. From the above expression, the relay operates in non-cooperative mode ($P_e(nc)$), when γ_1 is greater than $\gamma_{\text{th}(1)}$. The second parameter in the above expression is the density function $p_\gamma(\cdot)$ over the Rayleigh channel, which exploits the method of channel polarization of Bhattacharya parameter ($Z(W_N^{(i)})$) for “ r ” binary input channels in polar codes for higher-order multiple-antenna systems (y_{M_t}). Using the above parameters, the probability of error can be estimated between the source and destination. Substituting (26) and (29) in (28), the error probability for direct transmission becomes

$$P_e(nc) = \sum_{i \in F_3} Z(W_N^{(i)}) Q_{N_i}(\gamma) p_\gamma(\gamma | \gamma_1 > \gamma_{th(1)}) \frac{\bar{\gamma}_1}{\bar{\gamma}_1 + 1} \frac{1 - e^{-\gamma_{th(1)}(\frac{1}{\bar{\gamma}_1} + 1)}}{1 - e^{-\gamma_1/\bar{\gamma}_1}} + y_{M_r}. \tag{30}$$

When γ_2 exceeds $\gamma_{th(2)}$, the relaying system operates in decode-and-forward mode. At the same time, γ_1 should not exceed $\gamma_{th(1)}$. Therefore, the destination combines signals from the source as well as from the relay using the MRC technique through multiple antenna systems. Further, the destination decides to process the signals. After the combination of signals from the source and relay, there is a minimum amount of error present. Therefore, the error probability of the DF mode is given by

$$P_e(D) = \sum_{i \in F_3} \left[Z(W_N^{(i)}) Q_{N_i}(\gamma) P_e(det) P_e(er) + (1 - P_e(det)) P_e(c) + y_{M_r} \right]. \tag{31}$$

In (31), $P_e(det)$ represents the detection error probability of the DF relay. This probability depends on $\gamma_{th(2)}$ and the probability of error propagation $P_e(er)$. $P_e(er)$ is an event for which an error occurs after the combination of signals at the destination received from the source as well as from the relay. An error event occurs in cooperative mode. This error probability $P_e(c)$ occurs in a manner that is similar to the previous case, but the signals are a combination of the source and an acceptable signal from the relay. The above error events can be minimized using the polar-code-based IHDAF relaying protocol.

The probability of error for the cooperative mode is represented by

$$P_e(det) = \Pr(p_\gamma^{tot} | \gamma_2 > \gamma_{th(2)}) + y_{M_r}. \tag{32}$$

In the above equation, $(p_\gamma^{tot} | \gamma_2)$ is the conditional error probability and $(p_\gamma^{tot} | \gamma_2 > \gamma_{th(2)})$ is the conditional density function of γ_2 , provided that γ_2 is greater than $\gamma_{th(2)}$. γ^{tot} denotes the overall instantaneous SNR at the output of the combining technique at the destination.

The relay is still in a bottleneck situation in terms of whether to transmit the erroneous signal to the destination, even when the relay link estimates the quality of the incoming signal, before sending the signals to the destination. This kind of error propagation can be avoided by properly choosing γ_1 at the relay. Then, the error propagation probability can be calculated as [2]

$$P_e(er) = \frac{\bar{\gamma}_3}{\gamma_1 + \gamma_3} \frac{1 - e^{-\gamma_1(\frac{1}{\bar{\gamma}_1} + 1/\bar{\gamma}_3)}}{1 - e^{-\gamma_{th(1)}/\bar{\gamma}_{th(1)}}} + y_{M_r}. \tag{33}$$

The probability of error for the cooperative mode, which occurs when the relay decodes the desired signal at an acceptable level, is given by

$$P_e(c) = \sum_{i \in F_3} Z(W_N^{(i)}) Q_{N_i}(\gamma) p_\gamma(\gamma, \gamma_1 \leq \gamma_{th(1)}) + y_{M_r}. \tag{34}$$

During the next phase, γ_2 is less than $\gamma_{th(2)}$; then, the proposed system is switched over to AF mode. Therefore, the error probability in AF mode can be written as

$$P_e(A) = \sum_{i \in F_3} Z(W_N^{(i)}) Q_{N_i} p_\gamma(\gamma, \gamma_1 \leq \gamma_{th(1)}, (\gamma, \gamma_2 \leq \gamma_{th(2)})) + y_{M_r}. \tag{35}$$

In the proposed model, assume that the channel state information is available at the receiver, where the channel responses are necessary. Here, the destination would combine the signals using the MRC technique based on the outcome of the SNR performance. Equation (11) can be incorporated to make a decision about the system using an SC decoder. The signal received from the channel consists of noisy ones, and these signals are filtered out through the polar SC decoder by separating fixed and frozen sets to extract original information.

Using (32), the error-free signal received at the destination for the cooperative mode is given by

$$y_{ind}(r) = 1 - \left\{ \sum_{i \in F_3} Z(W_N^{(i)}) Q_{N_i}(\gamma) \Pr(p_{\gamma_2} | \gamma_2 > \gamma_{th(2)}) + y_{M_r} \right\} (\hat{u}_i). \tag{36}$$

Similarly, the received signal for the direct scenario can be denoted from (30) as

$$y_{dir}(nc) = 1 - \left\{ \sum_{i \in F_3} Z(W_N^{(i)}) Q_{N_i}(\gamma) p_\gamma(\gamma, \gamma_1 > \gamma_{th(1)}) P_1(e) + y_{M_r} \right\} (\hat{u}_i). \tag{37}$$

In order to prevent error propagation and to decrease the BER, the optimal threshold policy considered for $\gamma_{th(1)}$ and $\gamma_{th(2)}$ can be related as

$$\gamma_{th(i)}^{opt} = \underset{\gamma_{th(i)}}{\operatorname{argmin}} \left\{ P_e(IHDAF), \gamma_{th(i)} \right\}. \tag{38}$$

The optimal threshold expression minimizes the error probability for Rayleigh fading channels with different threshold values $\gamma_{th(i)}, i = 1, 2$. Moreover, the values of the thresholds $\gamma_{th(1)}$ and $\gamma_{th(2)}$ follow (38).

In the final stage, the original information “ K ” can be extracted using the SC decoder (\hat{K}) by eliminating erroneous values through SC operation.

In this work, the IHDAF relaying protocol is combined with polar codes. It is mainly used to increase channel capacity, and is denoted by C . Using the code rate $R_c = K/N$, the channel term towards the free polar set (F_3) and the remaining terms form the frozen polar set (F_1). The channel capacity for a higher-order MIMO channel through polar codes can be approximated as

$$C \leq \max_{\Pr(X_s, X_r^{\text{IHDAF}})} \min(I(X_s; Y_2 Y_1, X_1) + I(X_r^{\text{IHDAF}}; Y_1) + I(X_r^{\text{IHDAF}}; Y_3)). \quad (39)$$

In the above expression, $\Pr(X_s, X_r^{\text{IHDAF}})$ denotes the combined probability of the signals from the source and relay.

V. Simulation Results

To illustrate the performance of the proposed scheme, simulations were carried out over Rayleigh channels. The system was simulated using MATLAB. The simulation parameters are summarized in Table 1.

For the proposed system, Monte Carlo simulations were performed with code block length $N = 1,024$ bits and polar codes with $K = 512$ bits.

To investigate the outcome of $\gamma_{\text{th}(2)}$ on the end-to-end BER performance, the BER was simulated with respect to the system SNR for different values of $\gamma_{\text{th}(2)}$ and with optimal $\gamma_{\text{th}(1)}$. Figure 4 illustrates the error performance of the IHDAF relaying protocol based on polar codes for 2×2 MIMO configurations with $\gamma_{\text{th}(2)} = 0.5$, $\gamma_{\text{th}(2)} = 2$, $\gamma_{\text{th}(2)} = 3$, and $\gamma_{\text{th}(2)} = 6$. The corresponding values of $\gamma_{\text{th}(1)}$ can be obtained by substituting the $\gamma_{\text{th}(2)}$ values in (38). It can be seen that the BER decreases when the value of $\gamma_{\text{th}(2)}$ increases for an optimal $\gamma_{\text{th}(1)}$. Figure 4 shows the error performance with respect to the SNR for different $\gamma_{\text{th}(2)}$ values with equal power allocation. Further, it is proven that the gain with $\gamma_{\text{th}(2)} = 6$ and $\gamma_{\text{th}(1)} = (\text{optimal})$ is improved by 30% when compared with $\gamma_{\text{th}(2)} = 2$ and $\gamma_{\text{th}(1)} = 4$ at SNR = 17 dB.

Table 1. Simulation parameters.

Parameters	Values
Number of relay node	1
Code length	1,024
Channel coding	Polar coding
Modulation scheme	16-QAM
Order	2×2 MIMO 4×4 MIMO

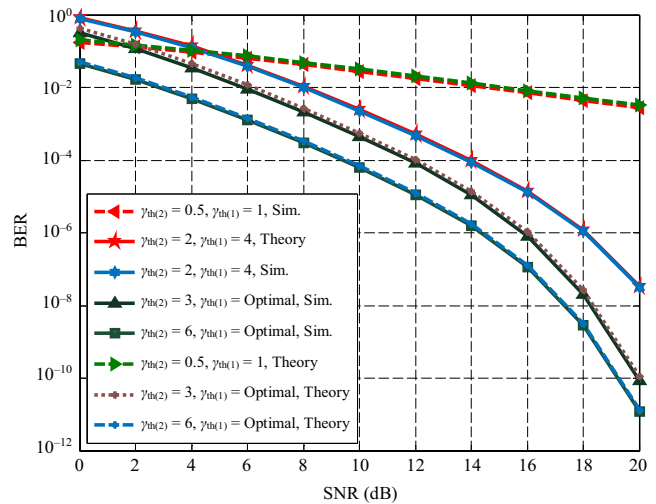


Fig. 4. SNR versus BER performance for different $\gamma_{\text{th}(2)}$ values and optimal $\gamma_{\text{th}(1)}$ of polar coded IHDAF relaying in 2×2 MIMO.

Figure 5 shows the BER for a polar-coded IHDAF relaying (PIHR) for different values of $\gamma_{\text{th}(1)}$ with optimal $\gamma_{\text{th}(2)}$ for a 2×2 MIMO system. Similar to Fig. 4, the simulation results show that the polar code with $(\gamma_{\text{th}(1)}) = 6$ performs better than other results in terms of error reduction.

Figure 6 depicts the BER of the polar-based IHDAF relaying protocol for 4×4 MIMO antenna configurations with $\gamma_{\text{th}(2)} = 0.5$, $\gamma_{\text{th}(2)} = 2$, $\gamma_{\text{th}(2)} = 3$, and $\gamma_{\text{th}(2)} = 6$. With respect to the optimal relationship in (38), the probability of error can be minimized at higher values of $\gamma_{\text{th}(2)}$. From Fig. 6, it is inferred that the error performance in the 4×4 MIMO configuration has been reduced

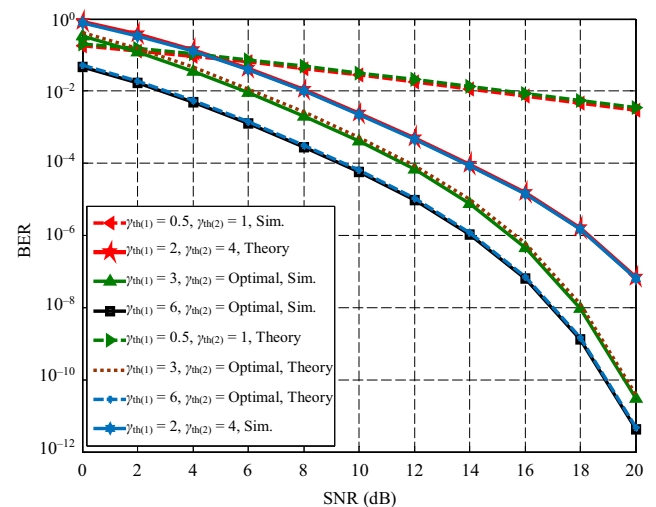


Fig. 5. SNR versus BER performance for different $\gamma_{\text{th}(1)}$ and optimal $\gamma_{\text{th}(2)}$ of polar coded IHDAF relaying.

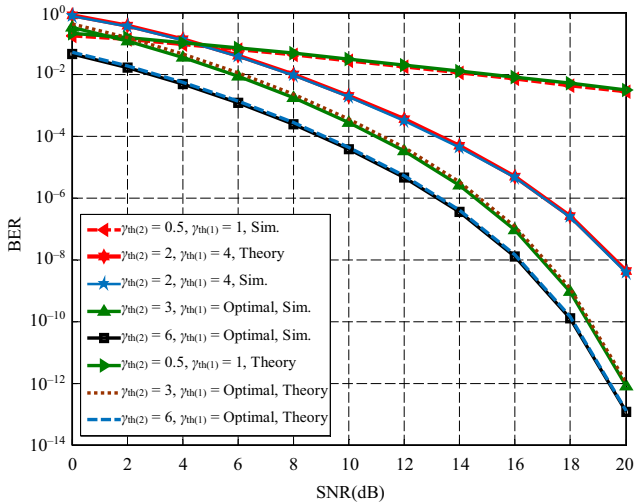


Fig. 6. SNR versus BER performance for different $\gamma_{th(2)}$ and optimal $\gamma_{th(1)}$ of PIHR relaying in 4×4 MIMO.

significantly. Further, it is proved that the gain with $\gamma_{th(2)} = 6$ and an optimal $\gamma_{th(1)}$ has been improved by around 38% when compared with $\gamma_{th(2)} = 2$ and $\gamma_{th(1)} = 4$ at SNR = 16 dB.

Figure 7 demonstrates the BER performance for incremental with polar-based HDAF for $\gamma_{th(1)}$ with optimal $\gamma_{th(2)}$ for 4×4 MIMO. Similar to Fig. 5, the simulation results proved that the 4×4 MIMO configuration performs better in terms of error and gain improvement when compared with 2×2 MIMO systems.

The error performance of channel coding schemes such as polar, turbo, and LDPC has been analyzed in [16]. The polar code is a power-efficient code when compared to other channel codes [20]. The channel capacity of the

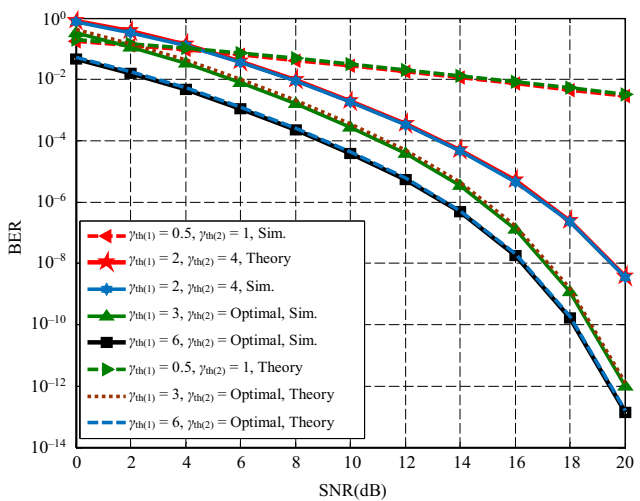


Fig. 7. SNR versus BER performance for different $\gamma_{th(1)}$ and optimal $\gamma_{th(2)}$ of polar coded IHDAF relaying.

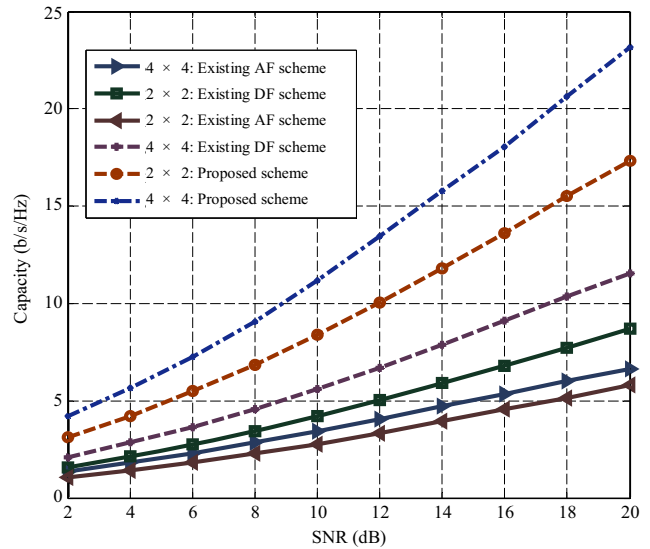


Fig. 8. SNR versus capacity performance of polar code-based IHDAF relaying.

proposed scheme is greatly improved when compared to existing AF and DF relaying schemes in Fig. 8. The value obtained in (39) is used to compute the channel capacity of the proposed system.

VI. Conclusion

In this paper, the performance of the proposed incremental-based hybrid decode-amplify-and-forward relaying scheme using polar codes over a Rayleigh channel has been evaluated with an SNR threshold. The results of various threshold values, such as $\gamma_{th(1)}$ and $\gamma_{th(2)}$, were compared with different optimal values. Simulation results shows that the polar codes through an IHDAF relay can provide a 30% gain for 2×2 MIMO and 38% for 4×4 MIMO configurations when the threshold values are optimal. Further, the channel capacity is improved by around 17.5 b/s/Hz for 2×2 MIMO and 23 b/s/Hz for 4×4 MIMO systems. The results prove that the capacity is increased when compared to AF and DF relaying schemes. In future work, there will be the incorporation of the IHDAF relaying protocol using polar codes through a large MIMO scheme along with a multicarrier modulation scheme to achieve better quality of service in terms of outage capacity and coverage for near-future cellular networks.

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