S-SHAPED CONNECTED COMPONENT FOR A NONLINEAR DIRICHLET PROBLEM INVOLVING MEAN CURVATURE OPERATOR IN ONE-DIMENSION MINKOWSKI SPACE

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ABSTRACT. In this paper, we investigate the existence of an S-shaped connected component in the set of positive solutions of the Dirichlet problem of the one-dimension Minkowski-curvature equation

$$\begin{cases} \left(\dfrac{u'}{\sqrt{1-u'^2}}\right)' + \lambda a(x) f(u) = 0, & x \in (0,1), \\ u(0) = u(1) = 0, \end{cases}$$

where λ is a positive parameter, $f \in C[0,\infty)$, $a \in C[0,1]$. The proofs of main results are based upon the bifurcation techniques.

1. Introduction

Hypersurfaces of prescribed mean curvature in flat Minkowski space $\mathbb{L}^{N+1} = \{(x,t) : x \in \mathbb{R}^N, t \in \mathbb{R}\}$, with the Lorentzian metric $\sum_{i=1}^{N} dx_i^2 - dt^2$, where $(x,t) = (x_1, x_2, \ldots, x_N, t)$, are of interest in differential geometry and in general relativity. It is well-known that the study of spacelike submanifolds of codimension one in $\mathbb{L}^{N+1}$ with prescribed mean extrinsic curvature leads to Dirichlet problems of the type

$$(1.1) \qquad \begin{cases} -\operatorname{div}\left(\dfrac{\nabla u}{\sqrt{1-|\nabla u|^2}}\right) = f(x,u) \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ and the nonlinearity $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is continuous, see [1, 2].

The existence, multiplicity and qualitative properties of solutions of (1.1) have been extensively studied by many authors in recent years, see Coelho

et al. [9], Treibergs [27], Cano-Casanova et al. [6], Pan et al. [23], López [17], Corsato et al. [10,11], Korman [14] as well as Ma et al. [20,22], and the references therein. It is worth pointing out that the starting point of this type of problems is the seminal paper [7], and from Bartnik and Simon [2] as well as Bereanu and Mawhin [5], we know (1.1) has a solution whatever f is. This can be seen as a universal existence result for the above problem. However, in our study problem (1.1) generally admits the null solution, it may be interesting to investigate the existence of non-trivial solutions, especially the positive solutions. However, there are few works on positive solutions of (1.1), see Coelho et al. [8], Bereanu et al. [3,4], Ma et al. [21] and Dai [12].

Specifically, depending on the behaviour of $f = f(x,s)$ near $s = 0$, Coelho et al. [8] discussed the existence of either one, or two, or three, or infinitely many positive solutions of the quasilinear two-point boundary value problem

$$\begin{cases} \left(\dfrac{u'}{\sqrt{1 - u'^2}}\right)' + f(x,u)) = 0, \ x \in (0,1), \\ u(0) = u(1) = 0, \end{cases}$$

where f is L^p-Carathéodory function, and the proof of main results are based upon the variational and topological methods. Bereanu et al. [3,4] obtained some important existence, nonexistence and multiplicity results for the positive radial solutions of problem (1.1) in a ball by using Leray-Schauder degree argument and critical point theory. Recently, Ma et al. [21] concerned the global structure of radial positive solutions for the problem (1.1) in a ball by using global bifurcation techniques, and extended the results of [3,4] to more general cases, all results, depending on the behavior of nonlinear term f near 0. Dai [12] investigated the intervals of the parameter λ in which the problem (1.1) has zero, one or two positive radial solutions corresponding to sublinear, linear, and superlinear nonlinearities f at zero, respectively. However, [12,21] only give a full description of the set of radial positive solutions of (1.1) for certain classes of nonlinearities f, and give no any information about the directions of a bifurcation.

In 2015, Sim and Tanaka [26] proved the existence of S-shaped connected component in the set of positive solutions for the one-dimensional p-Laplacian problem with sign-changing weight

$$(1.2) \qquad \begin{cases} (|u'|^{p-2}u')' + \mu m(x)f(u) = 0, & x \in (0,1), \\ u(0) = u(1) = 0, \end{cases}$$

where $p > 1$, $m \in C[0,1]$, $f \in C[0,\infty)$ and μ is a positive parameter. They obtained the following result by bifurcation techniques.

Theorem A ([26, Thoerem 1.1]). *Assume*

(H1) *there exist* $x_1, x_2 \in [0,1]$ *such that* $x_1 < x_2, m(x) > 0$ *on* (x_1, x_2) *and* $m(x) \leq 0$ *on* $[0,1]\backslash[x_1, x_2]$,

(F1') *there exist* $\alpha > 0, f_0 > 0$ *and* $f_1 > 0$ *such that* $\lim\limits_{s \to 0^+} \dfrac{f(s) - f_0 s^{p-1}}{s^{p-1+\alpha}} = -f_1$,

(F2') $f_\infty := \lim\limits_{s \to \infty} \frac{f(s)}{s^{p-1}} = 0$,

(F3') *there exists* $s_0 > 0$ *such that*

$$\min_{s \in [s_0, 2s_0]} \frac{f(s)}{s^{p-1}} \geq \frac{f_0(p-1)}{\mu_1 m_0} \left(\frac{\pi_p}{x_2 - x_1} \right)^p,$$

where $\mu_1 > 0$ *is the first eigenvalue of the linear problem associated to* (1.2), *and*

$$\pi_p := \frac{2\pi}{p \sin(\frac{\pi}{p})}, \quad m_0 = \min_{x \in [\frac{3x_1 + x_2}{4}, \frac{x_1 + 3x_2}{4}]} m(x).$$

Then there exist $\mu_* \in (0, \frac{\mu_1}{f_0})$ *and* $\mu^* > \frac{\mu_1}{f_0}$ *such that*
  (i) (1.2) *has at least one positive solution if* $\mu = \mu_*$;
  (ii) (1.2) *has at least two positive solutions if* $\mu_* < \mu \leq \frac{\mu_1}{f_0}$;
  (iii) (1.2) *has at least three positive solutions if* $\frac{\mu_1}{f_0} < \mu < \mu^*$;
  (iv) (1.2) *has at least two positive solutions if* $\mu = \mu^*$;
  (v) (1.2) *has at least one positive solution if* $\mu > \mu^*$.

Of course, the natural question is whether or not the similar result can be established for the prescribed mean curvature problem (1.1)?

The purpose of this paper is to show the existence of the S-shaped connected component in the set of positive solutions for nonlinear Dirichlet problem with mean curvature operator in one-dimension Minkowski space

$$(1.3) \qquad \begin{cases} \left(\dfrac{u'}{\sqrt{1 - u'^2}} \right)' + \lambda a(x) f(u) = 0, \ x \in (0, 1), \\ u(0) = u(1) = 0, \end{cases}$$

where λ is a positive parameter, $a \in C[0,1]$ and $f : [0, \infty) \to [0, \infty)$ is a continuous function. This is the special case of the one-dimensional version of (1.1). To the best of our knowledge, for problem (1.3), such bifurcation curve is completely new and has not been practically described before.

Let us set

$$\phi(s) = \frac{s}{\sqrt{1 - s^2}}, \ s \in \mathbb{R}.$$

It is obvious that $\phi : (-1, 1) \to \mathbb{R}$ is an odd, increasing homeomorphism and $\phi(0) = 0$. We say that a function $u \in C^1[0,1]$ is a solution of (1.3) if $\max\limits_{x \in [0,1]} |u'(x)| < 1$, $\phi \circ u' \in C^1[0,1]$, and satisfies (1.3). Let $X = C[0,1]$ with the norm $||u||_\infty := \max\limits_{x \in [0,1]} |u(x)|$. Let $E = \{u \in C^1[0,1] : u(0) = u(1) = 0\}$ with the norm $||u|| := ||u'||_\infty$.

Let λ_k be the k-th eigenvalue of the eigenvalue problem

$$(1.4) \qquad \begin{cases} u''(x) + \lambda a(x) u(x) = 0, \quad x \in (0, 1), \\ u(0) = u(1) = 0, \end{cases}$$

and ϕ_k be the eigenfunction corresponding to λ_k. It is well-known that

$$0 < \lambda_1 < \lambda_2 < \cdots < \lambda_k < \cdots, \quad \lim_{k \to \infty} \lambda_k = \infty,$$

and that ϕ_k has exactly $k - 1$ zeros in (0,1), see [28].

Assume that:

(A1) $a : [0, 1] \to [0, \infty)$ is a continuous function and $a(x) > 0$ in $(0, 1)$;

(F1) $f \in C([0, \infty), [0, \infty))$ with $f(s) > 0$ for $s > 0$;

(F2) there exist $f_0 \in (0, \infty)$, $\delta \in (0, \frac{1}{32})$ and $g \in C([0, \infty), [0, \infty))$ with $g(s) > 0$ for $s \in (0, \delta]$ such that

$$f(s) = f_0 s - g(s) \quad \text{for} \quad s \in [0, \delta],$$

where $\lim_{s \to 0^+} \frac{g(s)}{s} = 0$;

(F3) there exists $s_0 : s_0 \in (\frac{1}{32}, \frac{1}{12})$, such that

$$\min_{s \in [s_0, 4s_0]} \frac{f(s)}{s} \geq \frac{27 f_0}{5\sqrt{5}\lambda_1 a_0} \cdot (2\pi)^2,$$

where $a_0 = \min_{s \in [\frac{1}{4}, \frac{3}{4}]} a(s)$.

The main result of this paper is the following.

Theorem 1.1. *Assume that* (A1) *and* (F1)-(F3) *hold. Then there exist* $\lambda_* \in (0, \frac{\lambda_1}{f_0})$ *and* $\lambda^* > \frac{\lambda_1}{f_0}$ *such that*

(i) (1.3) *has at least one positive solution if* $\lambda = \lambda_*$;

(ii) (1.3) *has at least two positive solutions if* $\lambda_* < \lambda \leq \frac{\lambda_1}{f_0}$;

(iii) (1.3) *has at least three positive solutions if* $\frac{\lambda_1}{f_0} < \lambda < \lambda^*$;

(iv) (1.3) *has at least two positive solutions if* $\lambda = \lambda^*$;

(v) (1.3) *has at least one positive solution if* $\lambda > \lambda^*$;

(vi) $\lim_{\lambda \to \infty} \|u\|_\infty = \frac{1}{2}$ *and* $\lim_{\lambda \to \infty} \|u\| = 1$.

Remark 1.1. Let (λ, u) be a solution of (1.3), then it follows from $|u'(x)| < 1$ that

$$\|u\|_\infty < \frac{1}{2}.$$

This leads to the bifurcation diagrams mainly depend on the behavior of $f = f(s)$ near $s = 0$. This is a significant difference between the one-dimension Minkowski-curvature problems and the p-Laplacian problems.

Remark 1.2. In the special case $p = 2$, (F1') reduces to

$$(1.5) \qquad\qquad f(s) = f_0 s - f_1 s^{1+\alpha} \quad \text{for} \quad s \in [0, \chi],$$

where $\chi > 0$ is a sufficient small constant. It is easy to see that condition (F2) is weaker than (1.5), in fact $f_1 s^{1+\alpha}$ is a special case of $g(s)$.

The main result is obtained by reducing the problem (1.3) to an equivalent non-singular problem and use the Rabinowitz global bifurcation techniques [24]. Indeed, under (F1) and (F2) we get an unbounded connected component

which is bifurcating from $(\frac{\lambda_1}{f_0}, 0)$, and condition (F2) pushes the bifurcation to the right near $u = 0$. Condition (F3) leads the bifurcation curve to the left at some point, and finally to the right near $\lambda = \infty$.

For other results concerning the existence of an S-shaped connected component in the set of solutions for diverse boundary value problems, see [13,16,25, 30] for the semilinear boundary value problems, and [29] for the p-Laplacian boundary value problems.

The rest of the paper is organized as follows. In Section 2, we give an equivalent formulation of problem (1.3) and some preliminary results to show the change of direction of a bifurcation. Section 3 is devoted to proving the main result. Finally in Section 4, we shall give a further result in the case that $f_0 = \infty$.

2. Some preliminary results

2.1. An equivalent formulation

Let us define a function $\widetilde{f} : \mathbb{R} \to \mathbb{R}$ by setting

$$(2.1) \qquad \widetilde{f}(s) = \begin{cases} f(s), & 0 \le s \le \frac{1}{2}, \\ \text{linear}, & \frac{1}{2} < s < 1, \\ 0, & s \ge 1, \\ -\widetilde{f}(-s), & s < 0. \end{cases}$$

Notice that, within the context of positive solutions, problem (1.3) is equivalent to the same problem with f replaced by $\widetilde{f}$. In the sequel, we shall replace f with $\widetilde{f}$, however, for the sake of simplicity, the modified function $\widetilde{f}$ will still be denoted by f. Next, let us define h as follows

$$(2.2) \qquad h(s) = \begin{cases} (1 - s^2)^{3/2}, & |s| \le 1, \\ 0, & |s| > 1. \end{cases}$$

Now, we state one lemma which similar to the *step 1* of [8, Theorem 2.3]. For completeness and later references, let us recall and prove the lemma.

Lemma 2.1. *A function $u \in C^1[0,1]$ is a positive solution of (1.3) if and only if it is a positive solution of the problem*

$$(2.3) \qquad \begin{cases} -u'' = \lambda a(x) f(u) h(u'), & x \in (0,1), \\ u(0) = u(1) = 0. \end{cases}$$

Proof. It is clear that a positive solution $u \in C^1[0,1]$ of (1.3) is a positive solution of (2.3). Conversely, assume that $u \in C^1[0,1]$ is a positive solution of (2.3). We aim to show that $||u'||_\infty < 1$. Assume on the contrary that this is not true. Then we can easily find an interval $[c,d] \subseteq [0,1]$ such that, either $u'(c) = 0, 0 < |u'(x)| < 1$ in (c,d) and $|u'(d)| = 1$, or $|u'(c)| = 1, 0 < |u'(x)| < 1$

in (c, d) and $u'(d) = 0$. Assume the former case occurs. The function u satisfies the equation

$$(\phi(u'))' + \lambda a(x) f(u) = 0$$

in $[c, d]$. For each $x \in (c, d)$, integrating over the interval $[c, x]$, we obtain

$$|\phi(u'(x))| = \lambda \left| \int_c^x a(t) f(u(t)) dt \right| \leq C_1$$

for some constant $C_1 > 0$ and hence

$$|u'(x)| \leq \phi^{-1}(C_1)$$

for every $x \in [c, d]$. Since $\phi^{-1}(C_1) < 1$, taking the limit as $x \to d^-$ we obtain the contradiction $|u'(d)| < 1$. Therefore $||u'||_\infty < 1$ and, as a consequence, u is a positive solution of (1.3). $\qquad\square$

Lemma 2.2. *Assume that* (A1) *and* (F1) *hold. Let u be a positive solution of* (2.3). *Then*

$$(2.4) \qquad \frac{1}{4}\|u\|_\infty \leq u(x) \leq \|u\|_\infty, \quad x \in \left[\frac{1}{4}, \frac{3}{4}\right].$$

Proof. Since $u'' = -\lambda a(x) f(u) h(u')$, condition (A1), (F1) and (2.2) imply that $u'(x)$ is decreasing on $(0, 1)$. Combining the fact $u(0) = u(1) = 0$ and $u(x) > 0$ on $(0, 1)$, we have $u'(0) > 0$ and $u'(1) < 0$. Therefore, u is concave on $(0, 1)$.

Let us denote

$$u(\tau) = \max_{x \in [0,1]} u(x), \qquad \bar{x} = \frac{1}{2}.$$

If $\tau \in (0, \bar{x}]$, then

$$(2.5) \qquad u(x) \geq 2u(\bar{x})x, \quad x \in [0, \bar{x}].$$

Since u is concave on $[\tau, 1]$, it follows that

$$u(\bar{x}) \geq \frac{1}{2} \frac{||u||_\infty}{1 - \tau}.$$

Combining this fact with (2.5), we have

$$u(x) \geq x||u||_\infty, \quad x \in [0, \bar{x}].$$

It deduces that (2.4) holds for $x \in [\frac{1}{4}, \bar{x}]$.

If $\tau \in (\bar{x}, 1)$, then

$$u(x) \geq \frac{||u||_\infty}{\tau} x \geq x||u||_\infty, \quad x \in [0, \bar{x}].$$

Notice that (2.4) is valid for $x \in [\frac{1}{4}, \bar{x}]$ again.

By the same argument, we may deduce that (2.4) is also valid in $[\bar{x}, \frac{3}{4}]$. $\quad\square$

Next, we give some property of concave functions.

Lemma 2.3. *Let $\nu \in (0,1)$ and $\beta_0 \in (0, \frac{1-\nu}{8})$ be given. Let $I_{\nu,\beta_0} := \left[\frac{4\beta_0}{1-\nu}, 1 - \frac{4\beta_0}{1-\nu} \right]$. Then*

$$\frac{1}{2} \in I_{\nu,\beta_0},$$

and

$$|u'(s)| \leq 1 - \nu, \qquad \forall\, u \in \mathcal{A}, \quad \forall\, s \in I_{\nu,\beta_0},$$

where

$$\mathcal{A} := \{u \in E \mid u \text{ is concave in } [0,1],\ u'(0) < 1,\ u'(1) > -1,\ \|u\|_\infty \leq 4\beta_0\}.$$

Proof. Set $1 - \nu = \alpha$ and $\xi = \frac{4\beta_0}{1-\nu}$. Then the condition can be rewritten as

$$0 < \alpha < 1, \xi \in (0, \frac{1}{2}), \text{ and } I := I_{\nu,\beta_0} = [\xi, 1 - \xi].$$

Since $u \in C^1[0,1]$ and u is concave in $[0,1]$, u' is decreasing. If there exists $s \in I$ such that $|u'(s)| > 1 - \nu = \alpha$, then $u'(s) > \alpha$ or $u'(s) < -\alpha$. If $u'(s) > \alpha$, then $\frac{u(s)-u(0)}{s} = u'(t)$ for some $t \in (0,s)$. Hence $\frac{u(s)}{s} \geq u'(s) > \alpha$. Therefore $u(s) > \alpha s \geq \alpha \xi = 4\beta_0 \geq \|u\|_\infty$. It is a contradiction. Similarly, we have a contradiction for other case. □

If we let $\nu = \frac{1}{3}$ and $\beta_0 = \frac{1}{24} \in (0, \frac{1}{12})$. Then we have the following:

Corollary 2.1. *For any concave function $u \in E$ with*

$$u'(0) < 1, \quad u'(1) > -1, \quad \|u\|_\infty \leq \frac{1}{6},$$

we have

$$|u'(x)| \leq \frac{2}{3}, \qquad x \in \left[\frac{1}{4}, \frac{3}{4} \right].$$

2.2. The direction of bifurcation

For every $p \in X$, it is well-known that the solution u of problem

$$\begin{cases} -u''(x) = a(x)p(u(x)), & x \in (0,1), \\ u(0) = u(1) = 0 \end{cases}$$

can be expressed by

$$u(x) = \int_0^1 G(x,s)a(s)p(u(s))ds := \mathcal{L}(p),$$

where the Green's function $G(x,s)$ is explicitly given by

$$G(x,s) = \begin{cases} x(1-s), & 0 \leq x \leq s \leq 1, \\ s(1-x), & 0 \leq s < x \leq 1. \end{cases}$$

It is easy to check that $\mathcal{L} : X \to E$ is completely continuous and (1.4) is equivalent to

$$u = \lambda \mathcal{L}(u),$$

so that the eigenvalues of (1.4) are the characteristic values of $\mathcal{L}$.

If (F2) holds, then $\lim\limits_{s\to 0^+}\frac{f(s)}{s}=f_0$. This fact together with (F1), we have

$$f(s)=\left(f_0-\frac{g(s)}{s}\right)s,$$

where $g(s)$ is a continuous function and

(2.6) $$\lim_{s\to 0}\frac{g(s)}{s}=0.$$

Let us set, for convenience, $k(y)=h(y)-1$ for $y\in\mathbb{R}$. We have

(2.7) $$\lim_{y\to 0}\frac{k(y)}{y}=0.$$

Define the operator $\mathcal{H}:\mathbb{R}\times E\to E$ by

$$\mathcal{H}(\lambda,u)=\lambda\mathcal{L}\left(\left((f_0-\frac{g(u)}{u})k(u')-\frac{g(u)}{u}\right)u\right).$$

Clearly, $\mathcal{H}$ is completely continuous and, by (2.6) and (2.7), we have

$$\lim_{\|u\|\to 0}\frac{\|\mathcal{H}(\lambda,u)\|}{\|u\|}=0$$

uniformly with respect to λ varying in bounded intervals. Observe that, for any λ, $(\lambda,u)\in\mathbb{R}\times E$, with $u>0$, is a solution of the equation

(2.8) $$u=\lambda f_0\mathcal{L}(u)+\mathcal{H}(\lambda,u),$$

if and only if u is a positive solution of (2.3). Denote by $\mathcal{S}$ the closure in $\mathbb{R}\times E$ of the set of all non-trivial solutions (λ,u) of (2.8) with $\lambda>0$. Let $P=\{u\in E:u(x)\geq 0,x\in[0,1]\}$. Then P is a positive cone of E and $\mathrm{int}P\neq\emptyset$.

Notice that

$$\|u'\|_\infty<1,\quad(\lambda,u)\in\mathcal{S},$$

which implies that

$$\|u\|_\infty<\frac{1}{2},\quad(\lambda,u)\in\mathcal{S}.$$

Hence, by Theorem 1.3 in [24] or Theorem 2.3 in [8], we have the following result.

Lemma 2.4. *Assume that* (A1), (F1) *and* (F2) *hold. Then there exists an unbounded connected component $\mathcal{C}$ in $\mathcal{S}$ which is bifurcating from $(\frac{\lambda_1}{f_0},0)$ such that $\mathcal{C}\subseteq(([0,+\infty)\times\mathrm{int}P)\cup\{(\frac{\lambda_1}{f_0},0)\})$. Moreover, $\mathcal{C}$ joins $(\frac{\lambda_1}{f_0},0)$ with infinity in λ direction.*

Lemma 2.5. *Assume that* (A1), (F1) *and* (F2) *hold. Let $\{(\lambda_n,u_n)\}$ be a sequence of positive solutions of (2.8) which satisfies $\|u_n\|\to 0$ and $\lambda_n\to\frac{\lambda_1}{f_0}$. Let $\phi_1(x)$ be the first eigenfunction of (1.4) which satisfies $\|\phi_1\|=1$. Then there*

exists a subsequence of $\{u_n\}$, again denoted by $\{u_n\}$, such that $\frac{u_n}{\|u_n\|}$ converges uniformly to ϕ_1 on $[0,1]$.

Proof. Let $v_n := \frac{u_n}{\|u_n\|}$. Then $\|v_n\| = \|v'_n\|_\infty = 1$, consequently, $\|v_n\|_\infty$ is bounded. By the Ascoli-Arzela theorem, there exists a subsequence of v_n which uniformly converges to $v \in X$. We again denote the subsequence by v_n. For any (λ_n, u_n), we have

$$(2.9) \qquad u_n(x) = \lambda_n \int_0^1 G(x,s)a(s)f(u_n(s))h(u'_n(s))ds.$$

Multiplying both sides of (2.9) by $\|u_n\|^{-1}$, we have

$$v_n(x) = \lambda_n \int_0^1 G(x,s)\frac{f(u_n(s))}{u_n(s)}a(s)h(u'_n(s))v_n(s)ds.$$

Since $\|u_n\| \to 0$ implies $\|u_n\|_\infty \to 0$. From (F2) and (2.2), we conclude that $\frac{f(u_n(s))}{u_n(s)} \to f_0$ and $h(u'_n(s)) \to 1$ as $n \to \infty$ uniformly for $s \in [0,1]$. By Lebesgue's dominated convergence theorem we know that

$$v(x) = \frac{\lambda_1}{f_0} \int_0^1 G(x,s)f_0 a(s)v(s)ds,$$

which means that v is a nontrivial solution of (1.4) with $\lambda = \lambda_1$, and hence $v \equiv \phi_1$. □

Lemma 2.6. *Assume that* (A1), (F1) *and* (F2) *hold. Let $\mathcal{C}$ be as in Lemma 2.4. Then there exists $\sigma > 0$ such that $(\lambda, u) \in \mathcal{C}$ and $|\lambda - \frac{\lambda_1}{f_0}| + \|u\| \leq \sigma$ imply $\lambda > \frac{\lambda_1}{f_0}$.*

Proof. By (F1) and (F2), there exists a sufficiently small $\delta > 0$ such that

$$0 \leq f(s) \leq f_0 s \quad \text{for} \quad s \in [0, \delta].$$

This fact together with the definition of h, we have

$$(2.10) \qquad 0 \leq \frac{f(s)}{s}h < f_0 \quad \text{in} \quad (0, \delta).$$

Fixed $\sigma = 2\delta$, then for any $(\lambda, u) \in \mathcal{C}$ which satisfies $|\lambda - \frac{\lambda_1}{f_0}| + \|u\| \leq \sigma$, we known u is a positive solution of the problem

$$(2.11) \qquad \begin{cases} u''(x) + \lambda a(x)\dfrac{f(u(x))}{u(x)}h(u'(x))u(x) = 0, & x \in (0,1), \\ u(0) = u(1) = 0, \end{cases}$$

and

$$0 \leq u \leq \delta.$$

On the other hand,

$$\begin{cases} \phi''_1(x) + \lambda_1 a(x)\phi_1(x) = 0, & x \in (0,1), \\ \phi_1(0) = \phi_1(1) = 0. \end{cases}$$

Combining this fact with (2.10) and (2.11), we have

$$\lambda_1 \int_0^1 a(x)\phi_1(x)u(x)dx = \int_0^1 u'(x)\phi_1'(x)dx$$

$$= \lambda \int_0^1 a(x)\frac{f(u(x))}{u(x)}h(u'(x))\phi_1(x)u(x)dx$$

$$< \lambda \int_0^1 a(x)f_0\phi_1(x)u(x)dx,$$

that is $\lambda_1 - \lambda f_0 < 0$, and accordingly, $\lambda > \frac{\lambda_1}{f_0}$. $\qquad\qquad$ □

Lemma 2.7. *Assume that* (A1), (F1) *and* (F2) *hold. Let* $\mathcal{C}$ *be as in Lemma 2.4. Then there exists* $\lambda_\diamond > 0$ *such that* $\mathrm{Proj}_{\mathbb{R}}\mathcal{C} = [\lambda_\diamond, \infty) \subset (0, \infty)$.

Proof. Suppose on the contrary that $\lambda_\diamond = 0$. Then there exists a sequence $\{(\mu_n, u_n)\} \subset \mathcal{C}$ satisfying $u_n > 0$ such that

$$\lim_{n\to\infty} (\mu_n, u_n) = (0, u^*) \ \text{ in } \ \mathbb{R} \times X$$

for some $u^* \geq 0$. Then by the fact

$$-(\phi(u_n'(x)))' = \mu_n a(x)f(u_n(x)), \quad u_n(0) = u_n(1) = 0,$$

after taking a subsequence and relabeling, if necessary, we have $u_n \to 0$.

By Lemma 2.2, there exists $\tau \in (0, 1)$ such that $u'(\tau) = 0$ and

$$\phi(u_n'(x)) = \mu_n \int_x^\tau a(t)f(u_n(t))dt \quad \text{for} \ \ x \in [0, \tau].$$

This fact together with $f(0) = 0$ yield that

(2.12) $$\lim_{n\to\infty} \|u_n'\|_\infty = 0.$$

On the other hand

$$\begin{cases} - u_n''(x) = \mu_n a(x)f(u_n(x))h(u_n'(x)), & x \in (0, 1), \\ u_n(0) = u_n(1) = 0. \end{cases}$$

Let $v_n = \frac{u_n}{||u_n||}$ for all n. Then we have

(2.13) $$\begin{cases} - v_n''(x) = \mu_n a(x)\frac{f(u_n(x))}{u_n(x)}h(u_n'(x))v_n(x), & x \in (0, 1), \\ v_n(0) = v_n(1) = 0. \end{cases}$$

Similar to the proof of Lemma 2.5, by (2.12) and (2.13), it concludes that $\mu_n \to \frac{\lambda_1}{f_0}$, which contradicts $\mu_n \to 0$. $\qquad\qquad$ □

Lemma 2.8. *Assume that* (A1), (F1) *and* (F3) *hold. Let* $(\lambda, u) \in \mathcal{C}$ *with* $||u||_\infty = 4s_0$. *Then* $\lambda < \frac{\lambda_1}{f_0}$.

Proof. Let $(\lambda, u) \in \mathcal{C}$. Then by Lemma 2.2, we obtain

(2.14) $$s_0 \leq u(x) \leq 4s_0, \ x \in \left[\frac{1}{4}, \frac{3}{4}\right].$$

Fixed $s_0 = \frac{1}{24}$, then from Corollary 2.1, for any $(\lambda, u) \in \mathcal{C}$ with $||u||_\infty = 4s_0$, we have

$$0 \leq |u'(x)| \leq \frac{2}{3}, \ x \in \left[\frac{1}{4}, \frac{3}{4}\right].$$

Now we assume on the contrary that $\lambda \geq \frac{\lambda_1}{f_0}$. Then for $x \in \left[\frac{1}{4}, \frac{3}{4}\right]$, by (2.14) and (F3), we have

$$\lambda a(x)\frac{f(u(x))h(u'(x))}{u(x)} \geq \frac{\lambda_1}{f_0} \cdot a_0 \cdot \frac{27 f_0 (2\pi)^2}{5\sqrt{5}\lambda_1 a_0} \cdot \frac{5\sqrt{5}}{27} = (2\pi)^2.$$

Let $v(x) = \sin\left(2\pi(x - \frac{1}{4})\right)$. Then v is a positive solution of

$$\begin{cases} v''(x) + (2\pi)^2 v(x) = 0, & x \in \left(\frac{1}{4}, \frac{3}{4}\right), \\ v\left(\frac{1}{4}\right) = v\left(\frac{3}{4}\right) = 0. \end{cases}$$

We notice that u is a solution of

$$u''(x) + \lambda a(x)\frac{f(u(x))h(u'(x))}{u(x)}u(x) = 0$$

on $\left[\frac{1}{4}, \frac{3}{4}\right]$. Strum comparison theorem, implies that u has at least one zero on $\left[\frac{1}{4}, \frac{3}{4}\right]$, see [15, 28]. This contradicts the fact that $u(x) > 0$ on $\left[\frac{1}{4}, \frac{3}{4}\right]$. $\qquad\square$

Lemma 2.9. *Let $\mathcal{C}$ be as in Lemma 2.4. Then there exists a sequence $\{w_n\} \subset \mathcal{C}$ such that*

$$\lim_{n\to\infty} ||w_n||_\infty = 0,$$

implies $w_n' \to 0$ in measure as $n \to \infty$.

Proof. Since $||w_n||_\infty = w_n(\tau)$, where τ is given by Lemma 2.2, it follows that $\lim_{n\to\infty} w_n(\tau) = 0$. For any $\gamma > 0$, let

$$m_n(\gamma) = \left\{x \in [0, \tau] : w_n'(x) \geq \gamma\right\}.$$

Then

$$w_n(\tau) = \int_0^\tau w_n'(x)dx \geq \int_{m_n(\gamma)} w_n'(x)dx \geq \gamma \operatorname{meas} m_n(\gamma),$$

which implies that $\operatorname{meas} m_n(\gamma) \to 0$. Therefore, $w_n' \to 0$ in measure. $\qquad\square$

Lemma 2.10. *Assume that* (A1) *and* (F1) *hold. Let $\mathcal{C}$ be as in Lemma 2.4. Then $\lim\limits_{(\lambda,u)\in\mathcal{C},\lambda\to\infty} ||u|| = 1$ and $\lim\limits_{(\lambda,u)\in\mathcal{C},\lambda\to\infty} ||u||_\infty = 1/2$.*

Proof. We divide the proof into four steps.

Step 1. We claim that there exists a constant $B_0 > 0$ such that for every $(\lambda, u) \in \mathcal{C}$, if $\lambda \geq B_0$, then

$$||u||_\infty \geq \rho_*$$

for some $\rho_* > 0$.

Suppose on the contrary that there exists a sequence $\{(\mu_n, u_n)\} \subset \mathcal{C}$ satisfying

$$(\mu_n, u_n) \to (\infty, 0) \quad \text{in} \quad (0, +\infty) \times X.$$

Then by Lemma 2.9, we have u'_n converges to 0 in measure as $n \to \infty$. From this fact and (2.13), after taking a subsequence and relabeling, if necessary, we have $v_n \to v_*$ in X for some $v_* \in X$, and,

$$-v''_* = \mu_n a f_0 v_*, \quad x \in (0, 1), \quad v_*(0) = v_*(1) = 0,$$

that is $\mu_n \to \frac{\lambda_1}{f_0}$. This contradicts with the fact $\mu_n \to \infty$. Therefore, the claim is valid.

Step 2. Fixed $\varepsilon \in (\frac{\tau}{4}, \tau)$, we can show that for every $(\lambda, u) \in \mathcal{C}$, if $\lambda > B_0$, then

$$(2.15) \qquad\qquad \min_{x \in [\varepsilon, \tau]} u(x) \geq \frac{1}{4} \rho_*.$$

Step 3. It follows from (2.15) and (F1), there exists some constant $M_0 > 0$ such that

$$f(u(x)) \geq M_0 > 0 \quad \text{for} \quad x \in [\varepsilon, \tau],$$

and accordingly,

$$\lim_{\lambda \to \infty} \lambda \int_x^\tau a(t) f(u(t)) dt = +\infty \quad \text{uniformly in} \quad x \in [\varepsilon, \tau - \varepsilon_1]$$

for arbitrary fixed $\varepsilon_1 \in (0, \tau - \varepsilon)$. This together with (A1), (F1) and the relation

$$u'(x) = \phi^{-1} \left(\lambda \int_x^\tau a(t) f(u(t)) dt \right)$$

imply that

$$(2.16) \qquad\qquad u' \to 1 \quad \text{in} \quad C[\varepsilon, \tau - \varepsilon_1] \quad \text{as} \quad \lambda \to +\infty.$$

Therefore, by the arbitrariness of ε and ε_1, we get $\lim\limits_{(\lambda, u) \in \mathcal{C}, \lambda \to \infty} ||u|| = 1$.

Step 4. Since

$$u'(x) \geq 0, \quad x \in [0, \tau], \qquad -u'(x) \geq 0, \quad x \in [\tau, 1].$$

This fact together with (2.16) imply that for $(\lambda, u) \in \mathcal{C}$,

$$\lim_{\lambda \to \infty} ||u||_\infty = \lim_{\lambda \to \infty} u(\tau) = \lim_{\lambda \to \infty} \int_0^\tau u'(t) dt \geq \lim_{\lambda \to \infty} \int_\varepsilon^{\tau - \varepsilon_1} u'(t) dt = \tau - \varepsilon_1 - \varepsilon.$$

By the arbitrariness of ε and ε_1, we have $\lim\limits_{\lambda\to\infty}||u||_\infty \geq \tau$ for $\tau \in (0,1)$, and similarly, we have $\lim\limits_{\lambda\to\infty}||u||_\infty \geq 1-\tau$ for $\tau \in (0,1)$. Therefore, it deduces that

$$(2.17) \qquad \lim_{\lambda\to\infty}||u||_\infty \geq \frac{1}{2}.$$

On the other hand, it follows that

$$u(\tau) = \int_0^\tau u'(t)dt \leq \tau, \quad \tau \in (0,1),$$

and

$$u(\tau) = \int_\tau^1 (-u'(t))dt \leq 1-\tau, \quad \tau \in (0,1).$$

And accordingly,

$$(2.18) \qquad u(\tau) \leq \frac{1}{2}.$$

Therefore, from (2.17) and (2.18), we have

$$\lim_{\lambda\to\infty}||u||_\infty = \frac{1}{2}. \qquad\qquad \square$$

3. Proof of the main result

In this section, we shall prove Theorem 1.1. We divide the proof into three steps.

Step 1. Rightward bifurcation.

By Lemmas 2.4-2.6, there exists an unbounded connected component $\mathcal{C}$ in the set of positive solutions of (1.3), which is bifurcating from $(\frac{\lambda_1}{f_0}, 0)$, and for any $(\lambda, u) \in \mathcal{C}$ which satisfies $|\lambda - \frac{\lambda_1}{f_0}| + ||u|| \leq \sigma$, where $\sigma > 0$ is a sufficiently small constant, $\mathcal{C}$ goes rightward. Moreover, $\mathcal{C}$ joins $(\frac{\lambda_1}{f_0}, 0)$ with infinity in λ direction.

Step 2. Direction turn of bifurcation.

By Lemma 2.10, we have $\lim\limits_{(\lambda,u)\in\mathcal{C},\lambda\to\infty}||u|| = 1$ and $\lim\limits_{(\lambda,u)\in\mathcal{C},\lambda\to\infty}||u||_\infty = 1/2$. Then there exists $(\lambda_0, u_0) \in \mathcal{C}$ such that $||u_0||_\infty = 4s_0$. Lemma 2.8 implies that $\mathcal{C}$ goes leftward.

Step 3. Existence of λ_* and λ^*.

By Lemma 2.7, if $(\lambda, u) \in \mathcal{C}$, then there exists $\lambda_\diamond > 0$ such that $\lambda \geq \lambda_\diamond$. And by Lemma 2.6, Lemma 2.8 and Lemma 2.10 again, $\mathcal{C}$ passes through some points $(\frac{\lambda_1}{f_0}, v_1)$ and $(\frac{\lambda_1}{f_0}, v_2)$ with $||v_1||_\infty < 4s_0 < ||v_2||_\infty$, and there exist $\underline{\lambda}$ and $\overline{\lambda}$ which satisfy $0 < \underline{\lambda} < \frac{\lambda_1}{f_0} < \overline{\lambda}$ and both (i) and (ii):

(i) if $\lambda \in (\frac{\lambda_1}{f_0}, \overline{\lambda}]$, then there exist u and v such that $(\lambda, u), (\lambda, v) \in \mathcal{C}$ and $||u||_\infty < ||v||_\infty < 4s_0$;

(ii) if $\lambda \in [\underline{\lambda}, \frac{\lambda_1}{f_0}]$, then there exist u and v such that $(\lambda, u), (\lambda, v) \in \mathcal{C}$ and $\|u\|_\infty < 4s_0 < \|v\|_\infty$.

Define $\lambda^* = \sup\{\bar{\lambda} : \bar{\lambda} \text{ satisfies (i)}\}$ and $\lambda_* = \inf\{\underline{\lambda} : \underline{\lambda} \text{ satisfies (ii)}\}$. Then by the standard argument, (1.3) has a positive solution at $\lambda = \lambda_*$ and $\lambda = \lambda^*$, respectively. This completes the proof. $\qquad\square$

4. Further results and remarks

In this section, we show the existence of the S-shaped connected component in the set of positive solutions of (1.3), when $f_0 = \infty$.

Theorem 4.1. *Assume* (A1), (F1),

(F4) $f_0 = \lim\limits_{s \to 0^+} \frac{f(s)}{s} = \infty$,

(F5) *there exists $m_0 \in (0, \frac{1}{32})$ such that $0 \le s \le m_0$ implies that*

$$f(s) \le \frac{6}{a^*} m_0,$$

where $a^* = \max\limits_{s \in [0,1]} a(s)$,

(F6) *there exists $m^0 > m_0$ with $m^0 \in (\frac{1}{32}, \frac{1}{12})$, such that*

$$\min_{s \in [m^0, 4m^0]} \frac{f(s)}{s} \ge \frac{27}{5\sqrt{5}a_0}(2\pi)^2.$$

Then there exist $0 < \lambda_{**} < 1$ and $\lambda^{**} > 1$ such that

(i) *(1.3) has at least one positive solution if $0 < \lambda < \lambda_{**}$;*

(ii) *(1.3) has at least two positive solutions if $\lambda = \lambda_{**}$;*

(iii) *(1.3) has at least three positive solutions if $\lambda_{**} < \lambda < \lambda^{**}$;*

(iv) *(1.3) has at least two positive solutions if $\lambda = \lambda^{**}$;*

(v) *(1.3) has at least one positive solution if $\lambda > \lambda^{**}$;*

(vi) $\lim\limits_{\lambda \to \infty} \|u\|_\infty = \frac{1}{2}$ *and* $\lim\limits_{\lambda \to \infty} \|u\| = 1$.

Proof. (*Sketched*) We will use the similar argument of [18] and [19] to get the desired result. For each $j \in \mathbb{N}$, let us define a function $f^{[j]} : [0, \infty) \to \mathbb{R}$ by

(4.1)
$$f^{[j]}(s) = \begin{cases} f(s), & s \in \left(\frac{1}{j}, \infty\right), \\ jf\left(\frac{1}{j}\right)s, & s \in \left[0, \frac{1}{j}\right]. \end{cases}$$

Then for each $j \in \mathbb{N}$, $f^{[j]}$ are continuous functions, with

(4.2)
$$\limsup_{j \to \infty}\left[f^{[j]}(s) - f(s)\right] = 0 \quad \text{uniformly for } s \in [0, \infty),$$

and

(4.3)
$$(f^{[j]})_0 = \lim_{s \to 0} \frac{f^{[j]}(s)}{s} = jf\left(\frac{1}{j}\right).$$

By the same method to prove Lemma 2.4, with obvious changes, we get that for each $j \in \mathbb{N}$, the closure of the set of positive solutions for the auxiliary problem

$$(4.4)_j \qquad \begin{cases} -u'' = \lambda a(x) f^{[j]}(u) h(u'), & x \in (0,1), \\ u(0) = u(1) = 0 \end{cases}$$

possesses a connected component $\mathcal{C}^{[j]}$ which joins $\left(\frac{\lambda_1}{jf(\frac{1}{j})}, 0\right)$ to infinity in λ-direction.

From (F4), it follows that

$$(4.5) \qquad \lim_{j \to \infty} (f^{[j]})_0 = \lim_{j \to \infty} \frac{f(1/j)}{1/j} = \infty,$$

and accordingly,

$$\lim_{j \to \infty} \frac{\lambda_1}{jf(\frac{1}{j})} = 0.$$

According to [18, Lemma 2.2] and [19, Lemma 2.4], the set $\limsup \mathcal{C}^{[j]}$ contains an unbounded connected component $\mathcal{C}$:

$$(4.6) \qquad (0,0) \in \mathcal{C} \subset \limsup_{n \to \infty} \mathcal{C}^{[j]}$$

which joins $(0,0)$ to infinity.

Therefore, by an argument similar to that of Theorem 1.1 together with the following Lemma 4.1 and Lemma 4.2, we can get the results of Theorem 4.1. $\qquad\square$

Lemma 4.1. *Assume that* (A1), (F1) *and* (F5) *hold. Let* $(\lambda, u) \in \mathcal{C}$ *with* $||u||_\infty = m_0$. *Then* $\lambda > 1$.

Proof. From the property of the Green's function $G(x,s)$, we have

$$\begin{aligned} m_0 = ||u||_\infty &= \max_{x \in [0,1]} \lambda \int_0^1 G(x,s) a(s) f(u(s)) h(u'(s)) ds \\ &< \lambda a^* \frac{6}{a^*} m_0 \int_0^1 G(s,s) ds \\ &= \lambda m_0, \end{aligned}$$

therefore, $\lambda > 1$. $\qquad\square$

Lemma 4.2. *Assume that* (A1), (F1), (F5) *and* (F6) *hold. Let* $(\lambda, u) \in \mathcal{C}$ *with* $||u||_\infty = 4m^0$. *Then* $\lambda < 1$.

Proof. Similar to Lemma 2.2 and Corollary 2.1, we have

$$m^0 = \frac{1}{4} ||u||_\infty \le u(x) \le ||u||_\infty = 4m^0, \ x \in \left[\frac{1}{4}, \frac{3}{4}\right],$$

and

$$0 \le |u'(x)| \le \frac{2}{3}, \ x \in \left[\frac{1}{4}, \frac{3}{4}\right].$$

Assume on the contrary that $\lambda \geq 1$, similar to the proof of Lemma 2.8, we can get a contradiction, and subsequently, $\lambda < 1$. $\qquad\square$

Remark 4.1. If we use the stronger condition

(A_s) $a : [0,1] \to [0,\infty)$ is continuous, $a(x) > 0$ in $(0,1)$, and $a(x) = a(1-x)$ for all $x \in [0,1]$, and work in the smaller Banach space

$$E^* = \{u \in C^1[0,1] : u(0) = u(1) = 0 \text{ and } u(x) = u(1-x) \text{ for all } x \in [0,1]\},$$

then by using the same method to prove Theorem 1.1 and similar argument to prove [6, Theorem 3.2], we may prove the existence of symmetric positive solutions of (1.3), although we don't know the symmetry of all solutions of (1.3).

Remark 4.2. From Lemma 2.3, in the case $\nu = \frac{1}{3}$ and $\beta_0 = \frac{1}{24}$, we have to use the assumption (F3). By adjusting the parameters ν and β_0 in Lemma 2.3, we may obtain some new conditions. For example, when $\nu = \frac{1}{4}$ and $\beta_0 = \frac{3}{64}$, we may replace (F3) by

(F3") there exists $s_0 : s_0 \in (\frac{1}{32}, \frac{1}{16})$, such that

$$\min_{s \in [s_0, 4s_0]} \frac{f(s)}{s} \geq \frac{64 f_0}{7\sqrt{7} \lambda_1 a_0} \cdot (2\pi)^2,$$

where $a_0 = \min\limits_{s \in [\frac{1}{4}, \frac{3}{4}]} a(s)$.

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