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The Effects of Visual Representations on Learning Proportional Expressions and Distributions

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The purpose of this study is to provide a method to help elementary school students learn ratio-related concepts effectively through visual representations. This study was conducted to identify the differences in the composition of ratio-related concepts between Korean and Singaporean textbooks, reconstruct a unit of proportional expressions and distributions by using visual representations and confirm the differences in performance between an experimental and a comparison group of 6th grade students. While the experimental group mathematics lessons is from the reconstructed textbook, the comparison group lessons is from an existing textbook that does not include any reconstructive representations. A t-test of mean was applied to determine the differences between the experimental and comparison group. Analysis revealed significant differences in the mean between the experimental group and the comparison group, and the intermediate level group showed more improvement compared to the higher and lower level groups. An implication of this study is that the application of visual representations can assist students' understanding of ratio-related concepts.

I. Introduction

1. The Rationale and Purpose of the Research

It is important for students to have the ability to judge and compare the authenticity of information in order to fulfill their role as future members of society. Mathematics can be used as a framework to view the world (Ministry of Education, 2015). To use various values and information, particularly in real-life contexts, it is necessary to compare and interpret conflicting mathematical elements by applying specific criteria. For example, if the standard is ratio-related concepts, such as ratio, rate, and proportion, then it is important to learn these mathematical concepts, especially since proportional relationships are discovered in numerous real world patterns and are fundamental to learning higher-level mathematics. Although the meaning of ratio in elementary school curricula is an academic pursuit, it can also be linked to real-life problems. Lesh (1987) argues that proportional reasoning ability develops through the learning of ratio-related concepts and it is an essential element of many important mathematical concepts, such as quotients and fractions that students have studied in previous years. In addition, Lash suggests that proportional reasoning, as a form of mathematical reasoning influences content that students study while learning higher-level mathematics. In practical terms, Kim and Na (2008) insists that students must have the ability to interpret the meaning of ratio-related ideas by recognizing their concepts meaningfully through everyday materials.

Despite the importance of these concepts, they

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are complicated, hard to teach, and according to students, the most difficult ideas to understand in all areas of elementary mathematics (Smith, 2002). The degree of students' understanding of ratio-related concepts is revealed as insufficient; likewise, the achievement level on the unit ratio and proportion is the lowest among 6th grade units (Karplus, Pulos, & Stage, 1983; Noelting, 1980; Park & Pang, 2008; Park, Park, & Kim, 2011; Streetfland, 1985, Yu & Woo, 1995).

In this regard, Jeong (2003a) points out the following two reasons for the lack of high performance on ratio-related concepts. First, textbook presentation of ratio-related concepts is unique and abstract, which differs from other concepts that students have previously encountered in textbooks. Textbooks, however, deal with the external features of the concepts, demonstrate them as superficial, and intensively focus on algorithmic or procedural approaches, rather than conceptual approaches. A second problem involves the classroom setting; teachers often treat ratio-related concepts in the class in meaningless ways. As a result, this study focused on the problem of textbooks, rather than the classroom setting.

There have been many studies on the composition of textbooks regarding the effective teaching and learning processes of ratio-related concepts. The widely accepted method from many researchers is to utilize visual representation. Learning ratio-related concepts has to begin from concrete contexts through many types of representations that progress to processes of abstraction because these concepts are unfamiliar and abstract to students (Charles & Lester, 1982; Kim, 2011; Lee & Park, 2002; Shin, 2005). In addition, there is a variety of research attempting to verify the necessities of using visual representations in ratio-related units and apply them to judge their effectiveness in teaching and learning ratio-related concepts. However, these studies are limited to problem solving situations, which do not include classroom situations. Furthermore, they primarily concentrated on using visual representations completed by students in class, and then they analyzed students' classwork (Kim, 2007; Kim & Paik, 2010; Shin, 2005).

This study applied visual representations to a ratio-related unit (i.e. the proportional expression and distribution) with the following three steps. First, the units that deal with the concept of ratio and proportion among the 2009-revised Korean mathematics textbooks were compared and analyzed with Singaporean textbooks, which are well known for the appropriate and plentiful usage of visual representations. Next, a ratio-related unit from Korean textbooks was reconstructed based on a visual representation using implications from the previous comparison of Korean and Singaporean textbooks. Finally, the reconstructed unit was applied in an actual classroom to verify its effectiveness. Thus, this study should include an appropriate presentation of visual representations regarding ratio-related units in the newly developed 2015-revised Korean mathematics curriculum and textbooks.

2. Research Questions

The purpose of this study is to analyze the content system and visual representation of a proportional expression and distribution unit of a 2009-revised Korean elementary mathematics textbook. Based on this analysis, a unit is reconstructed and the results of applying the reconstructed textbook unit on students' learning are examined in a 6th grade classroom. For the purposes of this study, the following research questions were addressed.

1) What are the differences in visual representations between Korean and Singaporean

mathematics textbooks?

2) What procedures are needed to reconstruct the unit of proportional expressions and distributions in the 6th grade unit of the 2009 revised Korean mathematics textbook using visual representations?

3) Are there differences in learning between the experiment group using the reconstructed textbook and the comparison group using the non-revised existing textbook?

II. Review of Relevant Literature

This section explores relevant literature in mind two areas. First, the term representation is examined regarding the variety of meanings and interpretations, which mostly depend on the nature of the research investigation. Second, the types of visual representations are surveyed, considering their particular significance in developing students' mathematical understanding.

1. Representation

The expression representation can be defined and interpreted in a variety of ways, according to scholars in education. Mandler (1983) classified representation into two categories: an external representation expressing individual feelings and thoughts, and an internal representation as a system of inwardly formed knowledge. Goldin and Kaput (1996) and Yee and Bostic (2014) also categorized representation into two respective domains: symbolic and non-symbolic. The symbolic representation can be simply described as abstract, containing such forms as expressions, equations, and inequalities. The non-symbolic representation is all other forms of representations including diagrams, tables, graphs, pictures, and even concrete models.

In Korea, many contemplated the term's range because representation had not been precisely and accurately matched one-to-one when being translated from English into Korean. Furthermore, Kim and Chang (1996) pointed out that because the term's range was too broad to discuss, it was not convenient to share the same idea from one to another. For these reasons, they gave shape to it by quoting Janvier (1987) and Glasersfeld (1987). According to these researchers, the external representation refers to a physical object, such as a specific figure or symbol, and it is classified as pyohyon in Korean. The internal representation refers to an image or an idea, which is the spiritual reality and is classified as pyosang in Korean. Due to the fact that this study focuses on the visual representation, which is non-symbolic and external, this study was conducted by concentrating solely on pyohyon.

2. Types of Visual Representations

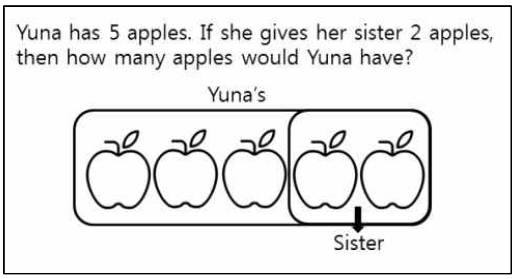
Skemp (1986) categorized visual representations into four types, according to problem situations: paintings, diagrams, graphs, and drawings. Similar to Skemp's categorization, Kang, Park, & Lew (1991) also had four types, which were closely related to the constructing strategy of a visual representation. Cho (2000) classifies visual representations themselves by their features and names, such as scenery, area, number line, and so on. Park's (2003) classification is similar to Cho (2000); however, there is a difference that includes tape diagrams. The types and composition strategies of the visual representations mentioned here are shown in Table 1.

[Table 1] Types and Strategies of Visual Representations (Jeon, 2005)

Kang (1991)	Skemp (1986)	Cho (2000)	Park (2003)
Draw a Figure Expression	Present Problem Situation as Picture	Scenery	Scenery
Draw Figures Sequentially		Area	Area
		Number Line	Number Line
		Tile	Tile
Represent the relation between components of the given problem		Segment	Segment
	Diagram	Venn Diagram	Venn Diagram
			Tape
	Graph		
		Structure (Tree)	Structure
Draw a Tree Diagram	Tree Diagram		

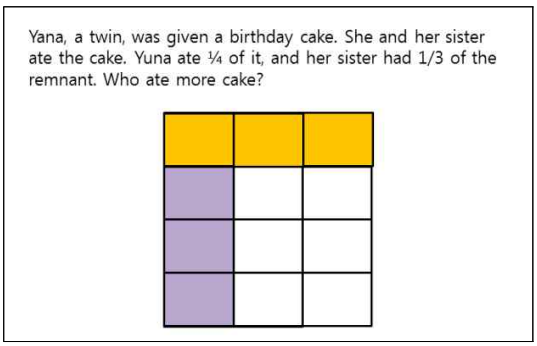
Yim and Lee (2015) discussed two types of visual representations that help develop proportional reasoning: dual number line and dual tape diagram. A dual number line is the response diagram in which corresponding numbers are above and below the number lines. A dual number line can represent relationships within a measurement-space and relationships between two measurement-spaces as well. If the conditions of upper line are doubled then the conditions of the lower line will be doubled as well. This helps students recognize the covariate relationship that both volumes are increasing and decreasing at the same time. On the other hand, a dual tape diagram is a visual representation with different characteristics from a dual number line. A dual tape diagram has tapes that are located above and below. The tapes are separated into portions (small rectangles) that are the same size both above and below. While the dual number line is an advantage in recognizing the dependent relationship of different variables, the dual tape is an advantage in comparing two quantities of the same kind to each other. In this regard, the dual tape has the benefit of presenting a variable partial perspective.

In this study, the visual representations were divided into 11 types, focusing on the classification of Jeong's (2014) results. The 11 types are presented in Figures 1 through 11.



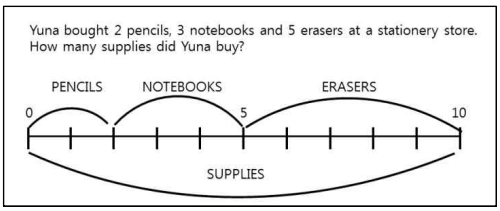
[Fig. 1] Scenery Diagram

Scenery diagram is a basic visual representation which helps the understanding of problems by translating information and relationships in problem contexts to more concrete.



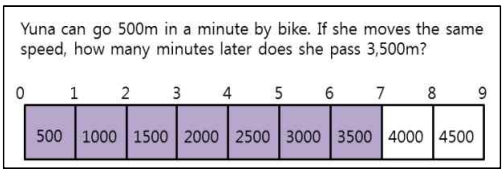
[Fig. 2] Area Diagram

Area diagram is a visual representation which is applied when transforming given numbers to rectangle shapes in order to compare their quantities. It is normally used in the comparison of fraction.



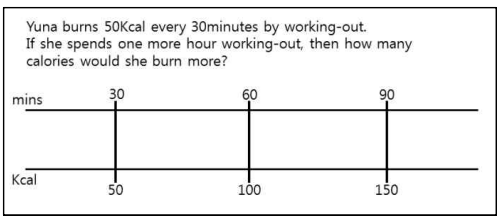
[Fig. 3] Number Line Diagram

Number line diagram is a representation which is divided straight line by a constant interval. It is an advantageous way to intuitively perceive the volume as a visual representation. Also it is useful for addition, subtraction, and even division by moving numbers. This representation is typical model to learn proportional distributions.



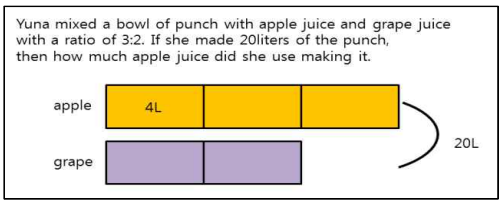
[Fig. 4] Tape Diagram

Tape diagram is a visual representation that divides a long tape strip and shades it to compare amounts.



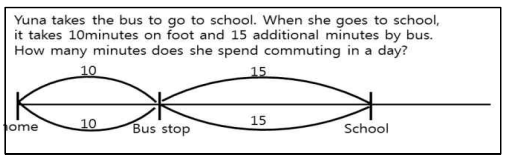
[Fig. 5] Dual Number Line

Dual number line is a visual representation in which corresponding number are located above and below the vertical lines. It may be different from the upper and lower line scale, however, both volumes have to increase and decrease at the same time. Students are able to explore proportion by manipulating numbers written on the lines and easily figure out their homogeneous.



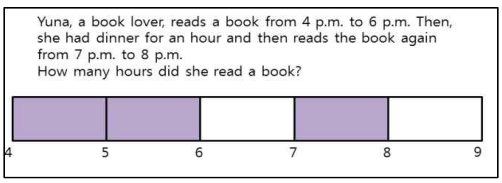
[Fig. 6] Dual Tape Diagram

Dual tape diagram is a representation presented to allow comparison of two or more quantities by presenting the same size tape on the top and bottom.



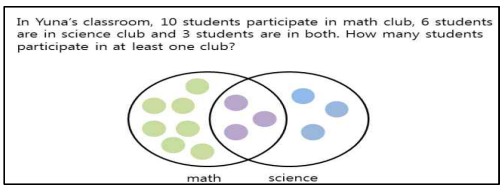
[Fig. 7] Segment Diagram

Segment diagram is a visual representation which helps to understand the problem by showing information and relationships included in the problem situations as segment, but it is not as accurate as number line diagram.



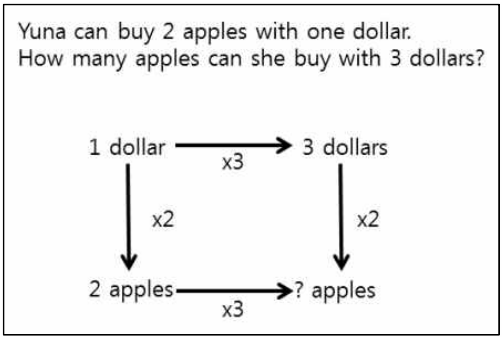
[Fig. 8] Tile Diagram

Tile diagram is a visual representation that enables intuitive reasoning using the shape of boxes with shaded rectangles. Unlike the tape diagram, tile diagram does not need to start from 0.



[Fig. 9] Venn Diagram

Venn diagram is a representation that lists related data and helps to identify the interrelationship or dependency of the data. It is not often used in elementary mathematics education.



[Fig. 10] Semantic Structure Diagram

Semantic Structure is a visual representation that is created to understand the meaning of problems by using necessary symbolic expressions such as arrows, segments, and operators. This can be useful because it exposes hidden operations between relations when solving multiplication or proportional expressions.

Yuna's brother are 2 years older than her. Yuna is 10 years old now.
When is the sum of Yuna and her brother's age greater than 30?

	THIS YEAR	1 YEAR	2 YEARS	3 YEARS	4 YEARS
YUNA	10	11	12	13	14
BRO	12	13	14	15	16
SUM	22	24	26	28	30

[Fig. 11] Re-statement

Re-Statement is a representation which is typically referred as table. This is widely applied in math education in order to handle raw data and organize them in aligned forms.

III. Methods

The methods for this study are implemented in several phases, a literary study and then a study for developing a unit consisting of visual representations, which is followed by an experimental study. These phases include the analysis of two different textbooks to examine the presentation of visual representations, using results from the analysis to reconstruct a ratio-related unit that would include visual representations, and finally applying the reconstructed unit in a 6th grade classroom. These phases are described in the following sections.

1. A Literary Study and Study for Developing a Unit

1.1. Research Objects

Research Problem 1 was executed as a literary study. The units of ratio and proportion and proportional expression and distribution in the 2009-revised Korean textbooks were analyzed by focusing on their use of visual representations (Ministry of Education, 2015). The subject of comparative analysis was the Singaporean textbook, which has been and continues to be widely known for its large quantity of visual representations. Previous studies evaluating elementary school Korean and Singaporean mathematics textbooks were compared using three out of four types of textbooks in Singapore (Choi et al., 2006; Kim & Paik, 2010). In particular, units related to the proportion of My Pals Are Here (Fong, Gan, & Chelvi, 2009), the most widely used units in Singapore, were selected as the study target.

Research Problem 2 was conducted based on the results from the analysis of Research Problem 1, as a unit development study of proportional expressions and distributions.

1.2. Analysis of Units

In analyzing the units related to proportion in the 2009-revised Korean and Singapore mathematics textbooks, the followings analysis criteria were applied.

(1) How many visual representations are used in each unit?

(2) What are the visual representations used in each unit?

(3) What is the most frequently used visual representation?

The analysis frame of the unit is shown in Table 2.

[Table 2] Analysis Frame of Unit

Standard	Analysis Unit	Analysis Method
Page Composition	Page	Average (Percentage)
Weight in Units		

The framework for analysis of the visual representation was constructed through Park (2003), Jeong (2014), Yim and Lee (2015), and Jeon (2005). This is shown in Table 3.

[Table 3] Framework for Analysis of Visual Representations

Standard	Content	Analysis Unit	Analysis Method
Types of Representations	Scenery	Number of Times	Weight (Percentage)
	Number Line		
	Dual Number Line		
	Tape		
	Dual Tape		
	Segment		
	Tile		
	Venn Diagram		
	Area		
	Semantic Structure		
	Re-statement		

1.3. Reconstruction of the Unit

The sequence of unit reconstruction using visual representations is shown in Table 4. When reconstructing the unit, the order and content of the 2009 revised mathematics textbooks were maintained. In addition, visual representations were added or replaced in learning materials and methods.

[Table 4] Sequence of Unit Reconstruction

Analysis	2009 Revised Mathematics Textbooks (KOR)	→	Drawing Implications	→ Reconstruction of the Unit
	Singapore Textbooks			
	The Preceding Studies	→	Representation Extraction	

2. An Experimental Study

2.1. Research Objects

The subjects were two randomly selected 6th grade elementary school classes located in Gangwon province, South Korea. The classes consisted of 30 students of average academic achievement from middle-class families.

2.2. Research Design

The purpose of the experimental study is to show how the use of reconstructed textbooks using visual representations affects the learning of proportional expressions and distributions. The experimental study was a nonequivalent control group design, which is shown in Table 5.

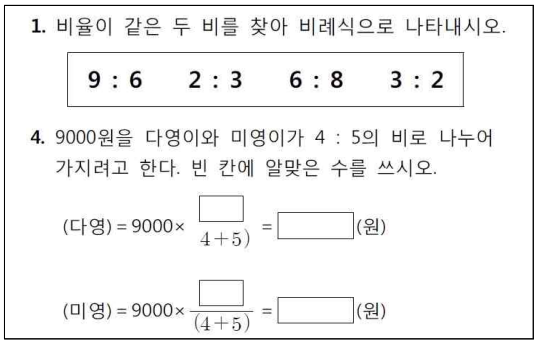
[Table 5] Design of the Experimental Study

Group	Pre-Test	Experimental Treatment	Post-Test
Experimental	Test A (Ratio and Proportion)	Apply Reconstructed Textbook	Test B (Proportional Expression and Distribution)
Comparison		Apply Existing Textbook	

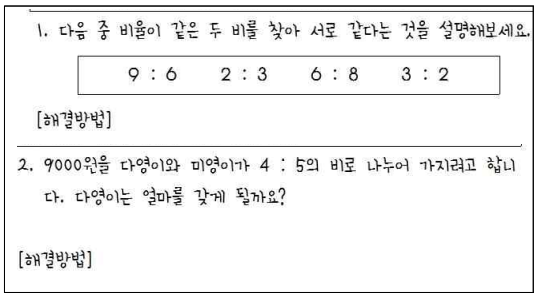
2.3. Inspection Tools

The tests used in this study were pre-test and post-test. The pre-test consisted of problems related to ratio and proportion previously studied. The post-test consisted of the problems related to the newly studied content of proportional expressions and distributions. The inspection tools were based on assessment papers related to ratio, proportion, and proportional expression and distribution registered with the Korea Education and Research Information Service (KERIS). All of the assessment items were given in sentences. Conversion to sentence form was verified by mathematics education experts and experienced teachers. Preliminary tests were also conducted to confirm the appropriateness of the questions or items. The example of inspection tools is

shown in Figure 12 and 13 respectively.



[Fig. 12] Original Inspection Tools



[Fig. 13] Edited Inspection Tools

2.4. Experimental Treatment and Analysis

The experimental treatment of this study was to teach the two groups by using a different textbook unit. In order to minimize influence of the various classroom management methods on the teaching and learning process, the same explanatory instruction model was applied to both the experimental group and the comparison group. Also, before the experiment, the teachers of the two groups were consulted in the teaching and learning process presented in the guidebook in order to control the methods and procedures of the lessons and the level and scope of questioning.

The experimental treatment was applied from October 10th to 28th in 2016. The treatment was executed on the same day of two groups due to control variable. The schedule of treatment is shown in 6.

[Table 6] Treatment Applied Date and Class

Date	Experimental	Comparison
Oct. 10. 2016	3rd	5th
Oct. 12. 2016	5th	2nd
Oct. 14. 2016	4th	4th
Oct. 17. 2016	3rd	5th
Oct. 19. 2016	5th	2nd
Oct. 21. 2016	4th	4th
Oct. 24. 2016	3rd	5th
Oct. 26. 2016	5th	2nd

The analysis of the experimental results was executed to confirm achievement, and the test results between the experimental and comparison group were analyzed using t-tested of mean. In addition, the Mann-Whitney U test was performed to determine the difference between the achievement levels of the two groups.

IV. Findings

1. The Comparison and Analysis of Korean and Singaporean Textbooks

1.1. Unit Configuration

Korean and Singaporean textbooks were analyzed, and the percentage of pages devoted to the units related to ratio and proportion in the textbooks and the content in each unit are summarized in Table 7 and 8. As illustrated in these two tables, the Korean textbooks offered two units of study in which ratio and proportion is complied as one unit and proportional expressions and distribution was the second unit with less slightly less emphasis, 16% and 21% respectively. In contrast, the Singaporean textbooks considered these concepts as four different units with more emphasis placed on ratio and percentage.

[Table 7] Analysis of Unit Configuration in Korean Textbooks Results

	Unit	Contents	Pages and Percentage
6-1	Ratio and Proportion	-The Meaning of Ratio -Reference Amount -Comparative Amount	P.94-135 (42p, 21%)
6-2	Proportional Expression and Distribution	-Proportional Expression -The Nature of Proportion -Proportional Distribution	P. 34-65 (32p, 16%)

[Table 8] Analysis of Unit Configuration in Singapore Textbooks Results

	Unit	Contents	Pages and Percentage
5-A	Ratio	-Meaning of Ratio -Proportional Expression	p.148-178 (31p, 17%)
5-B	Percentage	-Percent -Proportion	p. 64-91 (28p, 15%)
6-A	Ratio	-Ratio -The Value of Ratio -The Equivalence of Ratios	p.101-140 (40p, 22%)
6-A	Percentage	-Various Ratios -Percentage Utilization	p.141-178 (28p, 21%)

1.2. Analysis of Visual Representations

The visual representations of Korean and Singaporean textbooks were confirmed and classified based on the criteria presented in Tables 9 and 10. According to the results, the frequency of applying a visual representation was confirmed. As shown in these two tables, the Korean textbooks contained six types with 38 representations, whereas the Singaporean textbooks contained seven types with 152 representations. Also, Table 9 shows that re-statement is the most frequently used representation in the Korean textbooks. In contrast, as shown in Table 10, dual-tape is the most frequently representation in the Singaporean textbooks. Another interesting element about the various representations is that in the Singaporean textbook, they are more uniform. This is not the case for the Korean textbook, whereby one representation dominates all others displayed in the text.

[Table 9] Analysis of Visual Representation in Korean Textbooks Results

Summary	Types	Number of Times	Percentage(%)
6 Types 38 Representations	Scenery	8	21.1
	Area	1	2.6
	Segment	2	5.2
	Tape	3	7.9
	Re-statement	23	60.6
	Semantic Structure	1	2.6

[Table 10] Analysis of Visual Representation in Singapore Textbooks Results

Summary	Types	Number of Times	Percentage (%)
7 Types 152 Representations	Scenery	15	9.9
	Area	7	4.6
	Number Line	3	2.0
	Tape	21	14.5
	Dual Tape	67	44.0
	Semantic Structure	27	17.8
	Re-statement	11	7.2

2. Reconstruction of Unit

2.1. Reconstruction Focus and Method

The emphases of the reconstruction of the proportional expression and distribution unit of 2009-revised mathematics textbook are as follows.

· Maintain the material and storytelling of Korean mathematics textbooks. In this study, concepts presented and the activity promising are the same as the existing ones.

· The existing framework of the activities is maintained, however, the content that is not related to the storytelling element is converted into other content.

· The activities are integrated and displayed as

visual representations in the margins.

· Checking student comprehension with closing problems is maintained, but the problems' form or the teachers' approach to the problem can be changed.

· Reduce the use of re-statement diagrams.

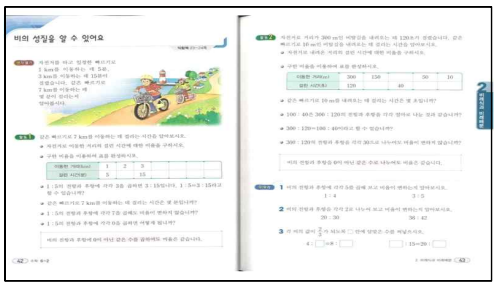
· Utilizing previously unused visual representations, such as dual tape, dual number line and semantic structure diagram.

· Use two or more visual representations per lesson.

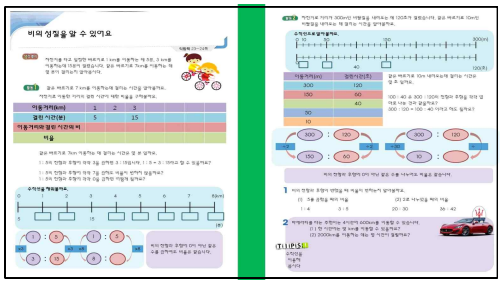
· Provide at least two visual representations for each activity.

2.2. Actual Reconstruction

An example from the existing textbook and the reconstructed textbook are shown in Figures 14 and 15, respectively.



[Fig. 14] Example of Existing Unit



[Fig. 15] Example of Reconstructed Unit

2.3. Revision of Re-statement

In the existing, Activity 1, there were re-statements and questions to fill a table using the given conditions. The re-statements were reconstructed, so that ratio and proportion could be obtained intuitively. Also, Activity 2 was modified with the intention to express the author's view of the ratio's co-variation.

2.4. Introduction of Number Line and Dual Number Line Diagrams

Since the situation given in the first half of the lesson, including the introduction, was about time and distance, a number line diagram was used. Activity 2 utilized a dual number line diagram to compare the external ratios.

2.5. Introduction of Semantic Structure

In order to visually confirm the equivalence of ratio and to estimate the magnitude of the change, the longitudinal proportional module, which is a semantic structure, is applied to both multiplication and division.

3. Analysis of Experimental Results

3.1 Analysis of Test Result

In the test, the independent samples t-test was conducted using the scores of the post-test as the dependent variable and the experimental treatment as the parameter.

[Table 11] The Result of Post-test T-test

Group	N	M	SD	df	t	p
Experimental	30	9.03	2.414	58	2.261	0.031
Comparison	30	7.70	2.261			

Since the significance level of the test result is 0.031, which is less than the significance level 0.05, the mean of the two groups is not the same at the significance level of 5%, hence the effect has occurred.

4.1. Analysis of differences between groups by level

The Mann-Whitney U-test was performed

because it was not possible to obtain regularity due to the small number of students in the subgroup.

[Table 12] The Result of Mann-Whitney U test the Higher Groups

Group	N	Ranks	Probability of Approximate Significance
Experimental	9	12.11	.027
Comparison	9	6.89	

[Table 13] The Result of Mann-Whitney U test the Intermediate Groups

Group	N	Ranks	Probability of Approximate Significance
Experimental	9	20.62	.007
Comparison	9	11.83	

The result of the Mann-Whitney U-test showed that for the higher-level students in the two groups, the probability of approximate significance was .027, the average ranking of the experimental group was 12.11, and the average ranking of the comparative group was 6.89. The results for the intermediate level students in the two groups, the probability of approximate significance was .007, the average ranking of the experimental group was 20.62, and the average ranking of the comparative group was 11.83. Hence, the effect of the experimental treatment was larger for the intermediate level students, rather than for the higher-level students.

V. Discussion

As a result of the comparative analysis of ratio-related units of 2009-revised Korean mathematics textbooks and Singaporean mathematics textbooks based on visual representations, there were 38 visual representations of 6 different kinds in 85 ratio related pages in Korean textbooks compared to 152 visual representations of 7 different kinds in 127 pages used in Singaporean textbooks. It was found that the number and types of visual representations used in Singaporean textbooks were much higher than Korean textbooks. In addition, among the visual representations of Korean textbooks, the re-statement diagram occupies an excessively large proportion (60.6%). Singaporean textbooks, despite using dual tape diagrams in which 44.0% of the textbooks managed to utilize diverse visual representations more consistently than Korean textbooks, which confirms that the use of visual representations was more uniform in Singaporean textbooks. This result was not surprising, because the Singaporean mathematics curriculum is closely connected to Bruner's (1964) theory of conceptual development that implies a clear progression of mathematics instruction. Bruner emphasizes that mathematical structures can be developed by learners when provided experiences that allow them to represent learning mathematics through appropriate concrete and pictorial examples, which ultimately helps them apply and understand mathematical abstractions.

Based on the results from the analysis of Research Problem 1, the unit of proportional expressions and distributions was reconstructed. In the reconstructed unit, 49 visual representations of 8 different kinds in 14 pages were used. In addition, the ratio of the restatement was lowered to 20.4%, while the ratio of semantic structure was increased to 41% and other visual representations were also used at an even rate.

Yim and Lee(2015) argued that dual number line and dual tape diagram had to apply more in order to enhance students' intuitive sense of proportion. According to this suggestion, 6 dual number lines and dual tape diagrams were presented in the revised unit (12.2%).

As a result of comparing the differences between how the experimental group was taught using the reconstructed textbooks and how the comparison group was taught using the existing textbooks, the level of significance in the unit achievement was 0.032. Thus, learning using the reconstructed

textbooks has a significant and positive impact on learning. Analysis revealed significant differences in the mean between the experimental group and the comparison group.

Also, the tendency of applying visual representations during problem solving was higher in experimental group. The most frequently used visual representations in this group were double number line and semantic structure. Students in this group preferred drawing representations to writing words in explanatory blanks. It seems that presenting representation on problem solving process clarifies problem contexts, helps to connect giving conditions, and even hinders mistakes. Furthermore, students used a few collection of representations repeatedly. It shows that students prefer a few specific visual representations and the use of visual representation as a strategy to approach the problems not only shortens the time to approach but also helps to solve the problems efficiently.

Particularly, when it was analyzed by focusing on the differences between the groups by level, it could be confirmed that the intermediate level students showed the most improvement compared to students in the higher and lower level groups.

There was a difference between the higher groups also, however, the difference between the intermediate levels was more dramatic. While the students in the intermediate experimental group used the visual representations in all of the problems, the higher experimental group used visual representation as one of the tools to identify their answers. The students in the higher level group have already formed abstract thinking about proportionality and seem to have strategies to solve problems already. On the other hand, the students of the intermediate group are forming the abstract concepts of the proportional expressions and distributions, and it can be considered that the insertion of the process of visually converting the abstraction of proportional expression helps this group to make sure what they are doing in the process. In addition, Seo, Pang, & Lee (2017) claims that it is important to know the multiplicative covariance of each quantity and the multiplicative constant between the two quantities in proportional reasoning. The visual representations are advantageous to showing the multiplicative relationships which are implied in the problem contexts. It might have helped the intermediate group students to consider the relationship and to obtain more improvement.

Jeong(2003a) also insists that ratio-related concepts have been understood superficially and thought focusing on its algorithmic aspects. It explains students' lack of high achievement in ratio-related concepts. By reflecting the idea, the result of this study, indicating higher achievement in the intermediate experimental group who have more opportunities to explore an intuitive and conceptual way of having many experiences and then introducing symbols or mechanical methods, supports and is consistent with Jeong's claim that it is necessary to learn mathematics more meaningfully and thus provide appropriate visual representations, which should be explored and developed by students.

VI. Conclusion

The results of this study show that a visual representation has a positive effect on students' performance when learning about ratio related concepts. However, current textbooks are not only lacking in the use of visual representations, but they also only present one specific representation repeatedly. Therefore, further research is needed to provide evidence that textbooks that utilize rich and diverse visual representations influence student learning, especially for ratio-related concepts. Although the use of textbooks with visual representations may lead to significant differences in

students' learning of proportional expressions and distributions, the results of this experiment are only a subset of the various learning topics associated with ratio, and a subset of the entire mathematics content. Therefore, it is necessary to study the entire elementary school textbook by widening the analysis and application of units to focus on visual representations.

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시각적 표현이 비례식과 비례배분 학습에 미치는 효과

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본 연구는 시각적 표현의 활용이 비례식과 비례배분 학습에 미치는 영향을 탐구하고자 수행하였다. 이를 위해 시각적 표현에 집중하여 싱가포르 교과서와 한국 교과서의 '비례식과 비례배분'단원을 분석하고, 분석 결과를 바탕으로 한국 교과서를 시각적 표현을 중심으로 재구성하였으며, 재구성한 교재를 활용한 집단과 기존의 한국 교과서를 활용한 집단의 성취도 차이를 분석하였다. 먼저 두 교과서의 분석에서 한국 교과서는 총 85면에 6종 38개의 시각적 표현이 제시되어 있었고, 싱가포르 교과서에는 127면에 8종 152개의 시각적 표현이 활용되고 있었다. 이 결과를 바탕으로 한국 교과서를 재구성하였는데, 재구성한 단원에는 14면에 8종, 49개의 시각적 표현이 포함되었다. 마지막으로 재구성한 교재를 활용한 집단과 기존 교과서를 활용한 집단 간의 학습의 차이를 알아보기 위한 평가 후 결과를 독립표본 t-검정으로 분석한 결과 두 집단 간의 평균에 유의미한 차이를 확인할 수 있었다. 그룹간 성취 변화를 비교하고자 Mann-Whiteny 검사를 실시하였고, 상·중·하 수준별 집단 중 중위집단에서 가장 많은 효과가 있는 것으로 드러났다. 본 연구는 교과서의 시각적 표현을 분석하고 이를 통해 교재를 재구성, 현장에 투입함으로써 시각적 표현이 비례식과 비례배분의 학습에 긍정적 영향을 준다는 것을 밝혀냈다는 점에서 의의를 찾을 수 있다.

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