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A Practical Algorithm for Selective Harmonic Elimination in Five-Level Converters

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Abstract

Multilevel converters are being widely used in medium-voltage high-power applications including motor drive systems, utility power transmission, and distribution systems. Selective harmonic elimination (SHE) is a well-known modulation method to generate high quality output voltage waveforms. This paper presents a new simple practical method for generating a generalized five-level waveform without selected low order harmonics. This method is based on a phase-shifted expression for the SHE problem, which can analytically calculate the exact values of switching angles and the feasible modulation index range for three-level and five-level waveforms. The proposed method automatically determines the number of transitions between levels and generates proper output waveform without solving complex trigonometric equations. Due to the simplicity of the computational burden, the real-time implementation of the proposed algorithm can be performed by a simple processor. Simulation and experiment results verify the correctness and effectiveness of the proposed method.

Key words: Diode-clamped inverter, Modulation techniques, Multilevel converters, Selective harmonic elimination (SHE)

I. INTRODUCTION

Nowadays, multilevel converters are widely used in high-power medium-voltage applications such as motor drives [1], distributed generation systems [2], photovoltaic systems [3], etc. [4], [5]. Improved waveform quality of the output voltage, reduced dv/dt, better electromagnetic capability, and lower switching losses are the most attractive features of multilevel converters compared with traditional two-level converters. Three conventional types of multilevel converters have been reported in previous studies: diode-clamped, capacitor-clamped and cascaded H-bridge converters.

To improve the output quality of multilevel converters, different modulation techniques have been suggested [4], [5] such as sinusoidal pulse width modulation (SPWM) [6], [7] and space vector modulation (SVM) [8]. These techniques are not able to completely remove low order harmonics. Another approach is to choose switching angles in a way that low order harmonics are completely eliminated, which is called selective harmonic elimination (SHE) [9]-[11].

Using the Fourier series, the SHE problem is formulated in a set of nonlinear trigonometric equations. However, the main challenge is to obtain solutions for these equations of switching angles, which has been the focus of a lot of investigations [9]-[13]. A larger number of switching angles creates more degree of freedom but increases the mathematical complexity and makes solving the problem more difficult [9], [12].

Several methods have been proposed in the literature to solve SHE nonlinear equations. Most of them can be classified into three main groups: numerical iterative methods, evolutionary algorithms, and algebraic methods. Numerical iterative methods such as the Newton–Raphson [9] and Homotopy [14] algorithms may end in local optima. Therefore, the initial values must be well selected. In addition, iterative methods can usually find one set of solutions. Unlike the numerical methods, evolutionary algorithms such as the genetic algorithm (GA) [15], [16] and particle swarm optimization (PSO) [17], [18] can introduce optimum angles for infeasible modulation indices. However, they are sensitive to their initial values and may end in local optima like numerical methods. Algebra based methods such as the resultant [19] and Groebner bases theories [20] do not need to choose initial

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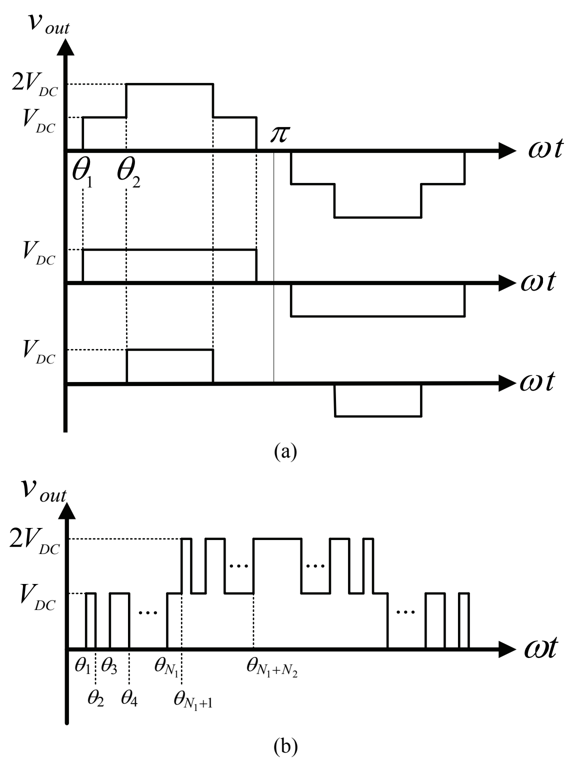

Fig 1. Output voltage: (a) Staircase five-level waveform; (b) Generalized five-level waveform.

values and can find all possible solutions. However, increasing the number of switching angles makes the computational burden very high.

The main drawback of the previously-mentioned techniques is the fact that they are computed off-line and stored in lookup tables, which are inherently discontinuous in nature. In addition, the angles are computed assuming pure sinusoidal waveform operation in the steady state. Hence, in dynamic operation for variable frequency and amplitude the angles are no longer optimal and low-order harmonics appear [21]. Consequently, finding a practical analytical algorithm for the SHE problem that can be implemented in real time is a challenging task.

In [22], an analytical method is proposed for only five-level waveform with two switching angles similar to Fig. 1(a). Therefore, this technique is not able to eliminate multiple harmonics. In addition, a new simple analytical method based on the phase-shifted expression is introduced in [23], [24] for both three-level and five-level waveforms with two switching angles.

In this paper, a new practical method for the elimination of multiple harmonics in five-level converters has been proposed. This method is based on the phased-shifted expression of the SHE problem. The proposed algorithm automatically determines the number of transitions between levels and is able to calculate the exact values of the switching angles if needed. There is no need to solve complex trigonometric equations, and real-time implementation of the proposed method is really simple and straightforward in compare to other methods.

II. Conventional SHE Problem in Five-Level Converters

Fig. 1(b) shows a generalized five-level waveform with N_1 transitions between zero and the first level, and N_2 transitions between the first and second level. In its simplest form, the output voltage has staircase waveform with only one transition between levels as shown in Fig. 1(a).

A. SHE Problem for Staircase Five-Level Waveform

Using Fourier series, the stepped multilevel voltage waveform of Fig. 1(a) can be expressed by (1):

$$v^{Out}(\omega t) = \sum_{n=1}^{\infty} V_n^{Out} \sin(n\omega t) \quad (1)$$

where n is the harmonic number, and V_n^{Out} represents the amplitude of the harmonic order n.

The amplitude for all of the even order harmonics is zero due to odd quarter wave symmetry. Since this waveform decomposes into two quasi-square waveforms, the amplitude of the odd order harmonics is calculated by (2).

$$V_n^{Out} = \frac{4V_{DC}}{n\pi}\left(\cos(n\theta_1) + \cos(n\theta_2)\right)\ ,\ n = 1,3,5,\ldots \quad (2)$$

Due to having two degrees of freedom, the objective of the SHE problem is to determine the switching angles (θ_1, θ_2) so that the expected amplitude of the fundamental harmonic is achieved and only one specified low order harmonic is eliminated [9], [22]. The SHE problem can be formulated by equation (3), where M is the modulation index defined by (4).

$$\begin{cases} \cos(\theta_1) + \cos(\theta_2) = 2M \\ \cos(n\theta_1) + \cos(n\theta_2) = 0 \end{cases} \qquad 0 \le \theta_1 \le \theta_2 \le \pi/2 \quad (3)$$

$$M = \frac{V_1^{Out}}{8V_{DC}/\pi} \qquad (0 \le M \le 1) \quad (4)$$

Finding an analytical solution for equation (3) is not simple. In [22], an analytical method has been introduced to solve equations (3) using *Chebyshev polynomials* and *Waring formulas* which is complex to understand.

B. SHE Problem for Generalized Five-Level Waveform

The amplitude of the odd order harmonics for generalized five-level waveform can be calculated by (5). In comparison with staircase waveform, there exists more degrees of freedom. The switching angles must be calculated so that the desired fundamental harmonic is attained and several of the harmonics are eliminated.

$$V_n^{Out} = \frac{4V_{DC}}{n\pi}\left(\sum_{i=1}^{N_1}(-1)^{i+1}\cos(n\theta_i) + \sum_{i=N_1+1}^{N_1+N_2}(-1)^{i}\cos(n\theta_i)\right) \quad (5)$$

Due to having $N = N_1 + N_2$ degrees of freedom, the maximum number of harmonics that can be eliminated is $N - 1$. Equations (6) and (7) describe the SHE problem for generalized five-level waveform [9].

$$\begin{cases} \sum_{i=1}^{N_1}(-1)^{i+1}\cos(\theta_i) + \sum_{i=N_1+1}^{N_1+N_2}(-1)^{i}\cos(\theta_i) = 2M \\ \sum_{i=1}^{N_1}(-1)^{i+1}\cos(3\theta_i) + \sum_{i=N_1+1}^{N_1+N_2}(-1)^{i}\cos(3\theta_i) = 0 \\ \vdots \\ \sum_{i=1}^{N_1}(-1)^{i+1}\cos(n\theta_i) + \sum_{i=N_1+1}^{N_1+N_2}(-1)^{i}\cos(n\theta_i) = 0 \end{cases} \quad (6)$$

$$0 \le \theta_1 \le \theta_2 \le \cdots \le \theta_N \le \pi/2 \quad (7)$$

Due to the complexity of these trigonometric equations, there is no mathematical method to solve them. Real-time implementation is a challenging task. Therefore, most of the solving techniques are based on off-line calculation [9].

III. PHASE-SHIFTED EXPRESSION FOR THE SHE PROBLEM IN FIVE-LEVEL CONVERTERS

In this section, the phase-shifted expression for the SHE problem in five-level converters is briefly explained. In [23], [24], this new expression is discussed in detail.

The staircase five-level waveform can be provided by subtracting two quasi-square waveforms - named *base waveform1* - with the same switching angles (α) but different phases (φ_1) as shown in Fig. 2. The n^{th} order harmonic amplitude of these waveforms (V_n^{Base1}) can be adjusted by α in equation (8).

$$V_n^{Base1} = \frac{4V_{DC}}{n\pi} cos(n\alpha) \quad (8)$$

In terms of the phase difference (φ_1) between these two quasi square waveforms, the n^{th} order harmonic amplitude of the output waveform (V_n^{Out1}) is calculated as follows:

$$v_n^{Out1}(t) = V_n^{Base1} sin(n\omega t) - V_n^{Base1} sin[n(\omega t - \varphi_1)]$$

$$\Rightarrow \ v_n^{Out1}(t) = V_n^{Base1}[2\, sin(n\,\varphi_1/2)\, cos(n\omega t - n\varphi_1/2)]$$

$$\Rightarrow \ V_n^{Out1} = \left[\frac{4V_{DC}}{n\pi} cos(n\alpha)\right][2sin(n\varphi_1/2)]$$

$$\Rightarrow \ V_n^{Out1} = \frac{8V_{DC}}{n\pi} cos(n\alpha)\, sin(n\varphi_1/2) \quad (9)$$

By considering $n = 1$:

$$V_1^{Out1} = \frac{8V_{DC}}{\pi} cos(\alpha)\, sin(\varphi_1/2) \quad (10)$$

Using (4) and due to having two degrees of freedom, the SHE problem for five-level converters is redefined by the following two equations:

$$\begin{cases} cos(\alpha)\, sin(\varphi_1/2) = M \\ cos(n\alpha)\, sin(n\varphi_1/2) = 0 \end{cases} \quad (11)$$

The arithmetic operation in equation (11) is multiplication unlike equation (3) which has a summation operation between cosine functions. Therefore, solving equation (11) compare to solving the conventional SHE problem is simpler.

To eliminate the n^{th} order harmonic of the output waveform, it is enough to choose the phase difference (φ_1) according to (12). The parameter k is a positive integer, and all results less than π are acceptable.

$$sin(n\,\varphi_1/2) = 0 \ \Rightarrow \ n\,\varphi_1/2 = k\pi \ \Rightarrow \ \varphi_1 = 2k\pi/n \quad (12)$$

In addition, the switching angle of both quasi-square waveforms can be calculated according to (13). Equation (13) indicates that the maximum acceptable value of the modulation index M_{max} is determined by (14).

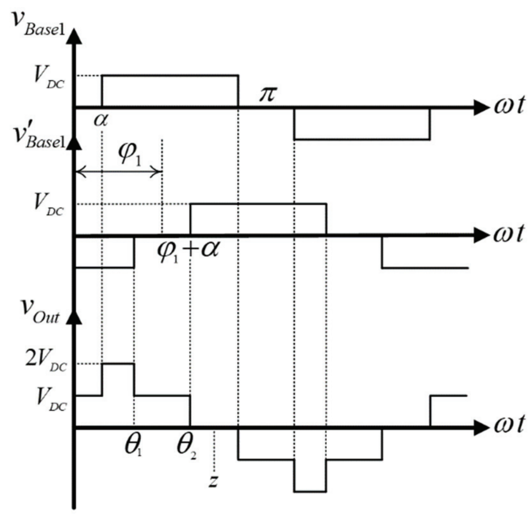

Fig. 2. Output voltage waveform of a five-level converter (phase-shifted expression).

$$\alpha = cos^{-1}\left(\frac{M}{sin(\varphi_1/2)}\right) \quad (13)$$

$$M_{Max} = sin(\varphi_1/2) \quad (14)$$

According to Fig. 2, the output voltage waveform has three levels if ($\alpha > \varphi_1/2$). In other words, the output voltage waveform is not a usual staircase waveform under this condition [23], [24]. Therefore, the border of five-level and three-level waveforms is determined by ($\alpha = \varphi_1/2$).

Using (11):

$$M_{Bdr} = cos(\varphi_1/2)\, sin(\varphi_1/2) \quad (15)$$

Equations (16) show the feasible modulation index intervals for the three-level and five-level output waveforms of an acceptable phase difference φ_1.

$$\begin{cases} 0 \le M \le M_{Bdr} \Rightarrow \text{three} - \text{level output waveform} \\ M_{Bdr} < M \le M_{Max} \Rightarrow \text{five} - \text{level output waveform} \end{cases} \quad (16)$$

All of the acceptable phase differences and modulation index intervals are shown in Fig. 3 for three-level and five-level waveforms. It is noticeable that under all conditions, the minimum value of the modulation index is zero due to generating a three-level waveform in the low modulation index range.

In order to clarify the simplicity of this method, the elimination of the fifth harmonic ($\text{n} = 5$) is discussed in detail.

Using (12):

$$n = 5 \ \Rightarrow \ \varphi_1 = 2k\pi/5$$

There exist two answers that are less than π, and the feasible modulation index interval for each answer is demonstrated as follows:

$$k = 1 \ \Rightarrow \ \varphi'_1 = 2\pi/5 \ \ \Rightarrow \ \alpha' = cos^{-1}\left(\frac{M}{sin(\pi/5)}\right)$$

$$M_{Max} = sin(\pi/5) \approx 0.5878$$

$$M_{Bdr} = cos(\pi/5)\, sin(\pi/5) \approx 0.4755$$

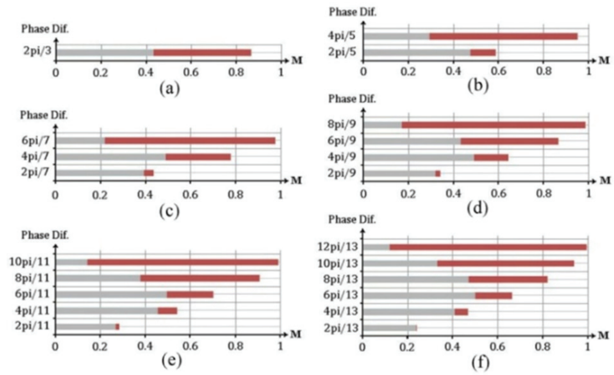

Fig. 3. Three-level and five-level modulation index intervals for the elimination of: (a) Third harmonic; (b) Fifth harmonic; (c) Seventh harmonic; (d) Ninth harmonic; (e) Eleventh harmonic; (f) Thirteenth harmonic.

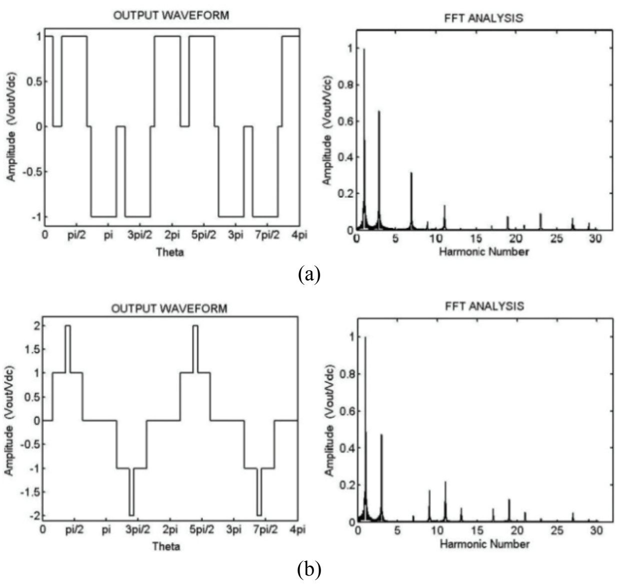

Fig. 4. Two different waveforms without the fifth harmonic for $M = \pi/8$: (a) Three-level waveform; (b) Five-level waveform.

$$\Rightarrow \begin{cases} 0 \le M \le 0.4755 \Rightarrow \text{three} - \text{level output waveform} \\ 0.4755 \le M \le 0.5878 \Rightarrow \text{five} - \text{level output waveform} \end{cases}$$

$$k = 2 \;\Rightarrow\; \varphi_1'' = 4\pi/5 \;\;\Rightarrow\;\; \alpha'' = cos^{-1}\left(\frac{M}{sin(2\pi/5)}\right)$$

$$M_{Max} = sin(2\pi/5) \approx 0.9511$$

$$M_{Bdr} = cos(2\pi/5)\,sin(2\pi/5) \approx 0.2939$$

$$\Rightarrow \begin{cases} 0 \le M \le 0.2939 \Rightarrow \text{three} - \text{level output waveform} \\ 0.2939 \le M \le 0.9511 \Rightarrow \text{five} - \text{level output waveform} \end{cases}$$

As an illustration, there are two different answers and two dissimilar waveforms for modulation index $M = \pi/8 \approx 0.3927$ or $(V_1^{Out} = V_{DC})$ without the fifth harmonic, which are determined by:

$$\varphi_1' = 2\pi/5 \;\;\Rightarrow\;\; \alpha' = cos^{-1}\left(\frac{\pi}{8sin(\pi/5)}\right) \approx 0.839 \text{ Rad}$$

$$\varphi_1'' = 4\pi/5 \;\Rightarrow\; \alpha'' = cos^{-1}\left(\frac{\pi}{8sin(2\pi/5)}\right) \approx 1.145 \text{ Rad}$$

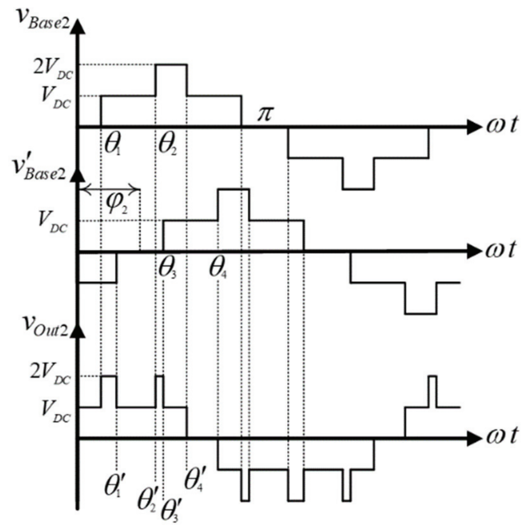

Fig. 5. Generating a generalized five-level waveform to eliminate two different harmonics.

These two waveforms are shown in Fig. 4. Note that the first one $(\varphi_1' = 2\pi/5)$ is a three-level waveform, and the second one $(\varphi_1'' = 4\pi/5)$ is a five-level waveform. In addition, as shown in Fig. 4, all of the odd multiples of the fifth harmonic (ex. 15th, 25th etc.) are eliminated too.

IV. Elimination of Multiple Harmonics in Five-Level Converters

As mentioned previously, having more switching angles make it possible to eliminate multiple harmonics. Under this condition, two questions must be answered:

- How many transitions are needed between zero and the first level ($N_1 = ?$), and between the first and second level ($N_2 = ?$)?
- What is the value of the switching angles?

Most of articles make assumptions about the number of transitions. However, these assumptions should change for different ranges of the modulation index. After that, evolutionary algorithms such as the genetic algorithm (GA) are used to calculate the optimum angles for a desired modulation index [25].

In this section, a simple algorithm is proposed to generate a five-level waveform without selected low order harmonics. This algorithm automatically determines the number of transitions between levels and is able to calculate the exact value of switching angles if needed.

Consider a three-level or five-level waveform without one selected harmonic, which is called *base waveform2*. This waveform can be generated by the method proposed in section III. The output waveform (v^{Out2}) is generated by subtracting two base waveforms with different phases (φ_2), as shown in Fig. 5. The n^{th} order harmonic amplitude of this new output waveform (V_n^{Out2}) can be calculated as follows:

$$v_n^{Out2}(t) = V_n^{Base2}\, sin(n\omega t) - V_n^{Base2}\, sin[n(\omega t - \varphi_2)]$$

$$\Rightarrow\ v_n^{Out2}(t) = V_n^{Base2}[2\, sin(n\,\varphi_2/2)\, cos(n\omega t - n\varphi_2/2)]$$

$$\Rightarrow V_n^{Out2} = V_n^{Base2}[2sin(n\varphi_2/2)] = V_n^{Out1}[2sin(n\varphi_2/2)]$$

$$\Rightarrow V_n^{Out2} = \frac{4V_{DC}}{n\pi} cos(n\alpha)\,[2^2 sin(n\varphi_1/2)\, sin(n\varphi_2/2)] \quad (17)$$

By choosing an appropriate value for (φ_2), the elimination of another low order harmonic is possible. For instance, by choosing $(\varphi_1 = 2\pi/3\ ,\ \varphi_2 = 2\pi/5)$, the third and fifth harmonics of the output waveform (v^{Out2}) are completely suppressed. This new output waveform can be used as a *base waveform3* to eliminate another low order harmonic.

This technique can be repeated again and again to eliminate multiple harmonics. After k iterations, the n^{th} order harmonic amplitude of the output waveform $\left(V_n^{Out_k}\right)$ can be written as follows:

$$V_n^{Out_k} = \frac{4V_{DC}}{n\pi} cos(n\alpha)\,[2^k \textstyle\prod_{i=1}^{k} sin(n\varphi_k/2)] \quad (18)$$

By considering $n = 1$, the first order harmonic of the output waveform can be calculated by (19).

$$V_1^{Out_k} = \frac{4V_{DC}}{\pi} cos(\alpha)\,[2^k \textstyle\prod_{i=1}^{k} sin(\varphi_k/2)] \quad (19)$$

$$\Longrightarrow \quad M = 2^{k-1} cos(\alpha)\,[\textstyle\prod_{i=1}^{k} sin(\varphi_k/2)] \quad (20)$$

According to (18), k different harmonics can be eliminated by choosing appropriate values for φ_1 to φ_k. Furthermore, the value of α is determined by (21).

$$\alpha = cos^{-1}\left(\frac{M}{2^{k-1}\, sin(\varphi_1/2)\ sin(\varphi_2/2) \dots sin(\varphi_k/2)}\right) \quad (21)$$

Totally, the proposed method can be explained by using three steps as follows:

1- Choose appropriate values for all of the phase differences $(\varphi_i\ ,\ 1 \le i \le k)$ before implementation. As a simple illustration, $(\varphi_1 = 4\pi/7\ ,\ \varphi_2 = 2\pi/5)$ are chosen to eliminate the fifth and seventh harmonics.

2- Calculate α by (21). In our example:

$$\alpha = cos^{-1}\left(\frac{M}{2\, sin(2\pi/7)\ sin(\pi/5)}\right) \Rightarrow \alpha = cos^{-1}\left(\frac{M}{0.9191}\right)$$

This final equation should be programmed in a processor to calculate α for different values of M.

3- Calculate the switching angles by (22).
According to Fig. 2: $\theta_1 = \varphi_1 - \alpha\ \ ,\ \ \theta_2 = \varphi_1 + \alpha$
According to Fig. 5: $\theta_3 = \theta_1 + \varphi_2\ ,\ \ \theta_4 = \theta_2 + \varphi_2$

$$\Rightarrow \begin{cases} \theta_1' = \varphi_2 - \varphi_1 + \alpha \\ \theta_2' = \varphi_1 + \alpha \\ \theta_3' = \varphi_2 + \varphi_1 + \alpha \\ \theta_4' = \pi - \varphi_1 - \alpha \end{cases} \quad (22)$$

For real-time implementation, a simple processor is needed to perform steps 2 and 3 for any desired value of the modulation index. Note that equation (23) is the Taylor series expansion of $\cos^{-1}(x)$ which can be easily calculated by a simple processor.

$$cos^{-1}(x) = \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1\times3}{2\times4}\frac{x^5}{5} + \frac{1\times3\times5}{2\times4\times6}\frac{x^7}{7} + \cdots\right) \quad (23)$$

As shown in Fig. 5, the number of switching angles needed for the elimination of two selected harmonics is four. By increasing the number of selected harmonics, the number of switching angles dramatically increases. However, any desired number of harmonics and their multiples can be completely eliminated without solving trigonometric equations. There is no need to make assumptions about the number of transitions between levels because this technique automatically generates the required number of transitions. Note that the value of the switching angles is related to α, and φ_1 to φ_k as shown in Fig. 2 and Fig. 5. Therefore, the exact value of the switching angles can be calculated. Furthermore, this technique can be implemented for all multilevel converter topologies.

V. Simulation Results

To investigate the validity of the proposed method, a five-level diode-clamped inverter is simulated in MATLAB/ SIMULINK. The characteristics of the converter are as follows:

- The total DC input voltage is 6 kV ($V_{DC} = 1.5$ kV).
- The output frequency is 50Hz.

Fig. 6 shows the schematic of the simulated inverter, which has four separate DC sources as the input of the inverter.

Several cases are examined in which selected harmonics are eliminated and the desired fundamental amplitude is achieved. In all cases, the phase voltage waveform of the converter is examined. Note that if a specific harmonic is eliminated from the phase voltage, the line voltage does not contain this harmonic either. Therefore, the proposed method is suitable for both three phase and single phase applications.

Case 1: Elimination of fifth and seventh harmonics for $M = 0.65$

By choosing $(\varphi_1 = 4\pi/7\ ,\ \varphi_2 = 2\pi/5)$, the value of the *base waveform1* switching angle (α) can be calculated by (21):

$$\alpha = cos^{-1}\left(\frac{0.65}{2\, sin(\pi/5)\ sin(2\,\pi/7)}\right) = 0.7852 \text{ Rad}$$

Fig. 7(a) shows the *base waveform1* with switching angle (α), and its shifted waveform with phase difference $\varphi_1 = 4\pi/7$. By subtracting these two base waveforms, *base waveform2* is generated as shown in Fig. 7(b). In addition, by subtracting *base waveform2* and its shifted waveform – with a phase difference of $\varphi_2 = 2\pi/5$ – an output waveform is generated which has three transitions between zero and the first level, and only one transition between the first and

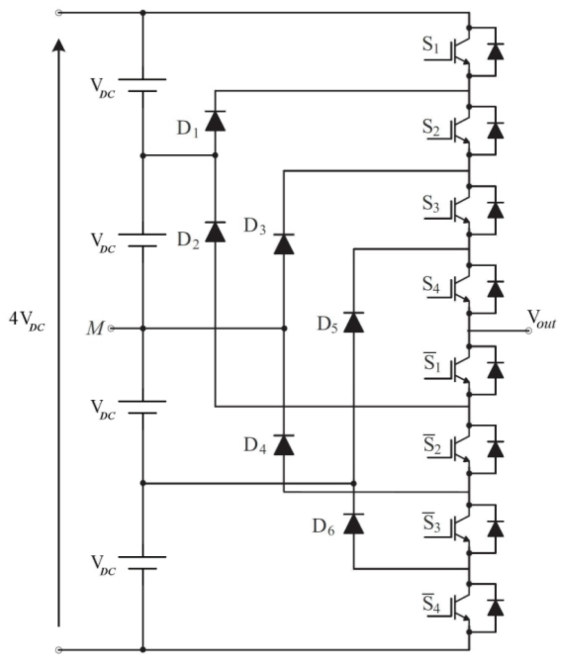

Fig. 6. One leg of a five-level diode-clamped inverter.

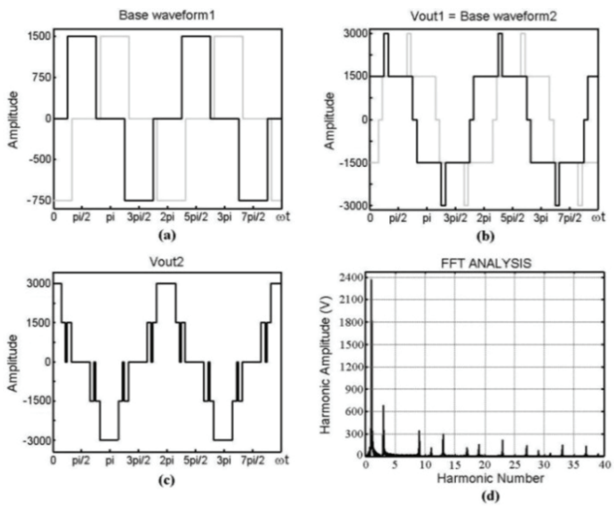

Fig. 7. Elimination of the fifth and seventh harmonics for $M = 0.65$ (simulation results).

second level as shown in Fig. 7(c). Fig. 7(d) shows the FFT spectrum of the output waveform.

The fifth and seventh harmonics are suppressed. In addition, all of the odd multiples of the fifth and seventh harmonics (ex. 15th, 21th, 25th, 35th etc.) are eliminated. Note that it is possible to find other solutions for this case by choosing different acceptable values for (φ_1, φ_2) such as $(\varphi_1 = 6\pi/7\ ,\ \varphi_2 = 2\pi/5)$ or $(\varphi_1 = 4\pi/7\ ,\ \varphi_2 = 4\pi/5)$.

Case 2: Elimination of fifth and seventh harmonics for $M = 0.85$

This case is similar to case 1. By choosing $(\varphi_1 = 4\pi/7\ ,\ \varphi_2 = 2\pi/5)$:

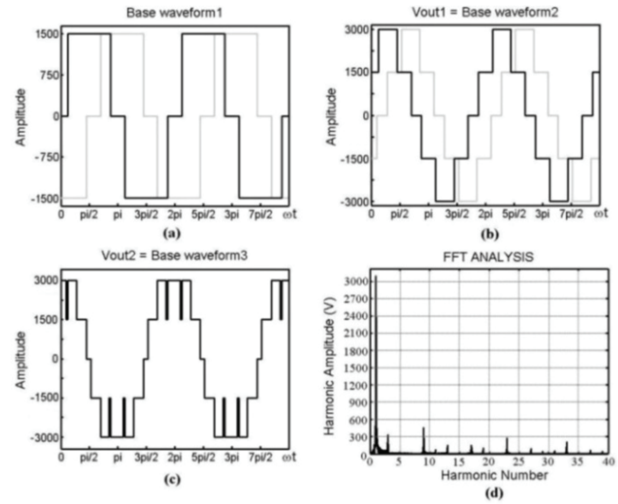

Fig. 8. Elimination of fifth and seventh harmonics for $M = 0.85$ (simulation results).

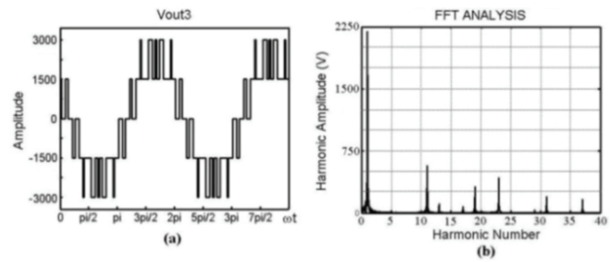

Fig. 9. Elimination of third, fifth and seventh harmonics for $M = 0.6$ (simulation results).

$$\alpha = cos^{-1}\left(\frac{0.85}{2\,sin(\pi/5)\ sin(2\,\pi/7)}\right) = 0.3902 \text{ Rad}$$

Fig. 8 shows simulation results of this case. According to Fig. 8(c), the output waveform has only one transition between zero and the first level, and three transitions between the first and second level. The FFT spectrum of the output waveform is shown in Fig. 8(d). The fifth and seventh harmonics and all of their odd multiples are eliminated.

It is considerable that in aforementioned cases, no initial assumptions are made for the number of transitions between the levels. Since the proposed phase-shifted method does not need to solve equations (5)-(7), it automatically determines the number of transitions between levels.

Case 3: Elimination of third, fifth and seventh harmonics for $M = 0.6$

$(\varphi_1 = 2\pi/3\ ,\ \varphi_2 = 2\pi/5\ ,\ \varphi_3 = 2\pi/7)$ was chosen to eliminate the third, fifth and seventh harmonics. Fig. 9 shows the output waveform and its FFT spectrum. Note that all of the triplen harmonics such as the ninth harmonic are also eliminated.

Case 4: Elimination of third, fifth, seventh and eleventh harmonics for $M = 0.75$

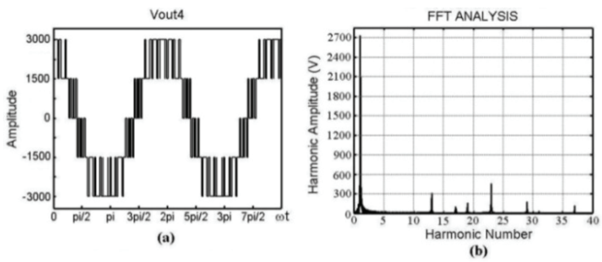

Fig. 10. Elimination of third, fifth, seventh and eleventh harmonics for $M = 0.75$ (simulation results).

In this case, four different harmonics are simultaneously eliminated. Fig. 10 shows simulation results of this case by choosing $(\varphi_1 = 2\pi/3\ ,\ \varphi_2 = 2\pi/5\ ,\ \varphi_3 = 6\pi/7\ ,\ \varphi_4 = 10\pi/11)$. According to Fig. 10(b) all of the harmonics mentioned above and their multiplicand are completely suppressed.

VI. Experimental Results

To demonstrate the validity of the proposed method, a scaled down five-level diode-clamped inverter prototype has been implemented with the input DC voltage of 680V (V_{DC} = 170 V) and 50Hz output frequency. The utilized controller is a TMS320F28335 Texas Instruments DSP. In addition, an auxiliary circuit is used to balance the input capacitor voltages. The modulation index and switching angles of the following cases are the same as those employed in the above simulations. The base scales for the FFT analysis are 100 V_{rms}/div and $f = 62.5\ \text{Hz/div}$ for all of the cases.

Fig. 11 shows experimental results of cases 1 and 2, both of which deal with fifth and seventh harmonics elimination under different modulation indices. According to the FFT spectrum, the selected harmonics are completely eliminated. Fig. 11(a) and 11(b) have good adaptation to Fig. 7 and 8.

The output voltage waveform of case3 and its FFT spectrum are shown in Fig. 12. The third, fifth and seventh harmonics are eliminated and the FFT spectrum has a good adaptation to Fig. 9(b). Due to the third harmonic elimination, all of the triplen harmonics such as the ninth harmonic are also suppressed.

Fig. 13 shows experimental results of case 4. This case is performed to eliminate the third, fifth, seventh and eleventh harmonics. According to Fig. 13(b), the selected harmonics are eliminated and the FFT spectrum has a good adaptation to Fig. 10(b).

For all of these cases, the calculation time of the switching angles is less than 200μs, which is suitable for the real-time implementation of the proposed method.

Despite measurement errors, the FFT spectrums of the simulation and experimental results are appropriately similar for all of these cases. In addition, both of them show the capability of the proposed method for the elimination of low order harmonics.

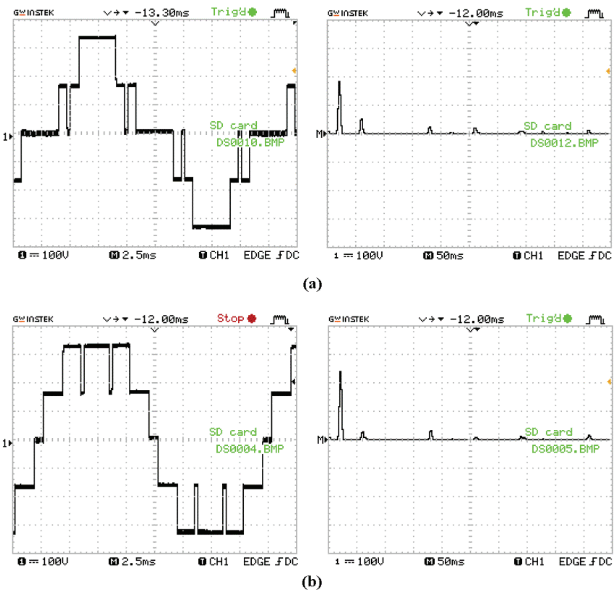

Fig. 11. Elimination of fifth and seventh harmonics for: (a) $M = 0.65$; (b) $M = 0.85$ (experimental results).

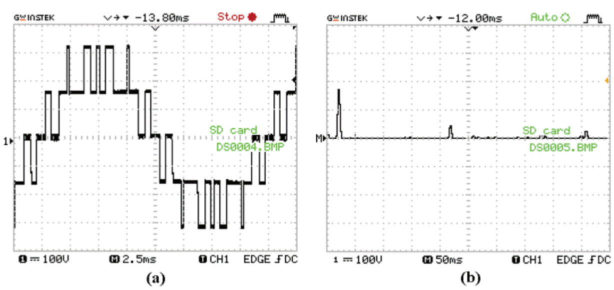

Fig. 12. Elimination of third, fifth and seventh harmonics for $M = 0.6$ (experimental results).

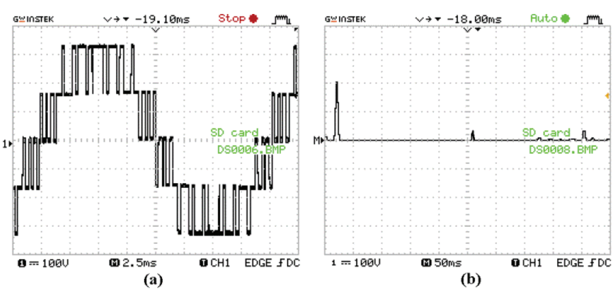

Fig. 13. Elimination of third, fifth, seventh and eleventh harmonics for $M = 0.75$ (experimental results).

VII. Discussion

Although the proposed method uses more switching angles compared to other methods, it operates for a wider range of modulation indices. Note that according to (21), the proposed method supports a low value of modulation index by automatically generating three-level output waveform, which cannot be achieved using other methods. In addition, this technique completely eliminates all of the selected harmonics and their multiplicands. Therefore, any desired value for the

THD can be obtained in theory. The main advantage of the proposed method is the ability of real-time implementation due to the simplicity of its arithmetic operations.

Numerical and evolutionary algorithms such as the GA require choosing initial values. In addition, these methods usually determine partial solutions [9], [16]. However, the proposed method does not need to choose initial values and is able to find all possible sets of solutions. Furthermore, the proposed method can find the exact value of switching angles as opposed to aforementioned methods. In addition, the calculation time of the proposed method is considerably less than other methods since they determine the switching angles after many iterations. However, the proposed method calculates the switching angles analytically.

Algebraic methods such as the resultant and Groebner bases theories can find all possible sets of solutions for an assumed pattern. However, the proposed method automatically generates different patterns according the value of the modulation index. In addition, real-time implementation of algebraic methods is very difficult due to their huge computational burden [9], [19], [20].

Finally, the proposed method, in comparison with the analytical method proposed in [22], has even more advantages. The proposed method is based on simple arithmetic operations. Therefore, it is simpler than the method in [22] where the modulation index intervals are only calculated for five-level staircase waveforms. However, the proposed method is able to determine modulation index intervals for both three-level and five-level waveforms, as shown in Fig. 3. The method proposed in [22] can only eliminate one selected harmonic. However, the method proposed in this paper is able to eliminate multiple selected harmonics.

VIII. CONCLUSIONS

In this paper, a practical method for real-time implementation of the SHE technique is proposed. The proposed method extends the phase-shifted expression of the SHE problem for multiple harmonic eliminations in five-level converters. The proposed method is able to calculate the exact value of the switching angles and it generates a proper waveform without the need to solve complex trigonometric equations. Compared to other methods, this method has the following advantages: 1) calculation of switching angles with reduced computational burden without the need for initial values; 2) easy real-time implementation; 3) ability to automatically determine the number of transitions between levels. Finally, the proposed method was applied to a five-level diode-clamped inverter, both in simulations and in an experimental setup. The results verify the correctness and effectiveness of the proposed method.

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