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Corrigendum to "On Soft Topological Space via Semi-open and Semi-closed Soft Sets, Kyungpook Mathematical Journal, 54(2014), 221–236"

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Abstract. In this manuscript, we show that the equality relations of the two assertions (ix) and (x) of [Theorem 2.11, p.p.224] in [3] do not hold in general, by giving a concrete example. Also, we illustrate that Example 6.3, Example 6.7, Example 6.11, Example 6.15 and Example 6.20 do not satisfy a soft semi T_0-space, a soft semi T_1-space, a soft semi T_2-space, a soft semi T_3-space and a soft semi T_4-space, respectively. Moreover, we point out that the three results obtained in [3] which related to soft subspaces are false, by presenting two examples. Finally, we construct an example to illuminate that Theorem 6.18 and Remark 6.21 made in [3] are not valid in general.

1. Introduction

The soft topological spaces concept was defined by Shabir and Naz [5] in 2011. They gave the fundamental soft topological notions such as soft open sets, soft closed sets and soft separation axioms. The soft semi-open sets concept was introduced by Mahanta and Das [3] in 2014. They investigated its main properties and formulated various soft topological notions depend on it such as soft semi-continuity, soft semi-compactness, soft semi-connectedness and soft semi T_i-spaces ($i = 0, 1, 2, 3, 4$).

But we observe that some results obtained in [3] are false. So the aims of this work are, first, to show some errors in [Theorem 2.11, p.p.224] in [3], by presenting an example. Second, to point out that Example 6.3, Example 6.7, Example 6.11, Example 6.15 and Example 6.20 do not satisfy a soft semi T_0-space, a soft semi T_1-space, a soft semi T_2-space, a soft semi T_3-space and a soft semi T_4-space, respectively. Third, to show some alleged results which related to soft subspaces with the help of two examples. Finally, to elucidate that Theorem 6.18 and Remark 6.21 of [3] are incorrect, by introducing an illustrative example.

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2. Preliminaries

In this section, we recollect the definitions that will be needed in this paper.

Definition 2.1.([4]) A pair (G, E) is said to be a *soft set* over X provided that G is a mapping of E into 2^X.

Remark 2.2.

(i) For short, we use the notation G_E instead of (G, E).

(ii) A soft set G_E can be expressed as a set of ordered pairs $G_E = \{(e, G(e)) : e \in E \text{ and } G(e) \in 2^X\}$.

Definition 2.3.([2]) A soft set G_A is a soft subset of a soft set F_B, denoted by $G_A \widetilde{\subseteq} F_B$, if $A \subseteq B$ and $G(a) \subseteq F(a)$, for all $a \in A$.

Definition 2.4.([5]) A collection τ of soft sets over X with a fixed set of parameters E is called a *soft topology* on X if it satisfies the following three axioms:

(i) The null soft set $\widetilde{\Phi}$ and the absolute soft set $\widetilde{X}$ are members of τ.

(ii) The soft union of an arbitrary number of soft sets in τ is a member of τ.

(iii) The soft intersection of a finite number of soft sets in τ is a member of τ.

The triple (X, τ, E) is called a soft topological space. Each soft set in τ is called *soft open* and its relative complement is called *soft closed*.

Definition 2.5.([5]) Let G_E be a soft subset of a soft topological space (X, τ, E). Then:

(i) A *soft interior* of G_E, denoted by $int(G_E)$, is the union of all soft open sets contained in G_E.

(ii) A *soft closure* of G_E, denoted by $cl(G_E)$, is the intersection of all soft closed sets containing G_E.

Definition 2.6.([1]) Let Y_A be a non-null soft subset of (X, τ, E). Then $\tau_{Y_A} = \{Y_A \widetilde{\bigcap} G_E : G_E \in \tau\}$ is called a *soft relative topology* on Y_A and (Y_A, τ_{Y_A}, A) is called a *soft subspace* of (X, τ, E).

Definition 2.7.([6]) A soft set G_E over X is called soft point if there is $e \in E$ such that $G(e) \neq \emptyset$ and $G(a) = \emptyset$, for each $a \in E \setminus \{e\}$. A soft point will be briefly denoted by e_G.

Definition 2.8.([6]) A soft point $e_F \in G_E$, if for an element $e \in E$, we have $F(e) \subseteq G(e)$.

Proposition 2.9.([6]) *If* $e_F \in G_E$, *then* $e_F \notin G_E^c$.

The converse of the above proposition is not always true as illustrated in Example 3.11 of [6].

Definition 2.10.([3]) A soft subset G_E of a soft topological space (X, τ, E) is said to be *soft semi-open* if $G_E \widetilde{\subseteq} cl(int(G_E))$. And its relative complement is said to be *soft semi-closed*.

Definition 2.11.([3]) Let G_E be a soft subset of a soft topological space (X, τ, E). Then:

(i) A *soft semi-interior* of G_E, denoted by $ssint(G_E)$, is the union of all soft semi-open sets contained in G_E.

(ii) A *soft semi-closure* of G_E, denoted by $sscl(G_E)$, is the intersection of all soft semi-closed sets containing G_E.

Definition 2.12.([3]) Two soft points e_G, e_H are said to be *distinct* if $G(e) \bigcap H(e) = \emptyset$.

Definition 2.13.([3]) A soft topological space (X, τ, E) is said to be:

(i) *Soft semi T_0-space* if for every pair of distinct soft points $e_H, e_G \in \widetilde{X}$, there exists a soft semi-open set U_E such that $e_H \in U_E$, $e_G \notin U_E$ or $e_G \in U_E$, $e_H \notin U_E$.

(ii) *Soft semi T_1-space* if for every pair of distinct soft points $e_H, e_G \in \widetilde{X}$, there exist soft semi-open sets U_E and V_E such that $e_H \in U_E, e_G \notin U_E$ and $e_G \in V_E$, $e_H \notin V_E$.

(iii) *Soft semi T_2-space* if for every pair of distinct soft points $e_H, e_G \in \widetilde{X}$, there exist disjoint soft semi-open sets U_E and V_E such that $e_H \in U_E$ and $e_G \in V_E$.

(iv) *Soft semi regular* if for every soft semi-closed set H_E and $e_H \in \widetilde{X}$ such that $e_H \notin H_E$, there exist disjoint soft semi-open sets U_E and V_E such that $e_H \in U_E$ and $H_E \widetilde{\subseteq} V_E$.

(v) *Soft semi normal* if for every two disjoint soft semi-closed sets H_E and F_E, there exist two disjoint soft semi-open sets U_E and V_E such that $H_E \widetilde{\subseteq} U_E$ and $F_E \widetilde{\subseteq} V_E$.

(vi) *Soft semi T_3-space* if it is both soft semi T_1 and soft semi regular.

(vii) *Soft semi T_4-space* if it is both soft semi T_1 and soft semi normal.

Definition 2.14.([3]) A soft topological space (X, τ, E) is said to be *soft semicompact* if every soft semi-open cover of $\widetilde{X}$ has a finite subcover.

3. Main Results

We begin this section with Theorem 3.1 below, originally proposed as two assertions (ix) and (x) of [Theorem 2.11, p.p.224] in [3].

Theorem 3.1. *Let G_C and K_D be two soft subsets of (X, τ, E). Then:*

(i) $sscl(G_C)\widetilde{\bigcup}sscl(K_D) = sscl(G_C\widetilde{\bigcup}K_D)$.

(ii) $ssint(G_C)\widetilde{\bigcap}ssint(K_D) = ssint(G_C\widetilde{\bigcap}K_D)$.

The next example clarifies that the two equality relations in the above theorem are not always true.

Example 3.2. Let $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{\Phi}, \widetilde{X}, F_E, G_E, H_E, L_E, M_E\}$ be a soft topology on $X = \{a, b, c\}$, where

$$\begin{aligned} F_E &= \{(e_1, \emptyset), (e_2, \{a\})\}; \\ G_E &= \{(e_1, \{a\}), (e_2, \{a\})\}; \\ H_E &= \{(e_1, \{b\}), (e_2, \{b\})\}; \\ L_E &= \{(e_1, \{b\}), (e_2, \{a, b\})\} \text{ and} \\ M_E &= \{(e_1, \{a, b\}), (e_2, \{a, b\})\}. \end{aligned}$$

Consider a soft set $N_E = \{(e_1, \emptyset), (e_2, \{b\})\}$. Then we can find that:

(i) $sscl(G_E) = G_E$ and $sscl(N_E) = H_E$,

(ii) $sscl(G_E)\widetilde{\bigcup}sscl(N_E) = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$,

(iii) $sscl(G_E\widetilde{\bigcup}N_E) = \widetilde{X}$.

From the above three cases, we conclude that $sscl(G_E\widetilde{\bigcup}N_E) \not\subseteq sscl(G_E)\widetilde{\bigcup}sscl(N_E)$. This illustrates that item **(ix)** of Theorem 2.11 of [3] is false.

Also, consider the two soft sets $O_E = \{(e_1, \{c\}), (e_2, \{a\})\}$ and $P_E = \{(e_1, \{b, c\}), (e_2, \{b, c\})\}$. Then we can find that:

(i) $ssint(O_E) = O_E$ and $ssint(P_E) = P_E$,

(ii) $ssint(O_E)\widetilde{\bigcap}ssint(P_E) = \{(e_1, \{c\}), (e_2, \emptyset)\}$,

(iii) $ssint(O_E\widetilde{\bigcap}P_E) = \widetilde{\Phi}$.

From the above three cases, we conclude that $ssint(O_E)\widetilde{\bigcap}ssint(P_E) \not\subseteq ssint(O_E\widetilde{\bigcap}P_E)$. This illustrates that item **(x)** of Theorem 2.11 of [3] is false as well.

The next theorem is the correct form of items (ix) and (x) of Theorem 2.11 of [3].

Theorem 3.3. *Let G_C and K_D be two soft subsets of (X, τ, E). Then:*

(i) $sscl(G_C)\widetilde{\bigcup}sscl(K_D) \subseteq sscl(G_C\widetilde{\bigcup}K_D)$,

(ii) $ssint(G_C\widetilde{\bigcap}K_D) \subseteq ssint(G_C)\widetilde{\bigcap}ssint(K_D)$.

In the following two remarks, we points out some mistakes made in [3].

Remark 3.4. In Example 2.2 of [3], it is obvious that $U_E = L_{3A}$, so we remove L_{3A} from the definition of the given soft topology.

Remark 3.5. We note that Example 6.3 of [3] does not satisfy a soft semi T_0-space, because the two soft points $e_G = \{(\{h_1\}, \emptyset)\}$ and $e_F = \{(\{h_2\}, \emptyset)\}$ are distinct and there does not exist a soft semi-open set L_E such that $[e_G \in L_E$ and $e_F \notin L_E]$ or $[e_F \in L_E$ and $e_G \notin L_E]$. So that we can conclude the following:

(i) Example 6.7 does not satisfy a soft semi T_1-space.

(ii) Example 6.11 does not satisfy a soft semi T_2-space.

(iii) Example 6.15 does not satisfy a soft semi T_3-space.

(iv) Example 6.20 does not satisfy a soft semi T_4-space.

Hereafter, we mention Theorem 3.6 below, originally proposed in [3] as [Theorem 6.5, p.p.232], [Theorem 6.9, p.p.233] and [Theorem 6.12, p.p.233].

Theorem 3.6. *A soft subspace of a soft semi T_i-space is soft semi T_i, for $i = 0, 1, 2$.*

The following two examples elucidate that the above theorem need not be true in general.

Example 3.7. Consider $E = \{e_1, e_2\}$ and let $\tau = \{\widetilde{\Phi}, \widetilde{X}, \{(e_1, \{a\}), (e_2, \{a\})\}\}$ be a soft topology on $X = \{a, b, c\}$. One can be checked that (X, τ, E) is a soft semi T_0-space. On the other hand, let $B_E = \{(e_1, \{b, c\}), (e_2, \{b, c\})\}$. Then $\tau_{B_E} = \{\widetilde{\Phi}, B_E\}$ be a soft topology on B_E. Obviously, (B_E, τ_{B_E}, E) is not a soft semi T_0-space.

Example 3.8. Consider $E = \{e\}$ and let $\tau = \{\widetilde{\Phi}, X, \{(e, \{a\})\}, \{(e, \{b\})\}, \{(e, \{a, b\})\}\}$ be a soft topology on $X = \{a, b, c\}$. One can be checked that (X, τ, E) is a soft semi T_2-space. On the other hand, let $B_E = \{(e, \{a, c\})\}$. Then $\tau_{B_E} = \{\widetilde{\Phi}, B_E, \{(e, \{a\})\}\}$ be a soft topology on B_E. Obviously, (B_E, τ_B, E) is not a soft semi T_1-space.

Finally, we mention Theorem 3.9 and Remark 3.10 below, originally proposed in [3] as [Theorem 6.18, p.p.234] and [Remark 6.21, p.p.234].

Theorem 3.9. *A soft topological space which is both soft semicompact and soft semi T_2-space is soft semi T_3.*

Remark 3.10. Every soft semi T_4-space is soft semi T_3.

The next example shows that the theorem and remark which above mentioned need not be true in general.

Example 3.11. Consider $E = \{e_1, e_2\}$ and let $\tau = \{\widetilde{\Phi}, \widetilde{X}, \{(e_1, \{a\}), (e_2, \{a\})\}, \{(e_1, \{b\}), (e_2, \{b\})\}\}$ be a soft topology on $X = \{a, b\}$.

Obviously, (X, τ, E) is both soft semicompact and soft semi T_2-space. Also, it is soft semi T_4-space. On the other hand, $L_E = \{(e_1, \{a\}), (e_2, \{a\})\}$ is a soft semi-closed set such that $e_G = \{(e_1, X), (e_2, \emptyset)\} \notin L_E$. Since there do not exist two disjoint soft semi-open sets such that one of them contains L_E and the other contains e_G, then (X, τ, E) is not a soft semi T_3-space.

4. Conclusion

The authors of [3] defined and studied some soft topological concepts based on a soft semi-open sets notion. They utilized a soft point notion which given in [6] to establish some soft separation axioms. In this work, we correct some alleged results which introduced in [3], by constructing illustrative examples.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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