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# Confluent Hypergeometric Distribution and Its Applications on Certain Classes of Univalent Functions of Conic Regions

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Abstract. The purpose of the present paper is to investigate Confluent hypergeometric distribution. We obtain some basic properties of this distribution. It is worthy to note that the Poisson distribution is a particular case of this distribution. Finally, we give a nice application of this distribution on certain classes of univalent functions of the conic regions.

## 1. Introduction

The confluent hypergeometric function is given by the power series

$$F(a;c;z)=\sum_{n=0}^{\infty}\frac{(a)_n}{(c)_n(1)_n}z^n, \tag{1.1}$$

where $a, c$ are complex numbers such that $c \neq 0,-1,-2,\ldots$ and $(a)_n$ is the Pochhammer symbol defined in terms of the Gamma function, by

$$(a)_n=\frac{\Gamma(a+n)}{\Gamma(a)}=\begin{cases}1, & \text{if } n=0\\ a(a+1)\ldots(a+n-1), & \text{if } n\in N=\{1,2,3,\ldots\}\end{cases}$$

is convergent for all finite values of $z$, see [12].

This suggests that the series

$$F(a;c;m)=\sum_{n=0}^{\infty}\frac{(a)_n}{(c)_n(1)_n}m^n$$

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is convergent for $a, c, m > 0$.

Very recently, Porwal and Kumar [11] introduced the confluent hypergeometric distribution (CHD) whose probability mass function is

$$P(n) = \frac{(a)_n m^n}{(c)_n n! F(a;c;m)}, \quad a, c, m > 0, \quad n = 0, 1, 2, \ldots . \tag{1.2}$$

It is easy to see that for $a = c$ it reduces to the Poisson distribution.

## 2. Properties of CHD

**Definition 2.1.** If $X$ is a discrete random variable which can take the values $x_1$, $x_2$, $x_3$, … with respective probabilities $p_1$, $p_2$, $p_3$, … then *expectation* of $X$, denoted by $E(X)$, is defined as

$$E(X) = \sum_{k=1}^{\infty} p_k x_k. \tag{2.1}$$

**Definition 2.2.** The $r^{th}$ *moment* of a discrete probability distribution about $X = 0$ is defined by

$$\mu_r^{'} = E(X^r).$$

Here $\mu_1^{'}$ and $\mu_2^{'} - \left(\mu_1^{'}\right)^2$ are known as the mean and variance of the distribution.

**Moments about the origin**

(1)

$$\mu_1^{'} = \sum_{n=0}^{\infty} nP(n) = \sum_{n=0}^{\infty} n \frac{(a)_n m^n}{(c)_n n! F(a;c;m)} = \frac{ma}{c} \frac{F(a+1;c+1;m)}{F(a;c;m)}.$$

Similarly

(2)

$$\mu_2^{'} = \frac{1}{F(a;c;m)} \left[ \frac{(a)_2}{(c)_2} m^2 F(a+2;c+2;m) + \frac{a}{c} m F(a+1;c+1;m) \right].$$

(3)

$$\mu_3^{'} = \frac{1}{F(a;c;m)} \left[ \frac{(a)_3}{(c)_3} m^3 F(a+3;c+3;m) + 3 \frac{(a)_2}{(c)_2} m^2 F(a+2;c+2;m) + \frac{a}{c} m F(a+1;c+1;m) \right].$$

(4)

$$\mu_4^{'} = \frac{1}{F(a;c;m)}\left[\frac{(a)_4}{(c)_4}m^4F(a+4;c+4;m)+6\frac{(a)_3}{(c)_3}m^3F(a+3;c+3;m)\right.$$
$$\left.+7\frac{(a)_2}{(c)_2}m^2F(a+2;c+2;m)+\frac{a}{c}mF(a+1;c+1;m)\right].$$

**Definition 2.3.** The *moment generating function (m.g.f.)* of a random variable $X$ is denoted by $M_X(t)$ and defined by

$$M_X(t) = E(e^{tX}). \tag{2.2}$$

The proof of the following theorem is straight forward so we only state the result.

**Theorem 2.1.** *The moment generating function of the confluent hypergeometric Distribution is given by*

$$M_X(t) = \frac{F(a;c;me^t)}{F(a;c;m)}.$$

**Remark 2.1.** If we put $a = c$ in the expressions $\mu_1^{'}$, $\mu_2^{'}$, $\mu_3^{'}$, $\mu_4^{'}$ and in Theorem 2.1, then we obtain the corresponding results of Poisson distribution.

## 3. Application of Confluent Hypergeometric Distribution on Certain Classes of Univalent Functions of Conic Regions

Let $\mathcal{A}$ denote the class of functions $f(z)$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{3.1}$$

which are analytic in the open unit disc $U = \{z : z \in C \text{ and } |z| < 1\}$ and satisfy the normalization condition $f(0) = f'(0) - 1 = 0$. Further, we denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting of functions of the form (3.1) which are also univalent in $U$.

In 1997, Bharti *et al.* [1] introduced the subclasses $k$-uniformly convex functions of order $\alpha$ and corresponding class of $k$-starlike functions of order $\alpha$ as follows

A function $f$ of the form (3.1) is in $k - UCV(\alpha)$, if and only if, it satisfy the following condition

$$\Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} \geq k\left|\frac{zf''(z)}{f'(z)}\right| + \alpha, \quad 0 \leq k < \infty, \quad 0 \leq \alpha < 1. \tag{3.2}$$

For $\alpha = 0$ the class $k - UCV(\alpha)$ reduce to the class $k - UCV$ introduced and studied by Kanas and Wisniowska [6] and for $k = 1, \alpha = 0$ it reduce to the class of uniformly convex functions $UCV$ studied by Goodman [3]. Using the Alexander transform we can obtain the class $k-S_p(\alpha)$ in the following way $f \in k-UCV(\alpha) \Leftrightarrow zf' \in k-S_p(\alpha)$. For more results on these directions we refer the reader to [4, 5, 7, 8, 14, 15] and references therein.

A function $f \in \mathcal{A}$ is said to be in the class $P_\gamma^\tau(\beta)$ if it satisfies the following inequality

$$\left| \frac{(1-\gamma)\frac{f(z)}{z} + \gamma f'(z) - 1}{2\tau(1-\beta) + (1-\gamma)\frac{f(z)}{z} + \gamma f'(z) - 1} \right| < 1,$$

where $0 \le \gamma < 1$, $\beta < 1$, $\tau \in C/\{0\}$ and $z \in U$. The class $P_\gamma^\tau(\beta)$ was introduced by Swaminathan [17].

Next, we introduce the classes $S_\lambda^*$ and $C_\lambda$ as follows

$$S_\lambda^* = \left\{ f \in A : \left| \frac{zf'(z)}{f(z)} - 1 \right| < \lambda, \quad (z \in U, \lambda > 0) \right\} \tag{3.3}$$

and

$$C_\lambda = \left\{ f \in A : \left| \frac{zf''(z)}{f'(z)} \right| < \lambda, \quad (z \in U, \lambda > 0) \right\}. \tag{3.4}$$

From (3.3) and (3.4) it is easy to see that

$$f(z) \in C_\lambda \Leftrightarrow zf'(z) \in S_\lambda^*, \quad (\lambda > 0).$$

The classes $S_\lambda^*$ and $C_\lambda$ were introduced by Ponnusamy and Rønning [9].

Recently, Porwal [10] introduce a power series whose coefficients are probabilities of Poisson distribution

$$K(m, z) = z + \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} z^n, \qquad (z \in U),$$

and we note that, by ratio test the radius of convergence of above series is infinity.

The convolution (or Hadamard product) of two series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=0}^{\infty} b_n z^n$ is defined as the power series

$$(f * g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n.$$

Now, we introduce a new series $I(a; c; m; z)$ whose coefficients are probabilities of confluent hypergeometric distribution

$$I(a; c; m; z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1} m^{n-1}}{(c)_{n-1}(n-1)! F(a; c; m)} z^n,$$

where $a, c, m > 0$.

Now, we consider a linear operator $\Omega(a;c;m) : \mathcal{A} \to \mathcal{A}$ defined by

$$\begin{aligned}\Omega(a;c;m)f &= I(a;c;m;z) * f(z) \\ &= z + \sum_{n=2}^{\infty} \frac{(a)_{n-1} m^{n-1}}{(c)_{n-1}(n-1)! F(a;c;m)} a_n z^n.\end{aligned}$$

The Poisson distribution series is a recent topic of study in Geometric Function Theory. It established a connection between probability distribution and Geometric Function Theory. Motivated by results of [10] and on connections between the various subclasses of analytic univalent functions by using hypergeometric functions (see [2], [9]), we establish a number of connections between the classes $P_\gamma^\tau(\beta)$, $k-UCV(\alpha)$, $k-S_p(\alpha)$, $C_\lambda$ and $S_\lambda^*$ by applying the convolution operator $\Omega(a;c;m)$.

## 4. Coefficient Conditions

To establish our main results, we shall require the following lemmas.

**Lemma 4.1.**([1]) *A function $f \in \mathcal{A}$ is in $k-UCV(\alpha)$, if it satisfies the following condition*

$$\sum_{n=2}^{\infty} n[n(1+k) - (k+\alpha)]|a_n| \leq 1-\alpha. \tag{4.1}$$

**Remark 4.1.** It was also found that the condition (4.1) is necessary if $f \in \mathcal{A}$ is of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n, \ a_n \geq 0. \tag{4.2}$$

**Lemma 4.2.**([1]) *A function $f \in \mathcal{A}$ is in $k - S_p(\alpha)$ if it satisfies the following inequality*

$$\sum_{n=2}^{\infty} [n(1+k) - (k+\alpha)]|a_n| \leq 1-\alpha. \tag{4.3}$$

*The condition* (4.3) *is also necessary for functions of the form* (4.2).

**Lemma 4.3.**([6]) *Let $f \in \mathcal{S}$ and have the form* (3.1). *If for some $k$, $0 \leq k < \infty$, the inequality*

$$\sum_{n=2}^{\infty} n(n-1)|a_n| \leq \frac{1}{(k+2)},$$

*holds, then $f \in k-UCV$. The number $1/k+2$ can not be increased.*

**Lemma 4.4.**([9]) *Let $f \in \mathcal{A}$ be of the form* (3.1). *If*

$$\sum_{n=2}^{\infty}(\lambda+n-1)|a_n| \leq \lambda, \quad (\lambda>0), \tag{4.4}$$

*then $f \in S^*_\lambda$.*

**Lemma 4.5.**([9]) *Let $f \in \mathcal{A}$ be of the form* (3.1). *If*

$$\sum_{n=2}^{\infty}n(\lambda+n-1)|a_n| \leq \lambda, \quad (\lambda>0), \tag{4.5}$$

*then $f \in C_\lambda$.*

We further note that when $f(z)$ is of the form (4.2), the conditions (4.4) and (4.5) are both necessary and sufficient for belonging to the classes $S^*_\lambda$ and $C_\lambda$, respectively.

**Lemma 4.6.**([17]) *If $f \in P^\tau_\gamma(\beta)$ is of the form* (3.1) *then*

$$|a_n| \leq \frac{2|\tau|(1-\beta)}{1+\gamma(n-1)}.$$

**Theorem 4.1.** *If $a, c, m > 0$, $k \geq 0$, $0 \leq \alpha < 1$, $f \in P^\tau_\gamma(\beta), 0 < \gamma \leq 1$, $0 \leq \beta < 1$ and the inequality*

$$(k+1)\frac{a}{c}mF(a+1;c+1;m) + (1-\alpha)\left(F(a;c;m)-1\right) \leq \frac{\gamma F(a;c;m)(1-\alpha)}{2|\tau|(1-\beta)} \tag{4.6}$$

*is satisfied then $\Omega(a;c;m)f(z) \in k-UCV(\alpha)$.*

*Proof.* Since

$$\Omega(a;c;m)f(z) = z + \sum_{n=2}^{\infty}\frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-1)!}\frac{1}{F(a;c;m)}a_n z^n.$$

To prove that $\Omega(a;c;m)f(z) \in k-UCV(\alpha)$, from Lemma 4.1, it is sufficient to show that

$$\sum_{n=2}^{\infty}n[n(1+k)-(k+\alpha)]|A_n| \leq 1-\alpha, \tag{4.7}$$

where

$$A_n = \frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-1)!}\frac{1}{F(a;c;m)}a_n, \quad n \geq 2.$$

Now, by using Lemma 4.6 and $1+\gamma(n-1)\geq \gamma n$, we have

$$\begin{aligned}
&\sum_{n=2}^{\infty} n[n(1+k)-(k+\alpha)]\frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-1)!}\frac{1}{F(a;c;m)}|a_n| \\
&\leq 2|\tau|(1-\beta)\sum_{n=2}^{\infty} n[n(1+k)-(k+\alpha)]\frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-1)!}\frac{1}{F(a;c;m)}\frac{1}{1+\gamma(n-1)}, \\
&\leq \frac{2|\tau|(1-\beta)}{\gamma F(a;c;m)}\sum_{n=2}^{\infty}[n(1+k)-(k+\alpha)]\frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-1)!}, \\
&= \frac{2|\tau|(1-\beta)}{\gamma F(a;c;m)}\left[(k+1)\sum_{n=2}^{\infty}\frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-2)!}+(1-\alpha)\sum_{n=2}^{\infty}\frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-1)!}\right] \\
&= \frac{2|\tau|(1-\beta)}{\gamma F(a;c;m)}\left[(k+1)\frac{a}{c}mF(a+1;c+1;m)+(1-\alpha)(F(a;c;m)-1)\right].
\end{aligned}$$

The last expression is bounded above by $1-\alpha$, if (4.6) holds.

This completes the proof of Theorem 4.1. □

**Theorem 4.2.** *If* $a,c>1,\ m>0,\ k\geq 0,\ 0\leq\alpha<1,\ f\in P_{\gamma}^{\tau}(\beta), 0<\gamma\leq 1,$ $0\leq\beta<1$ *and the inequality*

(4.8)
$$\begin{aligned}
&(k+1)\left(F(a;c;m)-1\right)-\frac{(k+\alpha)}{m}\frac{(c-1)}{(a-1)}\left(F(a-1;c-1;m)-1-\frac{(a-1)}{(c-1)}m\right) \\
&\leq \frac{\gamma(1-\alpha)F(a;c;m)}{2|\tau|(1-\beta)}
\end{aligned}$$

*is satisfied then* $\Omega(a;c;m)f(z)\in k-S_p(\alpha)$.

*Proof.* The proof of this theorem is much akin to that of Theorem 4.1 so we omit the details involved. □

**Theorem 4.3.** *Let* $a,c>1,\ m>0,\ f\in P_{\gamma}^{\tau}(\beta);\ \ 0<\gamma\leq 1,\ \beta<1,\ \lambda>0$ *and the inequality*

$$\frac{2|\tau|(1-\beta)}{\gamma F(a;c;m)}\left[(F(a;c;m)-1)+\frac{(\lambda-1)}{m}\frac{(c-1)}{(a-1)}\left(F(a-1;c-1;m)-1-\frac{(a-1)}{(c-1)}m\right)\right]\leq\lambda$$

*is satisfied then* $\Omega(a;c;m)f(z)\in S_{\lambda}^{*}$.

*Proof.* To prove that $\Omega(a;c;m)f(z)\in S_{\lambda}^{*}$, from Lemma 4.4 it is sufficient to prove that

$$\sum_{n=2}^{\infty}(n+\lambda-1)|A_n|\leq\lambda$$

where

$$A_n=\frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-1)!}\frac{1}{F(a;c;m)}a_n,\quad n\geq 2.$$

Since $f \in P_\gamma^\tau(\beta)$ using Lemma 4.6 and $1+\gamma(n-1) \geq \gamma n$ we need only to show that

$$\sum_{n=2}^{\infty}(n+\lambda-1)\frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-1)!}\frac{1}{F(a;c;m)}\frac{2|\tau|(1-\beta)}{1+\gamma(n-1)} \leq \lambda.$$

Now adopting the same technique of Theorem 4.1 and performing simple calculations we obtain the required result. □

**Theorem 4.4.** *Let $a, c, m > 0,\ f \in P_\gamma^\tau(\beta);\ \ 0 < \gamma \leq 1,\ \beta < 1$ and $\lambda > 0$ and the inequality*

$$\frac{2|\tau|(1-\beta)}{F(a;c;m)\gamma}\left[\frac{a}{c}mF(a+1;c+1;m)+\lambda\left(F(a;c;m)-1\right)\right] \leq \lambda$$

*is satisfied then $\Omega(a;c;m)f(z) \in C_\lambda$.*

*Proof.* The proof is similar to that of Theorem 4.3 therefore we omit the details. □

**Theorem 4.5.** *Let $a, c, m > 0,\ k \geq 0,\ 0 \leq \alpha < 1$ and the inequality*

(4.9)

$$(1+k)\frac{(a)_3}{(c)_3}m^3F(a+3;c+3;m)+(6+5k-\alpha)\frac{(a)_2}{(c)_2}m^2F(a+2;c+2;m) + (7+4k-3\alpha)\frac{a}{c}mF(a+1;c+1;m) \leq 1-\alpha$$

*is satisfied then $\Omega(a;c;m)f(z)$ maps $f(z) \in S$ of the form (3.1) into $k-UCV(\alpha)$.*

*Proof.* Let $f(z) \in \mathcal{S}$ be of the form (3.1). In view of Lemma 4.1 it is enough to show that

$$T = \sum_{n=2}^{\infty} n[n(k+1)-(k+\alpha)]\frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-1)!}\frac{1}{F(a;c;m)}|a_n| \leq 1-\alpha.$$

Now

$$\begin{aligned} T &= \sum_{n=2}^{\infty} n[n(k+1)-(k+\alpha)]\frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-1)!}\frac{1}{F(a;c;m)}|a_n| \\ &\leq \sum_{n=2}^{\infty} n^2[n(k+1)-(k+\alpha)]\frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{(n-1)!}\frac{1}{F(a;c;m)} \\ &= \frac{1}{F(a;c;m)}\Bigg[(1+k)\frac{(a)_3}{(c)_3}m^3F(a+3;c+3;m) \\ &+ (6+5k-\alpha)\frac{(a)_2}{(c)_2}m^2F(a+2;c+2;m) \\ &+(7+4k-3\alpha)\frac{a}{c}mF(a+1;c+1;m)+(1-\alpha)(F(a;c;m)-1)\Bigg] \end{aligned}$$

The last expression is bounded above by $1-\alpha$, if (4.9) holds. Thus the proof of Theorem 4.5 is established. □

**Theorem 4.6.** *Let* $a, c, m > 0,\ k \geq 0,\ 0 \leq \alpha < 1$ *and the inequality*

$$(1+k)\frac{(a)_2}{(c)_2}m^2F(a+2;c+2;m)+(3+2k-\alpha)\frac{a}{c}mF(a+1;c+1;m) \leq 1-\alpha, \tag{4.10}$$

*is satisfied, then* $\Omega(a;c;m)f(z)$ *maps* $f(z) \in S$ *of the form* (3.1) *into* $k-S_p(\alpha)$.

*Proof.* The proof of this theorem is much akin to that of Theorem 4.5. Therefore we omit the details involved. □

**Theorem 4.7.** *Let* $a, c, m > 0,\ \lambda > 0$ *and the inequality*

$$\frac{(a)_3}{(c)_3}m^3F(a+3;c+3;m)+(\lambda+5)\frac{(a)_2}{(c)_2}m^2F(a+2;c+2;m)+(3\lambda+4)\frac{a}{c}mF(a;c;m) \leq \lambda$$

*is satisfied then* $\Omega(a;c;m)f(z)$ *maps* $f(z) \in S$ *of the form* (3.1) *into* $C_\lambda$.

*Proof.* The proof of this theorem is similar to that of Theorem 4.3. Therefore we omit the details involved. □

**Theorem 4.8.** *Let* $a, c, m > 0,\ \lambda > 0$ *and the inequality*

$$\frac{(a)_2}{(c)_2}m^2F(a+2;c+2;m)+(\lambda+2)\frac{a}{c}mF(a+1;c+1;m) \leq \lambda F(a;c;m) \tag{4.11}$$

*is satisfied then* $\Omega(a;c;m)f(z)$ *maps* $f(z) \in \mathcal{S}$ *of the form* (3.1) *into* $S^*_\lambda$.

*Proof.* The proof of this theorem is similar to that of Theorem 4.3. Therefore we omit the details involved. □

## 5. An Integral Operator

In the following theorem, we obtain analogues results in connection with a particular integral operator $G(a,c,m,z)$ which is defined as follows

$$G(a,c,m,z) = \int_0^z \frac{\Omega(a;c;m)f(t)}{t}dt. \tag{5.1}$$

**Theorem 5.1.** *Let* $f$ *be of the form* (3.1) *is in the class* $P^\tau_\gamma(\beta)$ *with* $a, c, m > 0$ *and the inequality* (4.8) *is satisfied then* $G(a,c,m,z)$ *defined by* (5.1) *is in the class* $k-UCV(\alpha)$.

*Proof.* Since

$$G(a,c,m,z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}}\frac{m^{n-1}}{n!}\frac{1}{F(a;c;m)}a_nz^n. \tag{5.2}$$

To prove $G(a,c,m,z) \in k-UCV(\alpha)$, adopting the technique of Theorem 4.1 and performing simple calculatios we obtain the required result. □

The proof of following Theorems 5.2–5.5 are similar to Theorem 5.1 therefore we only state the results of these theorems.

**Theorem 5.2.** *Let* $f$ *be defined by* (3.1) *in the class* $\mathcal{S}$ *with* $a, c, m > 0$ *and the inequality* (4.10) *is satisfied then* $G(a, c, m, z)$ *defined by* (5.1) *is in* $k - UCV(\alpha)$.

**Theorem 5.3.** *Let* $f$ *be defined by* (3.1) *in the class* $\mathcal{S}$ *with* $a, c, m > 0$ *and the inequality*

$$(k+1)\frac{a}{c}mF(a+1; c+1; m) \leq 1 - \alpha$$

*is satisfied then* $G(a, c, m, z)$ *is in the class* $k - S_p(\alpha)$.

**Theorem 5.4.** *Let* $f$ *be defined by* (3.1) *in the class* $\mathcal{S}$ *with* $a, c, m > 0$ *and the inequality*

$$\frac{a}{c}mF(a+1; c+1; m) \leq \lambda$$

*is satisfied then* $G(a, c, m, z)$ *is in the class* $S^*(\lambda)$.

**Theorem 5.5.** *Let* $f$ *be defined by* (3.1) *in the class* $\mathcal{S}$ *with* $a, c, m > 0$ *and the inequality* (4.11) *is satisfied then* $G(a, c, m, z)$ *is in the class* $C(\lambda)$.

**Remark 5.1.** If we put $a = c$ in Theorems 4.1–5.5, then we obtain the corresponding results of Srivastava and Porwal [16].

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# References

[1] R. Bharati, R Parvatham and A. Swaminathan, *On subclasses of uniformly convex functions and corresponding class of starlike funcitons*, Tamkang J. Math., **28**(1997), 17–32.

[2] A. Gangadharan, T. N. Shanmugam and H. M. Srivastava, *Generalized hypergeometric function associated with k-uniformly convex functions*, Comput. Math. Appl., **44**(2002), 1515–1526.

[3] A. W. Goodman, *On uniformly convex functions*, Ann. Polon. Math., **56**(1991), 87–92.

[4] A. W. Goodman, *On uniformly starlike functions*, J. Math. Anal. Appl., **155**(1991), 364–370.

[5] S. Kanas and H. M. Srivastava, *Linear operators associated with k-uniformly convex functions*, Integral Transform. Spec. Funct., **9**(2000), 121–132.

[6] S. Kanas and A. Wisniowska, *Conic regions and k-uniform convexity*, J. Comput Appl. Math., **105**(1999), 327–336.

[7] S. Kanas and A. Wisniowska, *Conic domains and starlike functions*, Rev. Roumaine Math. Pures Appl., **45**(2000), 647–657.

[8] W. Ma and D. Minda, *Uniformly convex functions*, Ann. Polon. Math., **57**(1992), 165–175.

[9] S. Ponnusamy and F. Rønning, *Starlikeness properties for convolutions involving hypergeometric series*, Ann. Univ. Mariae Curie-Sklodawska Sect. A, **52**(1998), 141–155.

[10] S. Porwal, *An application of a Poisson distribution series on certain analytic functions*, J. Complex Anal., (2014), Art. ID 984135, 3 pp

[11] S. Porwal and S. Kumar, *Confluent hypergeometric distribution and its applications on certain classes of univalent functions*, Afr. Mat., **28**(2017), 1–8.

[12] E. D. Rainville, *Special functions*, The Macmillan Co., New York, 1960.

[13] M. S. Robertson, *On the theory of univalent functions*, Ann. Math., **37**(1936), 374–408.

[14] F. Rønning, *Uniformly convex functions and a corresponding class of starlike functions*, Proc. Amer. Math. Soc., **118(1)**(1993), 189–196.

[15] H. M. Srivastava and A. K. Mishra, *Applications of fractional calculus to parabolic starlike and uniformly convex functions*, Comput. Math. Appl., **39**(2000), 57–69.

[16] D. Srivastava and S. Porwal, *Some sufficient conditions for Poisson distribution series associated with conic regions*, Int. J. Advanced Technology in Enginering Sci., **3(1)**(2015), 229–236.

[17] A. Swaminathan, *Certain sufficiency conditions on Gaussian hypergeometric functions*, J. Inequal. Pure Appl. Math., **5(4)**(2004), Article 83, 10 pp.