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SHADOWABLE POINTS FOR FINITELY GENERATED GROUP ACTIONS

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Abstract. In this paper we introduce the notion of shadowable points for finitely generated group actions on compact metric spaace and prove that the set of shadowable points is invariant and Borel set and if chain recurrent set contained shadowable point set then it coincide with nonwandering set. Moreover an action $T \in Act(G, X)$ has the shadowing property if and only if every point is shadowable.

1. Introduction

The classical dynamical system can be considered as an action of the group $\mathbb{Z}$. The first shadowing result obtained by Pilyugin for actions of group $\mathbb{Z}^p$, $p > 1$, was a kind of reductive shadowing theorem (RST) in [7] - it was shown that if the action of a one-dimensional subgroup of $\mathbb{Z}^p$ is topologically Anosov, then the action of $\mathbb{Z}^p$ is topologically Anosov as well. Afterwards, Osipov and Tikhomirov [5] represent a finitely generated group action version of this theorem. Very recently Chung and Lee [1] introduced the notion of topological stability for an action of a finitely generated group on a compact metric space, and proved that if an action is expansive and has the shadowing property then it is topologically stable.

The definition of shadowing for homeomorphisms in a compact metric space was generalized by splitting the shadowing property into pointwise shadowings giving rise to the concept of shadowable points, which are points where the shadowing property holds for pseudo-orbits begining at the point. In [2] the author further extends this notion by introducing

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the concept of quantitative shadowable points for homeomorphism and give a quantitative version of a result in [3].

In this paper, we will mainly study the structure of orbits having shadowable points for finitely generated group actions. First of all, we round out the introduction with some notations that we will use in the paper.

Let G be a finitely generated group with the discrete topology and X be a compact metric space with a metric d. Put $Homeo(X)$ the space of all homeomorphisms of X. We denote by $Act(G, X)$ the set of all continuous actions T of G on X; *i.e.* $T : G \times X \to X$ is a continuous map such that $T(e, x) = x$ and $T(g, T(h, x)) = T(gh, x)$ for $x \in X$ and $g, h \in G$, where e is the identity element of G. For briefly, $T(g, x)$ will be denoted by $T_g(x)$. Let $Homeo(X)^G = \prod_G Homeo(X)$ be the set of homeomorphisms from G to $Homeo(X)$ with the product topology. Then $Act(G, X)$ can be considered as a subset of $Homeo(X)^G$. Let A be a finitely generating set of G. Without special mention, we will see A has the smallest cardinality. We define a metric d_A on $Act(G, X)$ by

$$d_A(T, S) = \sup\{\ d(T_a x, S_a x)\ \mid x \in X, a \in A\}$$

for $T, S \in Act(G, X)$.

We say that a sequence of points $\{x_g\}_{g\in G} \subset X$ is a δ*-pseudo orbit for* T *with respect to* A if $d(T_a(x_g), x_{ag}) < \delta$ for every $a \in A$, $g \in G$ and a δ-pseudo-orbit $\{x_g\}_{g\in G}$ for T with respect to A is ϵ*-traced* by some point $x \in X$ if $d(x_g, T_g x) < \epsilon$ for every $g \in G$. We say that an continuous action T has the *shadowing property with respect to* A if for every $\epsilon > 0$, there exists $\delta > 0$ such that each δ pseudo orbit for T with respect to A is ϵ-traced by some point of X [1, 6].

2. Preliminary

Definition 2.1. $x \in X$ is *shadowable with respect to* A if for every $\epsilon > 0$ there is $\delta > 0$ such that every δ-pseudo orbit $\{x_g\}_{g\in G}$ of T with respect to A with $x_e = x$ can be ϵ-shadowed. We denote by $Sh(T)$ the set of shadowable points of T with respect to A.

It is clear that the definition of shadowable point of T does not depend on the choice of a compatible metric d on X. Note that the definition of shadowable point of a homeomorphism introduced in [3] coincides with our definition when $G = \mathbb{Z}$ and $A = \{1\}$. Furthermore, we can see that shadowable point of T does not depend on the choice of a symmetric finitely generating set A of G. Recall that A is symmetric if for any $a \in A$, $a^{-1} \in A$.

LEMMA 2.2. *Let A and B be finitely symmetric generating sets of G. For any $T \in Act(G, X)$, if x is a shadowable point of T with respect to A if and only if it is a shadowable point of T with respect to B.*

Proof. The proof is similar to that in [5] to show that the definition of shadowing property does not depend on the choice of symmetric finitely generating sets. □

DEFINITION 2.3. We say that a point $x \in X$ is *shadowable* if x is a shadowable point of T with respect to a finitely generating set A of G. We denote by $Sh(T)$ the set of shadowable points of T.

Oprocha [4] introduced the notion of the behavior of multidimensional time discrete dynamical systems containing the concepts of nonwandering set and chain recurrent set. We introduced the notion of theses definition of finitely generated group action versions. Recall that let A be a finitely generating set of G. For any $k \in \mathbb{N}$, we put $B(k) = \{g \in G : l_A(g) \le k\}$ where $l_A(g)$ is the word length metric on G induced by A.

DEFINITION 2.4. A point $x \in X$ is *nonwandering for T with respect to A* if for any neighborhood U of x and $g \in G$ with $l_A(g) > 0$ there exists $h \in G$ such that $l_A(h) > l_A(g)$ and $T_h U \cap U \neq \emptyset$. The set of all nonwandering points of T with respect to A is denoted by $\Omega(T, A)$.

LEMMA 2.5. *A point $x \in \Omega(T, A)$ if and only if for any neighborhood U of x there exist $h \in G \setminus \{e\}$ such that $T_h(U) \cap U \neq \emptyset$.*

Proof. Suppose that for a neighborhood U of x there exist $h \in G$ with $l_A(h) > 0$ such that $T_g(U) \cap U = \emptyset$ for all $g \in G$ with $l_A(g) > l_A(h)$. Let V be a neighborhood of x such that $\overline{V} \subset U$. We have $T_g(V) \cap V \subset T_g(U) \cap U = \emptyset$ for all $g \in G$ with $l_A(g) > l_A(h)$. Let $l_A(h) = n$. Denote by $V_g = T_g(V) \cap V$ and $W = V - \bigcup_{g \in B(n)} \overline{V_g}$.

Case 1: $x \in W = V - \bigcup_{g \in B(n)} \overline{V_g}$.

By assumption, there is $g' \in G$ with $l_A(g') > 0$ such that

$$T_{g'}(W) \cap W \neq \emptyset \textit{ and } T_{g'}(W) \cap W \subset T_{g'}(V) \cap V = V_{g'}.$$

But if $l_A(g') \le l_A(h)$ then we get

$$T_{g'}(W) \cap W \subset V_g \cap W = \emptyset.$$

On the other hand, if $l_A(g') \geq l_A(h)$ then

$$T_{g'}(W) \cap W \subset T_{g'}(V) \cap V = \emptyset,$$

which is a contradiction.
Case 2: $x \in \bigcup_{g \in B(n)} \overline{V_g}$. Then $x \in \overline{V_g}$ for some $g \in B(n)$.
Consequently we have

$$x \in \overline{V_g} = \overline{T_g(V) \cap V} \subset T_g(U) \cap U.$$

Let $U_1 = T_g(U) \cap U$. Then we gain $g_1 \in G$ with $l_A(g_1) \geq 1$ such that

$$x \in T_{g_1} U_1 \cap U_1 \subset T_{g_1}(T_g(U)) \cap U = T_{g_1 g}(U) \cap U = U_2$$

. By simple calculation, we have $x \in T_{g_n \cdots g_1 g}(U) \cap U$ for $l_A(g_n \cdots g_1 g) > N$. It is a contradiction. □

LEMMA 2.6. *Let A and B be finitely symmetric generating sets of G. Then we have $\Omega(T, A) = \Omega(T, B)$*

Proof. It is straightforward by Lemma 2.5. □

We get the result that nonwandering set does not depend on the choice of symmetric finitely generating sets. So we denote $\Omega(T)$ by the set of nonwandering points of T.

DEFINITION 2.7. Let $T \in Act(G, X)$ and A a finitely generated group. A point $x \in X$ is said to be *chain recurrent for T with respect to A* if for every $\delta > 0$ there exists a δ-pseudo orbit $\{x_g\}_{g \in G}$ of T with respect to A such that

(1) $x_e = x$,
(2) if the equality $x_{g'} = x$ holds for some index $g' \in G$, then the set $\{h \in G : x_h = x,\ l_A(h) > g\}$ is infinite for all $g \in G$.

The set of all chain recurrent points with respect to A will be denoted by $CR(T, A)$. Every δ-pseudo orbit of T with respect to A satisfying condition (1) and (2) is said to be a δ-chain of T through x with respect to A.

Recall that $T \in Act(G, X)$ is uniformly continuous if for any $\epsilon > 0$ there is $\delta > 0$ such that $d(x, y) < \delta$ implies $d(T_a x, T_a y) < \epsilon$ for all $a \in A$, $x, y \in X$ where A be a finitely generating set of G. We can also get the result that chain recurrent set does not depend on the choice of symmetric finitely generating sets.

LEMMA 2.8. *Let A and B be finitely symmetric generating sets of G. Then we have $CR(T, A) = CR(T, B)$.*

Proof. Let $x \in CR(T,A)$ and $\delta > 0$. Put $m = \max_{b\in B} l_A(b)$. Choose $\delta' > 0$ such that $m\delta' < \delta$. Since X is compact, T_g is uniformly continuous. Given δ', we can choose δ_A such that

$$d(x,y) < \delta_A \text{ implies } d(T_g(x), T_g(y)) < \delta' \text{ for all } g \in B(m)$$

where $B(m) = \{g \in G : l_A(g) \le m\}$. Given δ_A, since $x \in CR(T,A)$ there is a δ_A-pseudo orbit $\{x_g\}_{g\in G}$ of T with respect to A satisfy conditions (1) and (2) in Definition 2.7. It suffices to show that $\{x_g\}_{g\in G}$ is a δ-pseudo orbit of T with respect to B. For any $b \in B$, we write b as $a_1 \cdots a_n$, where $n = l_A(b) \le m$, $a_i \in A$, $i = 1, \cdots, n$. Then we have

$$\begin{aligned} d(T_b(x_g), x_{bg}) &= d(T_{a_1\cdots a_n}(x_g), x_{a_1\cdots a_n g}) \\ &\le d(T_{a_1\cdots a_{n-1}}(T_{a_n}(x_g)), T_{a_1\cdots a_{n-1}}(x_{a_n g})) \\ &\quad + d(T_{a_1\cdots a_{n-2}}(T_{a_{n-1}}(x_{a_n g})), T_{a_1\cdots a_{n-2}}(x_{a_{n-1}a_n g})) \\ &\quad + \cdots + d(T_{a_1 a_2}(T_{a_3}(x_{a_4\cdots a_n g})), T_{a_1 a_2}(x_{a_3\cdots a_n g})) \\ &\quad + d(T_{a_1}(T_{a_2}(x_{a_3\cdots a_n g})), T_{a_1}(x_{a_2\cdots a_n g})) \\ &\quad + d(T_{a_1}(x_{a_2\cdots a_n g}), x_{a_1\cdots a_n g}) \\ &< m\delta' < \delta \end{aligned}$$

for all $g \in G$, $b \in B$. This means that $\{x_g\}_{g\in G}$ is a δ-pseudo orbit of T with respect to B, and $x \in CR(T,B)$. The proof of the other side is established similarly. □

LEMMA 2.9. *Let $T \in Act(G,X)$, and let A be a finitely generating set of G. Then the chain recurrent set $CR(T)$ is closed and $\Omega(T) \subset CR(T)$.*

Proof. Clearly $\Omega(T) \subset CR(T)$. Next we will show that $CR(T)$ is closed.
Let us take any $y \in \overline{CR(T)}$ where $\overline{CR(T)}$ is closure of $CR(T)$. Then there exists a sequence $\{y_n\}_{n\in\mathbb{N}} \subset CR(T)$ converges to y. Let $\varepsilon > 0$ and since X is compact, $T_a : X \to X$ is uniformly continuous for all $a \in A$. Then there exists $0 < \delta < \frac{\varepsilon}{3}$ such that if $d(x,y) < \delta$ then $d(T_a(x), T_a(y)) < \frac{\varepsilon}{3}$. Since $\{y_n\}_{n\in\mathbb{N}}$ converges to y, we can choose $N \in \mathbb{N}$ satisfying $d(y_N, y) < \delta$.
The point y_N is chain recurrent, so there exists δ-chain $\{x_g\}_{g\in G} \subset X$ for y_N ($i.e$ $y_N = x_e$). Let us define a sequence $\{z_g\}_{g\in G}$ by the formula:

$$z_g = \begin{cases} x_g & \text{if } x_g \ne y_N, \\ y & \text{if } x_g = y_N, \end{cases}$$

Then the following inequality holds:

$$d(T_a(z_g), z_{ag}) \le d(T_a(z_g), T_a(x_g)) + d(T_a(x_g), x_{ag}) + d(x_{ag}, z_{ag}) < \varepsilon.$$

This implies that the sequence $\{z_g\}_{g\in G}$ is ε-chain of T through y. As $\varepsilon > 0$ was chosen arbitrarily, we obtain that $y \in CR(T)$, thus the set $CR(T)$ is closed. □

3. Main Results

Morales [3] introduced the notion for the shadowable points of homeomorphisms on compact metric spaces, and obtained many interesting results. In this section, we extend the group action version of this result. But in general, the finitely generated group G is not necessarily abelian. Thus when we proved the (1) of Theorem 3.2, we will prove it in a different way than homeomorphism case.

Definition 3.1. Let $T \in Act(G, X)$, and let A be a finitely generating set of G. A set $E \subset X$ is called *T-invariant set with respect to A* if $T_{a^{-1}}(E) = E$ for all $a \in A$.

Theorem 3.2. *For any $T \in Act(G, X)$,*

(1) *$Sh(T)$ is an invariant set (empty or nonempty, possibly noncompact),*
(2) *$Sh(T)$ is Borel set.*
(3) *if $CR(T) \subset Sh(T)$ then $\Omega(T) = CR(T)$*

Proof. Proof of (1). Let A be a finitely generating set of G. It suffices to prove that if $x \in Sh(T)$ then $T_a(x) \in Sh(T)$ for all $a \in A$. Fix $a \in A$. For any $\varepsilon > 0$, since X is compact, T_a is uniformly continuous, and so there is $0 < \varepsilon' < \varepsilon$ such that $d(T_a(y), T_a(z)) \le \varepsilon$ whenever $y, z \in X$ satisfy $d(y, z) \le \varepsilon'$ for all $a \in A$. For this ε', we get $\delta' > 0$ by the shadowableness of the point x. Since X is compact and A is finite, there is $0 < \delta < \frac{\delta'}{2}$ such that any $y, z \in X$ with $d(y, z) < \delta$ satisfy $d(T_{a'}(y), T_{a'}(z)) < \frac{\delta'}{2}$ for any $a' \in A$ and $a' \ne a$. Let $\{x_g\}_{g\in G}$ be a δ-pseudo orbit of T with respect to A through $T_a(x)$. We define a new pseudo orbit $\{y_g\}_{g\in G}$ by

$$y_g = \begin{cases} T_{a^{-1}}(x_e), & \text{if } g = e, \\ x_{ga^{-1}}, & \text{if } g \ne e. \end{cases}$$

Then we can check $\{y_g\}_{g\in G}$ is a δ'-pseudo orbit of T with respect to A through x. Indeed, if $g = e$ and $a' = a$, then

$$\begin{aligned} d(T_{a'}(y_g), y_{a'g}) &= d(T_{a'}(y_e), y_{a'}) = d(T_{a'}(T_{a^{-1}}(x_e)), y_{a'}) \\ &= d(x_e, y_a) = d(x_e, x_e) = 0 < \delta'. \end{aligned}$$

If $g = e$ and $a' \neq a$, then

$$\begin{aligned} d(T_{a'}(y_g), y_{a'g}) &= d(T_{a'}(y_e), y_{a'}) = d(T_{a'}(T_{a^{-1}}(x_e)), y_{a'}) \\ &\leq d(T_{a'}(T_{a^{-1}}(x_e)), T_{a'}(x_{a^{-1}})) + d(T_{a'}(x_{a^{-1}}), y_{a'}) \\ &< \frac{\delta'}{2} + d(T_{a'}(x_{a^{-1}}), x_{a'a^{-1}}) < \frac{\delta'}{2} + \delta < \delta'. \end{aligned}$$

If $g \neq e$, then we get

$$d(T_{a'}(y_g), y_{a'g}) = d(T_{a'}(x_{ga^{-1}}), y_{a'ga^{-1}}) < \delta'.$$

Hence there exists $y \in X$ such that $d(T_g(y), y_g) < \varepsilon'$ for all $g \in G$. Then for $g = e$, we have

$$d(T_e(y), y_e) = d(y, T_{a^{-1}}(x_e)) = d(T_{a^{-1}}(T_a(y)), T_{a^{-1}}(x_e)) < \varepsilon'.$$

Hence we obtain, $d(T_a(y), x_e) = d(T_e(T_a(y)), x_e) < \varepsilon$. For $g \neq e$, we get

$$\begin{aligned} d(T_g(y), y_g) &= d(T_g(y), x_{ga^{-1}}) = d(T_{g'a}(y), x_{g'}) \\ &= d(T_{g'}(T_a(y)), x_{g'}) < \varepsilon' < \varepsilon. \end{aligned}$$

This means that $Sh(T)$ is invariant.

Proof of (2). For $\delta > 0$ and $c > 0$, let $S_{\delta,c}(T, A)$ be the set of points z such that for every δ-pseudo orbit $\{x_g\}_{g\in G}$ with respect to A satisfying $d(T_a(x_g), x_{ag}) < \delta$, $x_e = z$ is c-shadowed by some $y \in X$. Note that

$$Sh_c(T, A) = \bigcup_{m\in\mathbb{N}} S_{\frac{1}{m},c}(T, A), Sh(T, A) = \bigcap_{n\in\mathbb{N}} Sh_{\frac{1}{n}}(T, A).$$

Hence it suffices to show that $S_{\delta,c}(T, A)$ is a closed subset of X for all $\delta > 0$ and $c > 0$. Let $(z_n)_{n\in\mathbb{N}}$ be a sequence of points in $S_{\delta,c}(T, A)$ such that $\lim_{n\to\infty} z_n = z$ for some $z \in X$. Given a pseudo orbit $\{x_g\}_{g\in G}$ with respect to A with $d(T_a(x_g), x_{ag}) < \delta$ and $x_e = z$, we define a new sequence of pseudo orbits $\{x_g^{(n)}\}_{g\in G, n\in\mathbb{N}}$ by

$$x_g^{(n)} = \begin{cases} z_n \ \textit{if}\ g = e, \\ x_g \ \textit{if}\ g \neq e. \end{cases}$$

Then for sufficiently large $n \in \mathbb{N}$ we have $d(T_a(x_g^{(n)}), x_{ag}^{(n)}) < \delta$ for all $g \in G$ and $a \in A$. For such $n \in \mathbb{N}$, the pseudo orbit $\{x_g^{(n)}\}_{g\in G}$ is c-shadowed by some $y_n \in X$ since $x_e^{(n)} = z_n \in S_{\delta,c}(T, A)$. Take a sequence

$(y_{n_j})_{j\in\mathbb{N}}$ such that $\lim_{j\to\infty} y_{n_j} = \overline{y}$ for some $\overline{y} \in X$. Then we have

$$d(x_e, \overline{y}) = d(z, \overline{y}) = \lim_{j\to\infty} d(z_{n_j}, y_{n_j}) = \lim_{j\to\infty} d(x_e^{(n_j)}, y_{n_j}) \leq c$$

and

$$d(x_g, T_g\overline{y}) = \lim_{j\to\infty} d(x_g^{(n_j)}, T_g y_{n_j}) \leq c$$

for $g \neq e$. Thus $\overline{y}$ is a c-shadowing point of $\{x_g\}_{g\in G}$ and then $z \in S_{\delta,c}(T, A)$.

Proof of (3). Let A be a finitely generating set of G and $\varepsilon > 0$. By Lemma 2.9, we sufficiently prove that $CR(T, A) \subset \Omega(T)$. Let us take any $x \in CR(T, A)$. Since x is a shadowable point of T with respect to A, there exists $\delta_x > 0$ such that every δ_x-pseudo orbit of T with respect to A through x is ε-shadowed. Let $\{x_g\}_{g\in G}$ be a δ-chain for x. By assumption, we obtain a point y such that $d(T_g(y), x_g) < \epsilon$ for any $g \in G$. Since $x \in CR(T, A)$, there exists $h \in G$ such that $l_A(h) > l_A(g)$ and $x_h = x$ for all $g \in G$. Then $d(x, y) < \varepsilon$ and $d(T_h(y), x) < \varepsilon$ (*i.e* $y, T_g(y) \in B_d(x, \varepsilon)$). This implies that $T_g(y) \in B_d(x, \varepsilon) \cap T_g(B_d(x, \varepsilon))$ and then $x \in \Omega(T)$. □

We say that a sequence $\{x_g\}_{g\in G}$ of X is through some subset $K \subset X$ if $x_e \in K$. We shall use the following auxiliary definition.

Definition 3.3. We say that a continuous action $T \in Act(G, X)$ has the *shadowing property through* K *with respect to* A if for every $\epsilon > 0$ there is $\delta > 0$ such that every δ-pseudo orbit of T with respect to A through K can be ϵ-shadowed.

Clearly if T has the shadowing property through K then every point in K is shadowable. The converse is true when K is compact by the following result.

Lemma 3.4. *Let $T \in Act(G, X)$ has the shadowing property through a compact subset K if and only if every point in K is shadowable.*

Proof. We only have to prove the sufficiency. Let A be a finitely generating set of G. Suppose that there is a compact subset K of X such that every point in K is shadowable but T does not have the shadowing property through K. Then there are $\epsilon > 0$ and a sequence $\{x_g^k\}_{g\in G}$ of $\frac{1}{k}$-pseudo orbit with respect to A through K which cannot be 2ϵ-shadowed for all $k \in \mathbb{N}$. Since K is compact, we can assume that $x_e^k \to p$ for some $p \in K$. Since $p \in K$, we have that K is shadowable. Then we can take $\delta > 0$ from the shadowableness of p for the above ϵ. We define a sequence $\{z_g^k\}_{g\in G}$ by

$$z_g^k = \begin{cases} x_g^k & g \neq e,\ k \in \mathbb{N} \\ p & g = e. \end{cases}$$

Then we have

$$d(T_a z_g^k, z_{ag}^k) = \begin{cases} d(T_a x_g^k, x_{ag}^k) & g \notin \{e, a^{-1} : a \in A\}, \\ d(T_a x_{a^{-1}}^k, p) & g = a^{-1}, \\ d(T_a p, x_a^k) & g = e. \end{cases}$$

Hence we get

$$d(T_a z_g^k, z_{ag}^k) \leq \begin{cases} \frac{1}{k} & g \notin \{e, a^{-1} : a \in A\}, \\ d(x_e^k, p) + \frac{1}{k} & g = a^{-1}, \\ d(T_a p, T_a(x_e^k)) + \frac{1}{k} & g = e \end{cases}$$

As T is uniformly continuous and $x_e^k \to p$, we obtain that $\{z_g^k\}_{g\in G}$ is a δ-pseudo orbit for k large. Then for such a k it follows that there is a $y_k \in X$ such that $d(T_g(y_k), z_g^k) \leq \epsilon$ for every $g \in G$. It follows that $d(T_a(y_k), x_g^k) \leq \epsilon$ for $g \neq e$. Since

$$d(y_k, z_e^k) \leq d(y_k, p) + d(p, x_e^k) \leq \epsilon + d(p, x_e^k).$$

We have that $d(T_g(y_k), x_g^k) \leq 2\epsilon$ for $g = e$ with k large. Consequently we have that $\{x_g^k\}_{g\in G}$ can be 2ϵ-shadowed for k large. The contradiction completes the proof. □

But Morales [3] introduced an counter example where the above Lemma 3.4 is not established if K is noncompact.

THEOREM 3.5. *$T \in Act(G, X)$ has the shadowing property if and only if $Sh(T) = X$.*

Proof. By taking $K = X$ in Lemma 3.4, we have that T has the shadowing property if and only if $Sh(T) = X$. □

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