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A MATHEMATICAL MODEL OF HEAT EMISSION ON THE EPIDERMIS OF A HUMAN BODY

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Abstract. We develop a mathematical model of heat emission on the epidermis of a human body. We present a global existence theorem of solutions for a nonlinear model system of coupled partial differential equations.

1. Introduction

Methods of mathematical modeling of heat-exchange processes in the human body are used in various problems in the fields of medicine, physiology, athletics, the garment industry, and in the design of survival systems.

The aim of this paper is to construct an appropriate model to explain movements of body temperature on the epidermis of the human body.

Heat-exchange process models have been studied in a large scale since the 1940s. In recent years, with the development of modern computational techniques and the more detailed physiological information available, the mathematical models have become more refined and more complex. With the help of modern theoretical and applicable mathematical technique, complex models can be worked out taking into account the anatomical structure of the body, different heat-transport mechanisms, and the effect of the thermoregulatory system.

This paper presents a new approach to construct a thermodynamic model of the human body as an open system under steady state conditions, correlated with the effect of deep body influence on the heat fluxes at the shell surface (skin). This explains the heat-transfer processes from the inner layer to the epidermis of the human body.

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2. Derivation of a model of heat emission on the epidermis

Normal human body temperature, also known as normothermia or euthermia, is a concept that depends upon the place in the body at which the measurement is made, and the time of day and level of activity of the person.

The body's extremities are colder than the body core. The time of day and other circumstances also affect on the body's temperature. For example, the core body temperature of an individual tends to have the lowest value in the second half of the sleep cycle. The lowest point (called nadir) is known as one of the primary markers for circadian rhythms. The body temperature also changes when a person is hungry, sleepy, or cold. Body temperature can be normally affected by such things as extreme physical activity, ovulation and pregnancy in women, and smoking. There is no single number that represents a normal or healthy temperature for all people under all circumstances using any place of measurement. Let us say that a distributional measurement ρ represents in direct proportion to the body temperature, and m is a distributional measurement inverse proportional to the body temperature. For our analysis, we do not specify the exact factors of ρ and m.

Let Ω^λ be a dermis part of human body with thickness $\lambda > 0$. The dermis Ω^λ is bonded along the top by a very thin layer, so called, epidermis layer, and the skin S is the surface of the epidermis layer. We assume that S is flat, bounded open subset of 2 dimensional plane that can be considered as the xy-plane, and we will denote its points by $\tilde{x} \equiv (x_1, x_2) \in S$. We also assume that the 3 dimensional subset Ω^λ is a cylindrical region $\Omega^\lambda = S \times (-\lambda, 0)$, and denote its points by $x \equiv (x_1, x_2, x_3) \in \Omega^\lambda$. Since the real body skin is locally diffeomorphic to S, the constraints for S and Ω^λ are not artificial for the study on the emission of body temperature. The region Ω^λ is a good heat transfer of conductivity

$$\frac{k\rho}{\lambda^2 m},$$

where k is a proportional constant and we presume the heat conductivity is in inverse proportion to the *square* of the thickness λ.

The heat is transferred mainly through the skin pores S_1 of the skin surface S. In fact, the skin S is divided into two sub-regions, S_1, S_2 where the sub-region S_2 is assumed to be smooth, and the pores S_1 are isolated. Due to the λ-scaling of the thickness, the epidermis layer bonded along the top of the dermis layer Ω^λ has a (vertical) heat-resistance $\lambda\, r$ to a layer in S_1. The skin layer S has the horizontal heat field (flux) $\lambda\,\sigma$ and the (horizontal) heat-resistance λR.

Let $u(x,t)$ denote the temperature distribution in Ω^λ and $\bar{v}(\tilde{x},t)$ the temperature at the point $\tilde{x}$ and time t that goes across S_1. There is no temperature exchanging on S_2, so we put $\bar{v}(\tilde{x},t) = 0$ for $\tilde{x} \in \bar{S_2}$ – this only means that the swapping temperature between the dermis layer Ω^λ and skin S_2 is null, however, it does not mean that the temperature on S_2 is zero. The temperature on S is not reflected in $\bar{v}(\tilde{x},t)$(Remark 2.1).

The changing of storage temperature amount in time with respect to $\bar{v}$ is $\frac{1}{\lambda}\frac{\partial}{\partial t}C$, and so C is a function of $\bar{v}$, that is, $C(\bar{v})$. We observe that

$$\frac{\partial}{\partial t}C(\bar{v}) = C'(\bar{v})\frac{\partial}{\partial t}\bar{v},$$

where $C'(\bar{v})$ denotes the specific heat at the temperature, called the heat-capacitance, and $C'(\bar{v})$ is a positive quantity. It follows that the corresponding function $C(\cdot)$ is *monotone*. Also it is natural to say that the heat-flux σ depends on the amount of the field $\tilde{\nabla}\bar{v}$. The dependence on $\bar{v}$ for C and on $\tilde{\nabla}\bar{v}$ for σ delivers the *nonlinearities* of the system.

In summary, we have the following system of equations :

$$\begin{aligned}
-\nabla\cdot \frac{k\rho}{\lambda^2 m}\nabla u(x,t) &= 0, \qquad x \in \Omega^\lambda,\\
\frac{k\rho}{\lambda^2 m}\frac{\partial u}{\partial x_3} &= \begin{cases} \frac{1}{\lambda}f(\tilde{x},t), & x_3 = \lambda, \quad \tilde{x} \in S,\\ \frac{1}{r}u, & x_3 = 0, \quad \tilde{x} \in S_2,\\ \frac{1}{\lambda r}(u - \bar{v}), & x_3 = 0, \quad \tilde{x} \in S_1, \end{cases}\\
u &= 0 \qquad \textit{on} \quad \partial S \times (-\lambda, 0),\\
\frac{1}{\lambda}\frac{\partial}{\partial t}C(\bar{v}) - \tilde{\nabla}\cdot \lambda\sigma(\tilde{x},\tilde{\nabla}\bar{v}) + \frac{1}{\lambda r}(\bar{v} - u) + \frac{1}{\lambda R}\bar{v} &= g, \quad \tilde{x} \in S_1, x_3 = 0,\\
\bar{v} &= 0 \qquad \textit{on} \quad \partial S_1,
\end{aligned}$$

where f and g are the given external forces on the bottom and the top surfaces. The measurable function ρ, m are assumed to be bounded.

Remark 2.1. *1. We can say that skin is a closed 2-manifold without boundary, and the part $\partial S \times (-\lambda, 0)$ of the boundary is empty. The Dirichlet boundary condition on $\partial S \times (-\lambda, 0)$ means that no heat-exchange through $\partial S \times (-\lambda, 0)$ is assumed.*

2. The temperature component, say $\hat{v}(\tilde{x},t)$, on S which is not affected by the heat from skin pores is reflected in the function $g(\tilde{x},t)$, for example, as in a heat equation $g(\tilde{x},t) = \frac{\partial}{\partial t}\hat{v}(\tilde{x},t) - \Delta\hat{v}(\tilde{x},t)$.

3. Mathematical analysis: weak formulation

First, we rescale the vertical axis. For it, let $\Omega^1 = \Omega$ and we employ a change of variables $x_3 \equiv \lambda z$, $\frac{\partial}{\partial z} = \lambda \frac{\partial}{\partial x_3}$ to get

$$(3.1) \qquad -\tilde{\nabla}\cdot \frac{k\rho}{m}\tilde{\nabla} u(\tilde{x}, z, t) - \frac{\partial}{\partial z}\frac{k\rho}{\lambda^2 m}\frac{\partial}{\partial z} u(\tilde{x}, z, t) = 0, \quad (\tilde{x}, z) \in \Omega^1$$

$$\frac{k\rho}{\lambda^2 m}\frac{\partial u}{\partial x_3} = \begin{cases} f(\tilde{x}, t) & \text{on } S \times \{-1\}, \\ \frac{1}{r} u & \text{on } S_2 \times \{0\}, \\ \frac{1}{r}(u - \bar{v}) & \text{on } S_1 \times \{0\}, \end{cases}$$

$$u = 0 \qquad \text{on } \partial S \times (-1, 0),$$

$$(3.2) \qquad \frac{\partial}{\partial t} C(\bar{v}) - \tilde{\nabla}\cdot \lambda^2 \sigma(\tilde{x}, \tilde{\nabla}\bar{v}) + \frac{1}{r}(\bar{v} - u) + \frac{1}{R}\bar{v} = g, \quad \text{on } S_1 \times \{0\},$$

$$\bar{v} = 0 \qquad \text{on } \partial S_1$$

with appropriate initial and boundary conditions. Now the parameter $\lambda > 0$ affects the geometry only through the skin pore array S_1 at $z = 0$. All additional effects are contained in the size of various coefficients in the above system.

We put $v := C(\bar{v})$ and let s be the inverse function of C^{-1}, so that we have $\bar{v} = s(v)$. We assume that the continuously differentiable function s increases the $p - 1$ order, that is to say, $|s(\tau)| \le c|\tau|^{p-1}$ for all $\tau \in \mathbb{R}$. We also impose an assumption that $\sigma(\tilde{x}, \widetilde{\nabla} s(v)) = \alpha(\tilde{x}, \widetilde{\nabla} v)$ for some measurable function $\alpha : S \times \mathbb{R}^2 \to \mathbb{R}^2$ which satisfies Leray-Lions type conditions: that is, there are positive constants c_1, c_2 such that for all $\xi, \eta \in \mathbb{R}^2$ and almost every $\tilde{x} \in S$

$$(3.3) \qquad \alpha(\tilde{x}, \xi) \cdot \xi \ge c_1 |\xi|^p,$$

$$(3.4) \qquad |\alpha(\tilde{x}, \xi)| \le c_2 |\xi|^{p-1},$$

$$(3.5) \qquad (\alpha(\tilde{x}, \xi) - \alpha(\tilde{x}, \eta)) \cdot (\xi - \eta) > 0 \quad \text{for } \xi \ne \eta.$$

A fundamental example of the operator $-\widetilde{\nabla} \cdot \alpha(\tilde{x}, \widetilde{\nabla} u)$ is the p-Laplacian $-\widetilde{\Delta}_p\, u := -\widetilde{\nabla} \cdot |\widetilde{\nabla} u|^{p-2} \widetilde{\nabla} u$. Then we have a semi-linear equation

$$(3.6) \qquad \frac{\partial}{\partial t} v - \lambda^2 \tilde{\nabla}\cdot\ \alpha(\tilde{x},\ \tilde{\nabla} v) + \frac{1}{r}\{s(v) - u\} + \frac{1}{R} s(v) = g(\tilde{x}, t)$$

from the equation (3.2).

In order to obtain an appropriate weak formulation, we introduce two function spaces,

$$\mathcal{V}_0 \equiv \{u \in W^{1,2}(\Omega) : u = 0 \quad on\ \partial S \times (-1, 0)\},$$
$$\mathcal{V}_1 \equiv W_0^{1,p}(S_1) \qquad (p \geq 2).$$

By the zero-extension to S_2, we regard $v \in W_0^{1,p}(S_1)$ as an element in $L^p(S)$. Then for $\phi \in \mathcal{V}_0$, we multiply both sides of the equation (3.1) by ϕ and integrate over Ω to obtain

$$-\int_\Omega \tilde{\nabla} \cdot \frac{k\rho}{m} \tilde{\nabla} u(\tilde{x}, z, t) \phi(\tilde{x}, z) d\tilde{x} dz - \int_\Omega \frac{\partial}{\partial z} \frac{k\rho}{\lambda^2 m} \frac{\partial}{\partial z} u(\tilde{x}, z, t) \phi(\tilde{x}, z) d\tilde{x} dz = 0.$$

An application of Green's theorem yields

$$\int_\Omega \frac{k\rho}{m} \tilde{\nabla} u(\tilde{x}, z, t) \cdot \tilde{\nabla} \phi(\tilde{x}, z) d\tilde{x} dz - \int_{-1}^0 \int_{\partial S} \frac{k\rho}{m} \left(u(\tilde{x}, z, t) \cdot \nu\right) \phi(\tilde{x}, z) dS(\tilde{x}) dz$$
$$+ \int_\Omega \frac{k\rho}{\lambda^2 m} \frac{\partial}{\partial z} u(\tilde{x}, z, t) \frac{\partial}{\partial z} \phi(\tilde{x}, z) d\tilde{x} dz - \int_S \frac{k\rho}{\lambda^2 m} \frac{\partial}{\partial z} u(\tilde{x}, z, t)\, \phi \Big|_{z=0}^{z=1} d\tilde{x} = 0,$$

where ν is the unit outward normal vector. The boundary conditions show that this is equivalent to

$$\int_\Omega \frac{k\rho}{m} \tilde{\nabla} u(\tilde{x}, z, t) \cdot \tilde{\nabla} \phi(\tilde{x}, z) d\tilde{x} dz + \int_\Omega \frac{k\rho}{\lambda^2 m} \frac{\partial}{\partial z} u(\tilde{x}, z, t) \frac{\partial}{\partial z} \phi(\tilde{x}, z) d\tilde{x} dz$$
$$+ \int_S \frac{1}{r} \{u(\tilde{x}, 0, t) - s(v)(\tilde{x}, t)\} \phi(\tilde{x}, 0) d\tilde{x} = \int_S f(\tilde{x}, t) \phi(\tilde{x}, 1) d\tilde{x}. \tag{3.7}$$

For $\varphi \in \mathcal{V}_1$, multiplying the two sides of the equation (3.6) by φ and integrating over S show that

$$\int_{S_1} \frac{\partial v}{\partial t} \varphi d\tilde{x} + \lambda^2 \int_{S_1} \alpha(\tilde{x}, \tilde{\nabla} v) \cdot \tilde{\nabla} \varphi d\tilde{x} + \int_{S_1} \frac{1}{R} s(v) \varphi d\tilde{x}$$
$$+ \int_{S_1} \frac{1}{r} \{s(v) - u(\tilde{x}, 0, t)\} \varphi d\tilde{x} = \int_{S_1} g(\tilde{x}, t) d\tilde{x}. \tag{3.8}$$

Thus a solution of system (3.1), (3.6) satisfies $u(t) \in \mathcal{V}_0$, $v(t) \in \mathcal{V}_1$ for $0 < t < T$, and (3.7), (3.8) hold for each $\phi \in \mathcal{V}_0$, $\varphi \in \mathcal{V}_1$. Conversely, it follows directly that an appropriately smooth solution of (3.7) and (3.8) will satisfy the system (3.1), (3.6) above.

In order to more clearly display the structure of the equations (3.7) and (3.8), we specify some notation. Define two forms a_1, a_2 as follows. For u_1, $u_2 \in \mathcal{V}_0$,

$$a_1(u_1, u_2) \equiv \int_\Omega \left(\frac{k\rho}{m} \tilde{\nabla} u_1 \cdot \tilde{\nabla} u_2 + \frac{k\rho}{\lambda^2 m} \frac{\partial u_1}{\partial z} \frac{\partial u_2}{\partial z} \right) d\tilde{x} dz.$$

This determines a family of linear operators $\mathcal{A}_1 : \mathcal{V}_0 \to \mathcal{V}_0'$ by

$$\mathcal{A}_1 u_1(u_2) \equiv a_1(u_1, u_2), \qquad u_1, u_2 \in \mathcal{V}_0.$$

For v_1, $v_2 \in \mathcal{V}_1$,

$$a_2(v_1, v_2) \equiv \int_{S_1} \left(\lambda^2 \alpha(\tilde{x}, \tilde{\nabla} v_1) \cdot \tilde{\nabla} v_2 + \frac{1}{R} s(v_1) v_2 \right) d\tilde{x} \tag{3.9}$$

determines the family of nonlinear operators $\mathcal{A}_2 : \mathcal{V}_1 \to \mathcal{V}_1'$ by

$$\mathcal{A}_2 v_1(v_2) \equiv a_2(v_1, v_2), \qquad v_1, v_2 \in \mathcal{V}_1.$$

We introduce operators for the "trace" at $z = 0$ and $z = 1$, respectively. Define $\gamma_0 : \mathcal{V}_0 \to L^2(S)$, $\gamma_0^* : L^2(S) \to \mathcal{V}_0'$ (dual), $\gamma_1 : \mathcal{V}_0 \to L^2(S)$, and $\gamma_1^* : L^2(S) \to \mathcal{V}_0'$ as follows:

$$\gamma_0 u(\varphi) \equiv \int_S u(\tilde{x}, 0) \varphi(\tilde{x}) d\tilde{x}, \quad \gamma_0^* v(\phi) \equiv \int_S v(\tilde{x}) \phi(\tilde{x}, 0) d\tilde{x} = \langle v, \gamma_0 \phi \rangle_{L^2(S)},$$

$$\gamma_1 u(\varphi) \equiv \int_S u(\tilde{x}, 1) \varphi(\tilde{x}) d\tilde{x}, \quad \gamma_1^* f(\phi) \equiv \int_S f(\tilde{x}) \phi(\tilde{x}, 1) d\tilde{x} = \langle f, \gamma_1 \phi \rangle_{L^p(S)}.$$

Using these definitions we can rewrite the equations (3.7) and (3.8) in the following forms.
Find $u \in C([0, T]; \mathcal{V}_0)$ and $v \in C([0, T]; \mathcal{V}_1) \cap C^1((0, T); L^p(S_1))$:

$$u(t) \in \mathcal{V}_0 : \mathcal{A}_1 u(t) + \frac{1}{r} \gamma_0^* \{ \gamma_0 u(t) - s(v)(t) \} = \gamma_1^*(f(t)) \qquad \text{in } \mathcal{V}_0' \tag{3.10}$$

$$v(t) \in \mathcal{V}_1 : \frac{\partial}{\partial t} v(t) + \mathcal{A}_2 v(t) + \frac{1}{r} \{ s(v)(t) - \gamma_0 u(t) \} = g \qquad \text{in } \mathcal{V}_1' \tag{3.11}$$

with initial condition $v(\tilde{x}, 0) = v_0(\tilde{x})$.

In the following section, we assume that $\inf\{\rho(x), m(x) \,|\, x \in \Omega\}$ is a positive real number. The notation $X \lesssim Y$ means that $X \leq CY$, where C is a fixed but unspecified constant.

4. Existence theorem

First we notice that the family of linear operators $\mathcal{A}_1 : \mathcal{V}_0 \to \mathcal{V}_0'$ is uniformly $\mathcal{V}_0$-coercive for $0 < \lambda \leq 1$. Indeed, using the generalized Poincaré lemma in [6], we have for $u \in \mathcal{V}_0$,

$$\begin{aligned} \mathcal{A}_1 u(u) &\geq \int_\Omega \frac{k\rho}{m} \mid \tilde{\nabla} u \mid^2 d\tilde{x} dz \\ &\geq \frac{1}{2} \int_\Omega \frac{k\rho}{m} \left| \tilde{\nabla} u \right|^2 d\tilde{x} dz + \frac{1}{2} \int_\Omega \frac{k\rho}{m} \left| \frac{\partial u}{\partial x_1} \right|^2 d\tilde{x} dz \gtrsim \|u\|_{\mathcal{V}_0}^2. \end{aligned}$$

Thus the linear operator

$$\mathcal{A}_1 + \frac{1}{r}\gamma_0^*\gamma_0 : \mathcal{V}_0 \longrightarrow \mathcal{V}_0'$$

is uniformly coercive. Hence for any $\gamma_1^*(f) + \frac{1}{r}\gamma_0^*\bar{v} \in \mathcal{V}_0'$, there is a unique $u(t) \in \mathcal{V}_0$ satisfying the equation (3.10) by the Lax-Milgram Theorem. Therefore the equation (3.10) is equivalent to

$$u = \left(\mathcal{A}_1 + \frac{1}{r}\gamma_0^*\gamma_0\right)^{-1}\left(\gamma_1^*(f) + \frac{1}{r}\gamma_0^* s(v)\right). \tag{4.1}$$

Substituting (4.1) into equation (3.11), we get

$$\begin{aligned}\frac{\partial v}{\partial t} &+ \left\{\frac{1}{r}\left(s - \gamma_0\left(\mathcal{A}_1 + \frac{1}{r}\gamma_0^*\gamma_0\right)^{-1}\frac{1}{r}\gamma_0^* s\right) + \mathcal{A}_2\right\} v \\ &= \left\{\frac{1}{r}\gamma_0\left(\mathcal{A}_1 + \frac{1}{r}\gamma_0^*\gamma_0\right)^{-1}\gamma_1^*\right\} f + g \qquad \text{in } \mathcal{V}_1'.\end{aligned}$$

We define a corresponding unbounded operator $\mathbf{A} : D(\mathbf{A}) \to L^q(S_1)$, $\frac{1}{p} + \frac{1}{q} = 1$. The domain of $\mathbf{A}$ is $D(\mathbf{A}) \equiv \{v \in \mathcal{V}_1 : \mathbf{A}v \in L^q(S_1)\}$, and it is defined on this domain by

$$\mathbf{A} \equiv \frac{1}{r}\left(s - \gamma_0\left(\mathcal{A}_1 + \frac{1}{r}\gamma_0^*\gamma_0\right)^{-1}\frac{1}{r}\gamma_0^* s\right) + \mathcal{A}_2.$$

1. An application of Hölder's inequality and the condition (3.4) yield

$$\begin{aligned}\left|\int_{S_1} \alpha(\tilde{x}, \tilde{\nabla}v(\tilde{x})) \cdot \tilde{\nabla}\phi(\tilde{x})\, d\tilde{x}\right| &\lesssim \int_{S_1} |\tilde{\nabla}v(\tilde{x})|^{p-1}|\tilde{\nabla}\phi(\tilde{x})|\, d\tilde{x} \\ &\lesssim \|\tilde{\nabla}v\|_{L^p}^{p-1}\|\tilde{\nabla}\phi\|_{L^p} \\ &\lesssim \|v\|_{W^{1,p}}^{p-1}\|\phi\|_{W^{1,p}}\end{aligned}$$

for $v, \phi \in \mathcal{V}_1$. This implies that the nonlinear operator $\mathcal{A}_2 : \mathcal{V}_1 \to \mathcal{V}_1'$ is continuous.

2. Next, we explain that the operator $\mathcal{A}_2$ is monotone:

$$(\mathcal{A}_2 u - \mathcal{A}_2 v)(u - v) \geq 0 \tag{4.2}$$

for all $u, v \in \mathcal{V}_1$. Clearly, the second term $\frac{1}{R}s : \mathcal{V}_1 \to \mathcal{V}_1'$ of (3.9) is monotone because $s'(\cdot) > 0$. To clarify the monotonicity of the first term of (3.9), we can note that the condition (3.5) implies that

$$\lambda^2 \left\{ \alpha(\tilde{x}, \widetilde{\nabla} u(\tilde{x})) - \alpha(\tilde{x}, \widetilde{\nabla} v(\tilde{x})) \right\} \cdot \widetilde{\nabla}(u-v)(\tilde{x}) \geq 0. \tag{4.3}$$

Integrating both sides of (4.3) on S_1 together with the monotonicity of $\frac{1}{R}s$, we can get the monotonicity of the operator $\mathcal{A}_2$.

3. Now, we verify that $\mathcal{A}_2$ is a coercive operator. The condition (3.3) leads to

$$\begin{aligned} \mathcal{A}_2 v(v) &= \lambda^2 \int_{S_1} \alpha(\tilde{x}, \tilde{\nabla} v) \cdot \tilde{\nabla} v \, d\tilde{x} + \frac{1}{R} \int_{S_1} s(v) v \, d\tilde{x} \\ &\gtrsim \int_{S_1} |\tilde{\nabla} v(\tilde{x})|^p d\tilde{x} + \|v\|_{L^p}^p \gtrsim \|v\|_{W^{1,p}}^p. \end{aligned}$$

4. From the fact that for any $v, \phi \in L^p(S_1)$,

$$\left| \int_{S_1} s(v)(\tilde{x}) \phi(\tilde{x}) d\tilde{x} \right| \lesssim \|v\|_{L^p}^{p-1} \|\phi\|_{L^p},$$

we notice that $s : L^p(S_1) \to L^p(S_1)'$ is continuous. Furthermore, by virtue of the Sobolev imbedding theorem, we observe that $s(v) \in L^2(S_1)$ for any $v \in \mathcal{V}_1$.

5. From the fact that

$$\gamma_0 \left(\mathcal{A}_1 + \frac{1}{r} \gamma_0^* \gamma_0 \right)^{-1} \frac{1}{r} \gamma_0^* : L^2(S_1) \to L^2(S_1)$$

is a contraction, we can notice that $I - \gamma_0 \left(\mathcal{A}_1 + \frac{1}{r} \gamma_0^* \gamma_0 \right)^{-1} \frac{1}{r} \gamma_0^*$ is continuous and monotone, where I represents the identity map. It follows that the composition operator $s - \gamma_0 \left(\mathcal{A}_1 + \frac{1}{r} \gamma_0^* \gamma_0 \right)^{-1} \frac{1}{r} \gamma_0^* s$ is continuous and monotone. Adding a positive multiple of this to $\mathcal{A}_2$ still retains a monotonicity. This illustrates the m-accretivity of $\mathbf{A}$.

6. Letting $F \equiv \left\{ \frac{1}{r} \gamma_0 \left(\mathcal{A}_1 + \frac{1}{r} \gamma_0^* \gamma_0 \right)^{-1} \gamma_1^* \right\} f + g$, by virtue of the heuristic existence theorem of nonlinear evolution equations(for example, see page 122 in [6]), we notice that there exists the unique solution of initial value problem

$$\frac{\partial v}{\partial t} + \mathbf{A}\, v = F$$

with $v(0) = v_0 \in L^2(S_1)$. Also, we can recover $u(t)$ via (4.1). Thereby we obtain:

THEOREM 4.1 (Existence of the solution u, v). *There exists a solution* $u \in L^\infty([0,T];\mathcal{V}_0)$, $v \in L^p([0,T];\mathcal{V}_1)$ *to the system (3.10) and (3.11).*

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