논리연구 21-3(2018) pp. 353-371

Algebraic Routley-Meyer-style semantics for the fuzzy logic MTL*

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【Abstract】This paper deals with Routley-Meyer-style semantics, which will be called *algebraic* Routley-Meyer-style semantics, for the fuzzy logic system **MTL**. First, we recall the monoidal t-norm logic **MTL** and its algebraic semantics. We next introduce algebraic Routley-Meyer-style semantics for it, and also connect this semantics with algebraic semantics.

【Key Words】(Algebraic) Routley-Meyer-style semantics, Kripke-style semantics, Algebraic semantics, Fuzzy logic, Substructural logic.

Received: Jul. 22, 2018. Revised: Sep. 17, 2018. Accepted: Sep. 7, 2018.

* This research was supported by "Research Base Construction Fund Support Program" funded by Chonbuk National University in 2018. I must thank the referees for their helpful comments.

1. Introduction

This paper investigates Routley-Meyer-style semantics, which is said to be *algebraic* Routley-Meyer-style semantics, for the *substructural fuzzy* logic MTL (Monoidal t-norm logic). Note that *substructural* logics lack structural rules like weakening or contraction, and *fuzzy* logics deal with vagueness. (Logics complete with respect to (w.r.t.) linearly ordered algebras are *fuzzy* in Cintula's sense (Cintula (2006)).)

For this, we first recall some relationships between substructural fuzzy logics and Kripke-style semantics. After Kripke first introduced the so-called *Kripke semantics* for modal and intuitionistic logics in Kripke (1963; 1965a; 1965b) using binary accessibility relations, many semantics generalizing them, the so-called *Kripke-style semantics*, have been provided for many-valued logics. As mentioned in Yang (2014a), there are at least three trends in generalization for many-valued logics. One is to provide model structures with binary relations, but without operations. Another is to provide model structures with both operations and binary relations. The other is to provide model structures with generalizations of binary relations.

As the present author mentioned in Yang (2014a; 2014b), various types of semantics in the second trend have been provided for infinite-valued or fuzzy logics. In particular, after semantics for the infinite-valued Łukasiewicz logic **Ł** was introduced by Urquhart (1986), many Kripke-style semantics have been provided for fuzzy logics based on t-norms (so called t-norm-based logics) by Montagna & Ono (2002), Montagna & Sacchetti (2003; 2004), and Diaconescu & Georgescu (2007). The author called this kind of semantics *algebraic* Kripke-style semantics, which are Kripke-style semantics being equivalent to algebraic semantics in that completeness is provided by this equivalence. Moreover, recently he has introduced such semantics for fuzzy logics based on more general structures such as uninorms (see Yang (2014a; 2014b; 2014c; 2016a; 2016b; 2018)).

This work is related to the third trend. Note that among the generalizations in this trend, the most well-known one is *Routley-Meyer* semantics with ternary relations. This semantics was first introduced for relevance logics and then generalized for other non-classical logics. Let us call semantics with ternary relations *Routley-Meyer-style semantics*. While algebraic Kripke-style semantics for fuzzy logics have been introduced, such Routley-Meyer-style semantics have not yet been introduced. This gives rise to the following natural question:

● Do algebraically complete fuzzy logics also have algebraic Routley-Meyer-style semantics?

This paper gives a positive answer for this question. As its verification, we provide algebraic Routley-Meyer-style semantics for the monoidal t-norm-based logic **MTL**. For this, first, in Section 2 we recall the fuzzy logic **MTL** and its algebraic semantics. In Section 3, we introduce algebraic Routley-Meyer-style semantics for it and connect this semantics with algebraic semantics.

For convenience, we shall adopt the notations and terminology similar to those in Montagna & Sacchetti (2003; 2004) and Yang (2016b; 2017b), and assume reader familiarity with them (together with the results found therein).

2. Preliminaries: MTL and its algebraic semantics

Here we briefly recall the system **MTL** and its algebraic semantics introduced in Yang (2016b) as preliminaries. **MTL** is based on a countable propositional language with formulas *Fm* built inductively as usual from a set of propositional variables *VAR*, binary connectives $\rightarrow$, &, $\wedge$, $\vee$, and constants **T**, **F**. Further definable connectives are:

df1. $\neg\phi := \phi \rightarrow \mathbf{F}$,
df2. $\phi \leftrightarrow \psi := (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$.

We may define **T** as **F** $\rightarrow$ **F**. For the rest of this paper, we use the customary notations and terminology, and the axiom systems to provide a consequence relation.

We start with the following axiom schemes and rules for the monoidal t-norm logic **MTL**.

Definition 2.1 (Yang (2016b)) **MTL** consists of the following axiom schemes and rules:

A1. $\phi \rightarrow \phi$ (self-implication, SI)

A2. $(\phi \wedge \psi) \to \phi$, $(\phi \wedge \psi) \to \psi$ ($\wedge$-elimination, $\wedge$-E)

A3. $((\phi\to\psi)\wedge(\phi\to\chi)) \to (\phi\to(\psi\wedge\chi))$ ($\wedge$-introduction, $\wedge$-I)

A4. $\phi \to (\phi \vee \psi)$, $\psi \to (\phi \vee \psi)$ ($\vee$-introduction, $\vee$-I)

A5. $((\phi\to\chi)\wedge(\psi\to\chi)) \to ((\phi\vee\psi)\to\chi)$ ($\vee$-elimination, $\vee$-E)

A6. $\mathbf{F} \to \phi$ (ex falsum quodlibet, EF)

A7. $\varphi \to \mathbf{T}$ (verum ex quodlibet, VE)

A8. $\phi \to (\psi \to \chi) \leftrightarrow \psi \to (\phi \to \chi)$ (permutation, PM)

A9. $(\varphi \to (\psi \to \chi)) \leftrightarrow ((\varphi \,\&\, \psi) \to \chi)$ (residuation, RES)

A10. $(\varphi \to \psi) \to ((\psi \to \chi) \to (\varphi \to \chi))$ (suffixing, SF)

A11. $(\varphi \,\&\, \psi) \to \varphi$ (weakening, W)

A12. $(\varphi \to \psi) \vee (\psi \to \varphi)$ (prelinearity, PL)

$\varphi \to \psi, \varphi \vdash \psi$ (mp)

$\varphi, \psi \vdash \varphi \wedge \psi$ (adj).

A *theory* over **MTL** is a set T of formulas. A *proof* in a sequence of formulas whose each member is either an axiom of **MTL** or a member of T or follows from some preceding members of the sequence using a rule of **MTL**. $T \vdash \phi$, more exactly $T \vdash_{\mathbf{MTL}} \phi$, means that ϕ is *provable* in T w.r.t. **MTL**, i.e., there is an **MTL**-proof of ϕ in T. A theory T is *inconsistent* if $T \vdash \mathbf{F}$; otherwise it is *consistent*.

The deduction theorem for **MTL** is as follows:

Proposition 2.3 (Hájek (1998)) Let T be a theory, and ϕ, ψ formulas.

$T \cup \{\phi\} \vdash_{\mathbf{MTL}} \psi$ iff there is n such that $T \vdash_{\mathbf{MTL}} \phi^n \to \psi$.

For convenience, "$\neg$", "$\wedge$", "$\vee$", and "$\rightarrow$" are used ambiguously as propositional connectives and as algebraic operators, but context should clarify their meanings.

Suitable algebraic structures for **MTL** are obtained as varieties of residuated lattice-ordered monoids (briefly, residuated monoids) in the sense of Galatos et al. (2007).

Definition 2.4 (i) An *integral commutative residuated monoid* is a structure $\mathbf{A} = (A, \top, \bot, \wedge, \vee, *, \rightarrow)$ such that:

(Ⅰ) $(A, \top, \bot, \wedge, \vee)$ is a bounded lattice with top element $\top$ and bottom element $\bot$.

(Ⅱ) $(A, *, \top)$ is a commutative monoid.

(Ⅲ) $y \leq x \rightarrow z$ iff $x * y \leq z$, for all $x, y, z \in A$ (residuation).

(ii) An *MTL-algebra* is an integral commutative residuated monoid satisfying: for all $x, y \in A$,

(PL^A) $(x \rightarrow y) \vee (y \rightarrow x) = \top$.

By x^n, we denote $x * \cdots * x$, n factors. Using $\rightarrow$ and $\bot$ we can define $\top$ as $\bot \rightarrow \bot$, and $\neg$ as in (df1).

An MTL-algebra is said to be *linearly ordered* if the ordering of its algebra is linear, i.e., $x \leq y$ or $y \leq x$ (equivalently, $x \wedge y = x$ or $x \wedge y = y$) for each pair x, y.

Definition 2.6 (Evaluation) Let $\mathcal{A}$ be an MTL-algebra. An *$\mathcal{A}$-evaluation* is a function $v : FOR \rightarrow \mathcal{A}$ satisfying: $v(\phi \rightarrow \psi) = v(\phi) \rightarrow v(\psi)$, $v(\phi \wedge \psi) = v(\phi) \wedge v(\psi)$, $v(\phi \vee \psi) = v(\phi) \vee$

$v(\psi)$, $v(\phi \ \& \ \psi) = v(\phi) * v(\psi)$, $v(\mathbf{T}) = \top$, $v(\mathbf{F}) = \bot$, (and hence $v(\neg\phi) = \neg v(\phi)$).

Definition 2.7 Let $\mathcal{A}$ be an MTL-algebra, T be a theory, ϕ be a formula, and K be a class of MTL-algebras.
(i) (Tautology) ϕ is a *tautology* in $\mathcal{A}$, briefly an *$\mathcal{A}$-tautology* (or *$\mathcal{A}$-valid*), if $v(\phi) = \top$ for each $\mathcal{A}$-evaluation v.
(ii) (Model) An $\mathcal{A}$-evaluation v is an *$\mathcal{A}$-model* of T if $v(\phi) = \top$ for each $\phi \in$ T. We denote the class of $\mathcal{A}$-models of T, by *Mod(T, $\mathcal{A}$)*.
(iii) (Semantic consequence) ϕ is a *semantic consequence* of T w.r.t. K, denoting by T $\vDash_{K} \phi$, if Mod(T, $\mathcal{A}$) = Mod(T $\cup$ $\{\phi\}$, $\mathcal{A}$) for each $\mathcal{A} \in$ K.

Definition 2.8 (**MTL**-algebra) Let $\mathcal{A}$, T, and ϕ be as in Definition 2.7. $\mathcal{A}$ is an ***MTL**-algebra* iff, whenever ϕ is MTL-provable in T (i.e. T $\vdash_{\mathbf{MTL}} \phi$), it is a semantic consequence of T w.r.t. the set $\{\mathcal{A}\}$ (i.e. T $\vDash_{\{A\}} \phi$), $\mathcal{A}$ a corresponding MTL-algebra). By *$MOD^{l}(MTL)$*, we denote the class of linearly ordered **MTL**-algebras. Finally, we write T $\vDash^{1}_{\mathbf{MTL}} \phi$ in place of T $\vDash_{MOD^{1}(\mathbf{MTL})} \phi$.

Theorem 2.9 (Strong completeness, Jenei & Montagna (2002)) Let T be a theory, and ϕ a formula. T $\vdash_{\mathbf{MTL}} \phi$ iff T $\vDash_{\mathbf{MTL}} \phi$ iff T $\vDash^{1}_{\mathbf{MTL}} \phi$.

Definition 2.10 An MTL-algebra is *standard* iff its lattice reduct is [0, 1].

Theorem 2.11 (Strong standard completeness, Jenei & Montagna (2002)) For **MTL**, the following are equivalent:
(1) $T \vdash_{\mathbf{MTL}} \phi$.
(2) For every standard **MTL**-algebra and evaluation v, if $v(\psi) = 1$ for all $\psi \in T$, then $v(\phi) = 1$.

3. Algebraic Routley-Meyer-style semantics

3.1 Semantics

We first introduce several Routley-Meyer-style frames.

Definition 3.1 (i) (Algebraic Kripke frame) An *algebraic Kripke frame* is a structure **X** = (X, ⊤, ⊥, ≤, *) such that (X, ⊤, ⊥, ≤, *) is a linearly ordered integral residuated monoid. The elements of **X** are called *nodes*.

(ii) (Algebraic Routley-Meyer frame) An *algebraic Routley-Meyer frame* is a structure **X** = (X, ⊤, ⊥, ≤, *, R) such that (X, ⊤, ⊥, ≤, *) is an algebraic Kripke frame and R ($\subseteq X^3$) satisfies the following postulates: for all $a \in X$,

p1. $R\top\top\top$

p2. $R\top aa$

p3. $Ra\top a$.

(iii) (MTL frame) An *MTL frame* is an algebraic Routley-Meyer frame, where * is conjunctive (i.e., $\bot * \top = \bot$)

and left-continuous (i.e., whenever $\sup\{x_i : i \in I\}$ exists, $x * \sup\{x_i : i \in I\} = \sup\{x * x_i : i \in I\}$), and so its residuum $\rightarrow$ is defined as $x \rightarrow y := \sup\{z: x * z \leq y\}$ for all x, y $\in$ X, and the following definitions and postulate hold: for all a, b, c, d $\in$ X,

df3. $R^2abcd := (\exists x)(Rabx \wedge Rxcd)$

df4. $R^2a(bc)d := (\exists x)(Raxd \wedge Rbcx)$

p_l. $R\top ab$ or $R\top ba$

p_i. Rabc implies $R\top bc$.

p_e. Rabc implies Rbac.

p_a. R^2abcd iff $R^2a(bc)d$.

Definition 3.1 (iii) ensures that an MTL frame has a supremum w.r.t. *, i.e., for every x, y $\in$ X, the set $\{z: x * z \leq y\}$ has the supremum. **X** is said to be *complete* if $\leq$ is a complete order.

An *evaluation* or *forcing* on an algebraic Routley-Meyer frame is a relation $\Vdash$ between nodes and propositional variables, and arbitrary formulas subject to the conditions below: for every propositional variable p,

(AHC) if $x \Vdash p$ and $y \leq x$, then $y \Vdash p$;

(min) $\bot \Vdash p$; and

for arbitrary formulas,

($\bot$) $x \Vdash$ **F** iff $x = \bot$;

(∧) x $\Vdash$ $\phi \wedge \psi$ = iff x $\Vdash$ ϕ and x $\Vdash$ ψ;

(∨) x $\Vdash$ $\phi \vee \psi$ iff x $\Vdash$ ϕ or x $\Vdash$ ψ;

(&) x $\Vdash$ ϕ & ψ iff there are y, z ∈ X such that Rzyx, y $\Vdash$ ϕ, and z $\Vdash$ ψ;

(→) x $\Vdash$ $\phi \to \psi$ iff for all y, z ∈ X, if Ryxz and y $\Vdash$ ϕ, then z $\Vdash$ ψ.

An evaluation or forcing on an MTL frame is an evaluation or forcing further satisfying that (max) for every atomic sentence p, {x : x $\Vdash$ p} has a maximum.

Definition 3.2 (i) (Algebraic Routley-Meyer model) An *algebraic Routley-Meyer model* is a pair (**X**, $\Vdash$), where **X** is an algebraic Routley-Meyer frame and $\Vdash$ is a forcing on **X**.
(ii) (MTL model) An *MTL model* is a pair (**X**, $\Vdash$), where **X** is an MTL frame and $\Vdash$ is a forcing on **X**. An MTL model (**X**, $\Vdash$) is said to be *complete* if **X** is a complete frame and $\Vdash$ is a forcing on **X**.

Definition 3.3 Given an algebraic Routley-Meyer model (**X**, $\Vdash$), a node x of **X** and a formula ϕ, we say that *x forces ϕ* to express x $\Vdash$ ϕ. We say that ϕ is *true* in (**X**, $\Vdash$) if t $\Vdash$ ϕ, and that ϕ is *valid* in the frame **X** (expressed by **X** $\vDash$ ϕ) if ϕ is true in (**X**, $\Vdash$) for every forcing $\Vdash$ on **X**.

Definition 3.4 An MTL frame **X** is an **MTL** frame if all axioms of **MTL** are valid in **X**. We say that an algebraic

Routley-Meyer model **X** is an ***MTL** model* if **X** is an **MTL** frame.

3.2 Soundness and completeness

For soundness and completeness for **MTL**, let $\vdash_{\mathbf{MTL}} \phi$ be the theoremhood of ϕ in **MTL**. For this, we first define R as follows:

(df5) Rabc := $c \leq b * a$.

We can easily show the following lemmas.

Lemma 3.5 (Cf, Yang (2016b)) (i) (Hereditary Lemma, HL) Let **X** be an algebraic Routley-Meyer frame. For any sentence ϕ and for all nodes x, y $\in$ **X**, if $x \Vdash \phi$ and $y \leq x$, then $y \Vdash \phi$.
(ii) Let $\Vdash$ be a forcing on an **MTL** frame, and ϕ a sentence. Then the set $\{x \in X : x \Vdash \phi\}$ has a maximum.

Lemma 3.6 $\top \Vdash \phi \to \psi$ iff for all $x \in X$, if $x \Vdash \phi$, then $x \Vdash \psi$.

Proof: By the condition ($\to$), we have that $\top \Vdash \phi \to \psi$ iff for all $x \in X$, if $Rx\top x$ and $x \Vdash \phi$, then $x \Vdash \psi$. Then, since we have $Ra\top a$ by the postulate (p3), we can ensure the claim by the condition ($\to$). □

Proposition 3.7 (Soundness) If $\vdash_{\mathbf{MTL}} \phi$, then ϕ is valid in every MTL frame.

Proof: We prove the validity of (p_i) and (p_e) as examples.

(p_i) We need to show that $\top \Vdash \phi \rightarrow (\psi \rightarrow \phi)$. By Lemma 3.6, it suffices to assume $x \Vdash \phi$ and show $x \Vdash \psi \rightarrow \phi$. To show this, using the condition ($\rightarrow$), we further assume that Ryxz and $y \Vdash \psi$, and show that $z \Vdash \phi$. By the suppositions and the postulate (p_i), we have $R\top xz$. Then, by (df5), we further have that $z \leq x * \top = x$; therefore, $z \Vdash \phi$ by Lemma 3.5 (i).

(p_e) We need to show that $\top \Vdash (\phi \,\&\, \psi) \rightarrow (\psi \,\&\, \phi)$. By Lemma 3.6, it suffices to assume $x \Vdash \phi \,\&\, \psi$ and show $x \Vdash \psi \,\&\, \phi$. To show this, using the condition (&), we may instead assume that Ryzx, $z \Vdash \psi$, and $y \Vdash \phi$. By the suppositions and the postulate (p_e), we have Rzyx. Then, by the condition (&), we further have $x \Vdash \psi \,\&\, \phi$.

The proof for the other cases is left to the interested reader. □

Now, we introduce an important result between postulates for MTL frames and algebraic (in)equations corresponding to the structural axioms of **MTL**.

Proposition 3.8 The postulates for MTL frames introduced in Definition 3.1 are reducible to algebraic (in)equations corresponding to the structural axioms of **MTL** introduced in Definition 2.1 (see Definition 2.4).

Proof: We consider (p_l) and (p_i) as examples.

(p_l) We show that this postulate corresponds to linearly orderedness. Using (p_l) and (df5), we obtain that $b \leq a * \top = a$ or $a \leq b * \top = b$, i.e., $b \leq a$ or $a \leq b$.

(p_i) We show that this postulate corresponds to the integral property, i.e., $a \leq \top$ for any a. Using (p_i) and (df5), we obtain that $c \leq b * a$ implies $c \leq b * \top = b$; therefore, $a \leq \top$ since $b * a \leq b = b * \top$.

The proof for the other cases is left to the interested reader. □

By a *chain*, we mean a linearly ordered algebra. Note that the relation R can be defined as in (df5) and the postulates for MTL frames introduced in Definition 3.1 are reducible to their corresponding algebraic (in)equations (see Proposition 3.8). The next proposition connects algebraic Routley-Meyer semantics and algebraic semantics for **MTL**.

Proposition 3.9 (i) The $\{\top, \bot, \leq, *\}$ reduct of an MTL chain A is an MTL frame, which is complete iff A is complete.

(ii) Let **X** = $(X, \top, \bot, \leq, *)$ be an MTL frame. Then the structure **A** = $(X, \top, \bot, \max, \min, *, \rightarrow)$ is an MTL-algebra (where *max* and *min* are meant w.r.t. $\leq$).

(iii) Let **X** be the $\{\top, \bot, \leq, *\}$ reduct of an MTL chain A, and let v be an evaluation in A. Let for every atomic formula p and for every $x \in A$, $x \Vdash p$ iff $x \leq v(p)$. Then (**X**, $\Vdash$) is an MTL model, and for every formula ϕ and for every $x \in$

A, we obtain that: $x \Vdash \phi$ iff $x \leq v(\phi)$.

(iv) Let $(\mathbf{X}, \Vdash)$ be an MTL model, and let A be the MTL-algebra defined as in (ii). Define for every atomic formula p, $v(p) = \max\{x \in X : x \Vdash p\}$. Then for every formula ϕ, $v(\phi) = \max\{x \in X : x \Vdash \phi\}$.

Proof: The proof for (i) and (ii) is easy. We prove (iii) and (iv).

As regards to claim (iii), we consider the induction steps corresponding to the cases where $\phi = \psi \,\&\, \chi$ and $\phi = \psi \to \chi$. (The proof for the other cases are trivial.)

Suppose $\phi = \psi \,\&\, \chi$. By the condition (&), $x \Vdash \psi \,\&\, \chi$ iff there are $y, z \in X$ such that $z \Vdash \psi$, $y \Vdash \chi$, and Ryzx, hence by the induction hypothesis, $z \Vdash \psi$ and $y \Vdash \chi$ iff $z \leq v(\psi)$ and $y \leq v(\chi)$. Then, by (df5), we further have that $x \leq z * y$. Therefore, it holds true that $x \leq z * y \leq v(\psi) * v(\chi) = v(\psi \,\&\, \chi)$. Conversely, if $x \leq v(\psi) * v(\chi) = v(\psi \,\&\, \chi)$, then take $z = v(\psi)$ and $y = v(\chi)$. Then we have $x \leq z * y$, $y \Vdash \psi$, and $z \Vdash \chi$, therefore $x \Vdash \psi \,\&\, \chi$ since Ryzx := $x \leq z * y$ by (df5).

Suppose $\phi = \psi \to \chi$. By the condition (→), $x \Vdash \psi \to \chi$ iff for all $y, z \in X$, if Ryxz and $y \Vdash \psi$, then $z \Vdash \chi$, hence by the induction hypothesis and (df5), iff $z \leq x * y$ and $y \leq v(\psi)$ only if $z \leq v(\chi)$, hence iff $x * v(\psi) \leq v(\chi)$ and thus iff $v(\psi) * x \leq v(\chi)$, therefore by residuation, iff $x \leq v(\psi) \to v(\chi) = v(\psi \to \chi)$, as desired.

For claim (iv), as in (iii), we consider the induction steps corresponding to the cases where $\phi = \psi \,\&\, \chi$ and $\phi = \psi \to \chi$.

For this, for any formula ϕ, by ϕ°, we denote the set $\{x : x \Vdash \phi\}$.

Suppose $\phi = \psi \& \chi$. Let $b = \max(\psi^{\circ})$, $c = \max(\chi^{\circ})$, and $a = b * c$. We show that $a = \max(\phi^{\circ})$. First, we have that $b \Vdash \psi$, $c \Vdash \chi$, and $a \leq b * c$. Thus, because of (iii), we can ensure that $a \Vdash \psi \& \chi$. To show that a is a maximum element, let $a < x$. Suppose that there are y, z such that $y \Vdash \psi$, $z \Vdash \chi$, and $x \leq y * z$. We have that $b * c < y * z$. But, since $y \Vdash \psi$ and $z \Vdash \chi$, we also obtain that $y \leq b$ and $z \leq c$; therefore, $y * z \leq b * c$, a contradiction. Hence $a \Vdash \phi$ but $x \nVdash \phi$.

Suppose $\phi = \psi \rightarrow \chi$. Let $b = \max(\psi^{\circ})$, $c = \max(\chi^{\circ})$, and $a = \sup\{x : b * x \leq c\}$. We show that $a = \max(\phi^{\circ})$. First, we have that $b * a \leq c$. Since for every x, $x \Vdash \psi$ implies $x \leq b$, we have $x * a \leq b * a \leq c$ and thus $x * a \Vdash \chi$; therefore, $a \Vdash \phi$. To show that a is a maximum element, let $a < x$. Then, $b * x > c$ and thus $b * x \nVdash \chi$. Moreover, since $b \Vdash \psi$, we further have that $x \nVdash \phi$. □

Theorem 3.10 (Strong completeness)
(i) **MTL** is strongly complete w.r.t. the class of all MTL frames.
(ii) **MTL** is strongly complete w.r.t. the class of complete MTL frames.

Proof: (i) and (ii) follow from Proposition 3.9 and Theorem 2.9, and from Proposition 3.9 and Theorem 2.11, respectively. □

4. Concluding remark

Here the present author investigated just algebraic Routley-Meyer-style semantics for some **MTL**. Note that the author provided not only algebraic but also *set-theoretic* Kripke-style semantics for fuzzy logics in Yang (2015a; 2016c, 2017a, 2017b, 2018). This gives rise to a question to consider set-theoretic Routley-Meyer-style semantics for **MTL**. I leave this for another work.

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퍼지 논리 MTL을 위한 대수적 루트리-마이어형 의미론

양 은 석

이 글에서 우리는 대수적 루트리-마이어형 의미론이라고 불릴 의미론을 연구한다. 이를 위하여 먼저 퍼지 논리 체계 MTL과 대수적 의미론을 소개한다. 다음으로 이 체계를 위한 대수적 루트리-마이어형 의미론을 제공한 후, 이를 대수적 의미론과 연관 짓는다.

주요어: (대수적) 루트리-마이어형 의미론, 크립키형 의미론, 대수적 의미론, 퍼지 논리, 준구조 논리