

Temperature Dependence of Efficiency Droop in GaN-based Blue Light-emitting Diodes from 20 to 80°C

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We investigate the temperature dependence of efficiency droop in InGaN/GaN multiple-quantum-well (MQW) blue light-emitting diodes (LEDs) in the temperature range from 20 to 80°C. When the external quantum efficiency (EQE) and the wall-plug efficiency (WPE) of the LED sample were measured as injection current and temperature varied, the droop of EQE and WPE was found to be reduced with increasing temperature. As the temperature increased from 20 to 80°C, the droop ratio of EQE was decreased from 16% to 14%. This reduction in efficiency droop with temperature can be interpreted by a temperature-dependent carrier distribution in the MQWs. When the carrier distribution and radiative recombination rate in MQWs were simulated and compared for different temperatures, the carrier distribution was found to become increasingly homogeneous as the temperature increased, which is believed to partly contribute to the reduction in efficiency droop with increasing temperature.

Keywords : GaN, Light-emitting diode, Quantum well, Efficient droop, Temperature
OCIS codes : (230.3670) Light-emitting diodes; (160.6000) Semiconductor materials

I. INTRODUCTION

Recently, the use of light-emitting diodes (LEDs) has considerably increased in general lighting and display applications, thanks to the high efficiency and eco-friendliness of these sources [1-3]. The peak external quantum efficiency (EQE) of InGaN/GaN-based blue LEDs has been demonstrated to be more than 80% [4]. Despite such high peak efficiency at relatively low current density, InGaN blue LEDs undergo significant efficiency droop as the current density increases, which could limit their use in high-current-driven applications [5-8]. Several mechanisms, such as Auger recombination [9], electron leakage [10], saturation of spontaneous emission rate [11], and reduction in effective active volume [12], have been proposed to explain the efficiency droop phenomenon. However, the true origin of the efficiency droop has not been clearly identified yet.

In addition to this *current droop*, the issue of temperature-dependent reductions in EQE, known as *thermal droop*, has attracted increasing attention [13-16]. It is important to

understand the mechanisms for thermal droop as well as for current droop in InGaN LEDs, for use in high-power, temperature-stable lighting applications. There have been a number of studies on the mechanisms of temperature-dependent carrier recombination in InGaN quantum wells (QWs) to study the thermal droop of LED efficiency [17-20]. These previous studies have measured temperature-dependent EQE curves over a wide temperature range that can be extended below 100 K on the low-temperature side or above 400 K on the high-temperature side. However, the temperature-dependence of efficiency droop in the temperature range between 20 and 80°C has not been very closely investigated, though most applications require LEDs to operate in this temperature range.

In this paper, we experimentally investigate the temperature dependence of efficiency droop in (In,Ga)N multiple-quantum-well (MQW) blue LEDs, focusing on the temperature range from 20 to 80°C. It will be shown that the efficiency droop can be reduced as the temperature increases, which has not been demonstrated in previous

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works on the temperature-dependence of LED efficiency. To interpret the measured temperature-dependent efficiency droop, the carrier distribution in InGaN MQWs is investigated using numerical simulation, and the role of carrier distribution in mitigating the problem of efficiency droop with increasing temperature will be discussed.

II. EXPERIMENT

The LED epilayers were grown on a *c*-plane sapphire substrate by metal-organic chemical vapor deposition. The LED layer structure consisted of a Si-doped *n*-GaN layer, MQW active layers, a 15-nm-thick *p*-type Mg-doped AlGaIn electron-blocking layer (EBL), and a 150-nm-thick Mg-doped *p*-GaN layer. The MQW active layers were composed of five 2.5-nm-thick InGaIn QWs separated by 8-nm-thick GaN barriers. The LED chip was fabricated in a vertical-injection structure with dimensions of $1 \times 1 \text{ mm}^2$. The fabricated LED chip was then encapsulated with an epoxy resin and packaged as a surface-mount device. The packaged LED sample was soldered onto a thick copper block, the temperature of which is controlled by a thermoelectric cooler (TEC). The electrical and optical characteristics were measured using an LED characterization system with a calibrated integrating sphere, as the TEC temperature varied from 20 to 80°C. The LED sample was operated under pulsed mode with a pulse width of 0.1 ms and duty cycle of 1%, to minimize self-heating effects. The light output power (LOP) and forward voltage were measured as the current increased to 350 mA.

The EQE can be obtained from the measured LOP. EQE is defined as the ratio of the number of photons emitted from the LED per second to the number of electrons injected into the LED per second:

$$EQE = \frac{\int (\lambda/hc) S(\lambda) d\lambda}{I/q}, \quad (1)$$

where q , h , and c are respectively the elementary charge, Planck's constant, and speed of light in vacuum. I is the current injected into the LED sample, and λ is the wavelength of light. $S(\lambda)$ is the spectral power distribution. LOP (P_{out}) is obtained by integrating the spectral power distribution over wavelength:

$$P_{out} = \int S(\lambda) d\lambda. \quad (2)$$

Using Eqs. (1) and (2), EQE can be written as

$$EQE = \frac{q\bar{\lambda}}{hc} \frac{P}{I}, \quad (3)$$

where $\bar{\lambda}$ is the centroid wavelength of the emission spectrum, which is defined as

$$\bar{\lambda} \equiv \frac{\int \lambda S(\lambda) d\lambda}{\int S(\lambda) d\lambda} = \frac{1}{P_{out}} \int \lambda S(\lambda) d\lambda. \quad (4)$$

Figure 1(a) shows the LOP of the LED sample as a function of injection current, at temperatures of 20, 40, 60, and 80°C. LOP at 350 mA decreased from 352 to 319 mW as the temperature increased from 20 to 80°C. Figure 1(b) shows an EQE versus current relation (EQE curve) at 20, 40, 60, and 80°C. For the EQE calculation using Eq. (3), the temperature dependence of $\bar{\lambda}$ as well as P_{out} was taken into account. $\bar{\lambda}$ was observed to increase from 452 to 455 nm as the temperature increased from 20 to 80°C. The peak EQE was found to decrease from 56.1% to 49.9% as the temperature increased from 20 to 80°C. The decrease in EQE with temperature is attributed to the increase in the Shockley-Read-Hall recombination rate, and

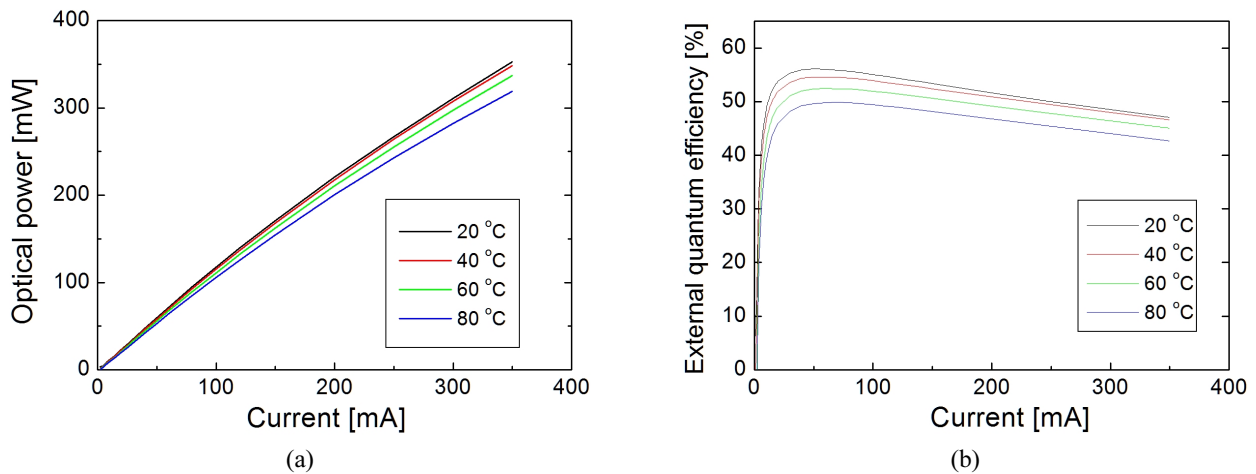


FIG. 1. (a) Light output power (LOP) of the LED sample as a function of current, at temperatures of 20, 40, 60, and 80°C. (b) External quantum efficiency (EQE) of the LED sample as a function of current, at temperatures of 20, 40, 60, and 80°C.

the decrease in the radiative recombination rate, with increasing temperature [21, 22].

Figure 2(a) shows the EQE curves normalized to the peak EQE values for each temperature. The current where the peak EQE was obtained increased from 50 to 70 mA as the temperature increased from 20 to 80°C. For currents larger than these values, significant efficiency droop was observed. Interestingly, the droop of the normalized EQE was reduced as the temperature increased. As a measure of the degree of efficiency droop, the droop ratio is introduced, which is defined as the difference between peak EQE and the EQE at 350 mA, normalized to the peak EQE [23].

$$\text{Droop ratio} = \frac{\text{peak EQE} - \text{EQE}(350 \text{ mA})}{\text{peak EQE}} \quad (5)$$

The droop ratio of EQE for the LED sample is plotted as a function of temperature in Fig. 2(b). As the temperature increased from 20 to 80°C, the droop ratio decreased from 0.161 to 0.141. That is, the efficiency droop problem can

be improved to some extent as temperature increases.

We also investigated the temperature-dependent droop of wall-plug efficiency (WPE). Figure 3(a) shows normalized WPE curves at temperatures 20, 40, 60, and 80°C. Each WPE curve is normalized to its peak value. The temperature dependence of the normalized WPE curves is similar to that of the normalized EQE curves in Fig. 2(a). Again, the normalized WPE at a given current increased as the temperature increased, implying that the droop in WPE decreased with increasing temperature. Figure 2(b) shows the droop ratio of WPE as a function of temperature. The droop ratio of WPE can be obtained from Eq. (5), by replacing EQW with WPE. As the temperature increased from 20 to 80°C, the droop ratio of WPE decreased from 0.271 to 0.254. To our knowledge, the reduction of EQE droop and WPE droop with increasing temperature has not been reported in previous works. It is expected that increasing the temperature can mitigate the droop problem to some extent, although the overall efficiency decreases with increasing temperature.

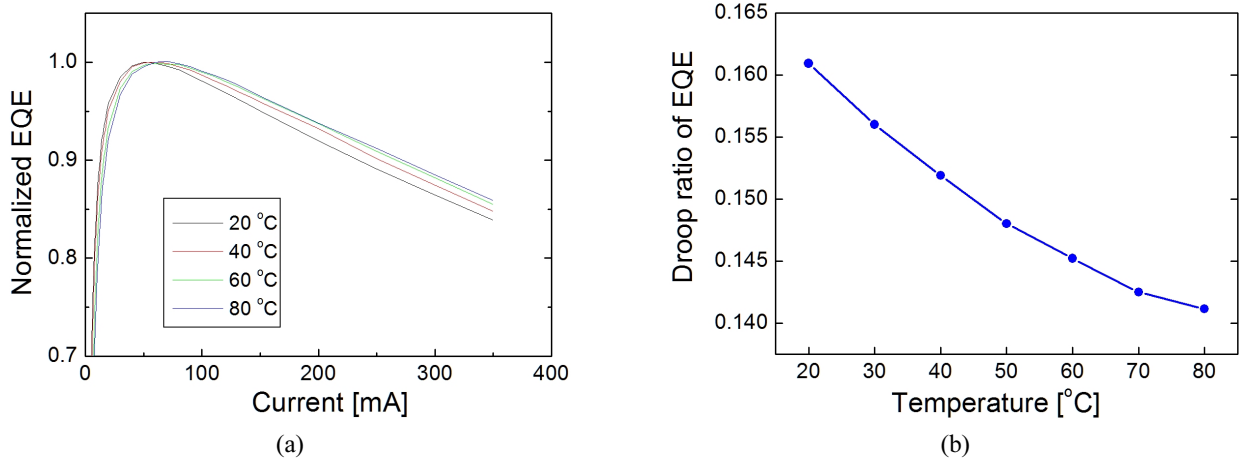


FIG. 2. (a) Normalized EQE as a function of current, at 20, 40, 60, and 80°C. (b) Droop ratio of EQE as a function of temperature.

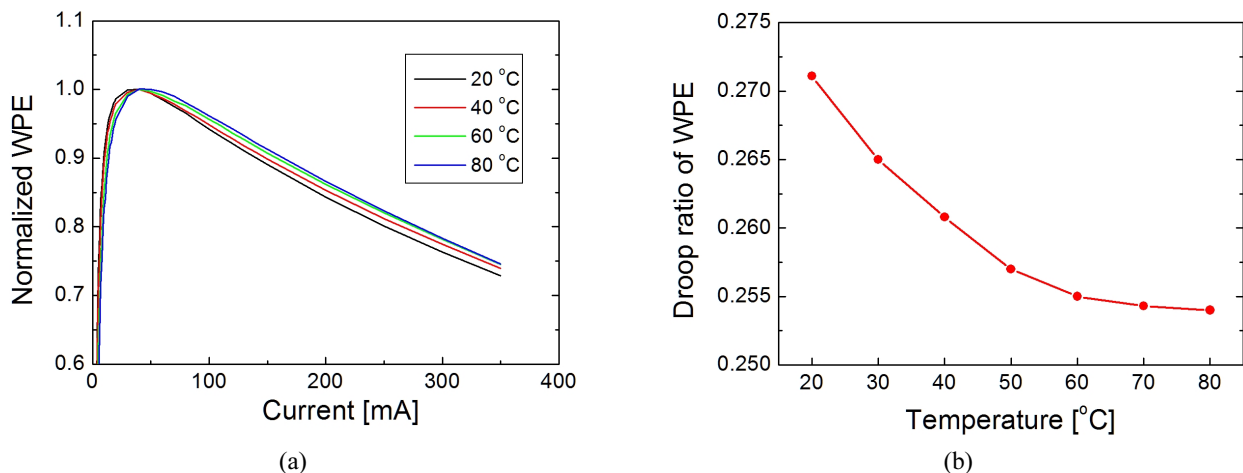


FIG. 3. (a) Normalized wall-plug efficiency (WPE) as a function of current, at 20, 40, 60, and 80°C. (b) Droop ratio of WPE as a function of temperature.

III. SIMULATION AND DISCUSSION

To understand the origin of droop reduction with increasing temperature, numerical simulation of the measured LED sample was performed. We focused on the simulation of the carrier distribution in InGaN/GaN MQWs, because the carrier distribution is known to have an important role in the efficiency droop of InGaN LEDs [24-28] and can be strongly influenced by temperature. For the simulation, the semiconductor device simulation software APSYS was employed. This simulation program self-consistently solves QW band structures, radiative and nonradiative carrier recombination, and the carrier drift and diffusion equation [29]. It has been widely used for simulating device characteristics of GaN-based LEDs.

In the simulation, the MQW structures were basically identical to those mentioned in Section II. The concentration of Si donors in *n*-GaN was $5 \times 10^{18} \text{ cm}^{-3}$, and that of Mg acceptors in both the EBL and the *p*-GaN cap layer $1 \times 10^{19} \text{ cm}^{-3}$. In the APSYS simulation, incomplete ionization of Mg acceptors and the field-ionization model were included, and the AlGaIn acceptor energy was linearly scaled from 170 meV in *p*-GaN to 470 meV in *p*-AlN [30, 31]. The built-in electric fields induced by spontaneous and piezoelectric polarizations at the heterointerfaces InGaN/GaN and AlGaIn/GaN were also included, using the model described in Ref. [32]. The polarization-induced internal electric field was modeled by placing the surface charge density at the heterointerfaces. The conduction-band offset of $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}/\text{In}_{0.02}\text{Ga}_{0.98}\text{N}$ active layers and AlGaIn/GaN layers was set at 0.7 [33]. The mobility model of Refs. [34, 35] was used for the mobility of carriers, which gave an electron mobility of $\sim 500 \text{ cm}^2/(\text{V s})$ for *n*-GaN with a doping concentration of $1 \times 10^{18} \text{ cm}^{-3}$. The hole mobility in the AlGaIn, InGaIn, and GaN layers was assumed to be $5 \text{ cm}^2/(\text{V s})$ [36]. In the temperature range from 20 to 80°C, electron leakage from the MQWs to the *p*-GaN layer

was not observed in the simulation. The recombination coefficients *A*, *B*, and *C* were assumed to be $1 \times 10^7 \text{ s}^{-1}$, $2 \times 10^{-11} \text{ cm}^3/\text{s}$, and $2 \times 10^{-31} \text{ cm}^6/\text{s}$ respectively [31]. In general, the recombination coefficients vary with temperature [17-20], but temperature dependence of *A*, *B*, and *C* was not included in this simulation, because the carrier and recombination-rate distributions in the MQWs would not change much with the variation of these coefficients from 20 to 80°C.

Figure 4(a) shows the hole concentration distribution in InGaN MQWs at temperatures of 20, 40, 60, and 80°C. Here the injection current was 350 mA. At 20°C, the distribution of hole concentration was quite inhomogeneous, decreasing rapidly as hole carriers moved from the *p*-side to the *n*-side QW. This inhomogeneity in hole distribution results from inefficient hole transport through Qws, caused by low hole mobility [24, 25]. As the temperature increased from 20°C, the hole concentration distribution became increasingly homogeneous, due to thermally enhanced hole transport from the *p*-side to the *n*-side QW [37, 38]. Similarly, the electron concentration distribution was also found to become homogeneous as the temperature increased. Figure 4(b) shows the distribution of radiative recombination rate at temperatures of 20, 40, 60, and 80°C. The radiative recombination rate distribution also became increasingly homogeneous as the temperature increased, similar to the case of the hole distribution. As the temperature increased, the distribution of the radiative recombination rate was more homogeneous than that of the hole distribution in Fig. 4(a), because the radiative recombination rate is proportional to the product of hole and electron distributions.

As the carrier distribution or radiative recombination rate distribution becomes homogeneous, efficiency droop can be reduced. The inhomogeneous carrier distribution results in a large increase of the Auger recombination rate at *p*-side QWs, which enhances the efficiency droop [25, 28]. As the temperature increases, the carrier distribution

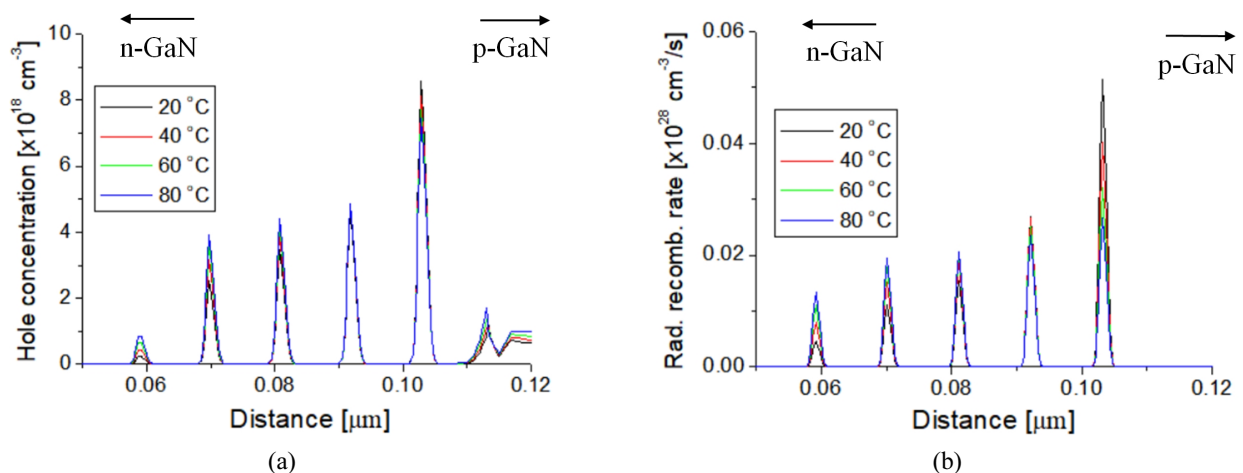


FIG. 4. (a) Hole concentration distribution in InGaN MQWs at 20, 40, 60, and 80°C. (b) Distribution of radiative recombination rate in InGaN MQWs at 20, 40, 60, and 80°C.

becomes increasingly homogeneous and the overall Auger recombination decreases, leading to the reduction in efficiency droop. Therefore, the reduced droop of EQE and WPE with increasing temperature shown in Figs. 2 and 3 can be explained, at least in part, by the temperature-dependent carrier distribution in the MQWs.

IV. CONCLUSION

The temperature dependence of efficiency droop in InGaN MQW blue LEDs was investigated in the temperature range from 20 to 80°C. The EQE and WPE of an LED sample were measured as temperature and current varied, and the temperature-dependent droop of these efficiencies was evaluated. When the efficiency curve was normalized to its peak value for each temperature, the droop in both EQE and WPE was observed to decrease as temperature increased. The droop ratio of EQE decreased from 16.1% to 14.1%, while that of WPE decreased from 27.1% to 25.4%, as the temperature increased from 20 to 80°C. To interpret the measured temperature-dependent efficiency droop, carrier distribution and radiative recombination rate in the MQWs were investigated using numerical simulation, and compared for different temperatures. The carrier distribution was found to become increasingly homogeneous as the temperature increased, as a result of thermally enhanced carrier transport, which is believed to partly contribute to the reduction in the efficiency droop with increasing temperature. It was found that increasing the temperature could mitigate the droop problem of InGaN blue LEDs to some extent, despite the decrease in overall efficiency. We expect that the abatement of efficiency droop with increasing temperature can be advantageously used in some temperature-stable application of LEDs.

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