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Laser Phase Noise to Electronic Phase Noise Conversion in Optical Links Comprising Optical Resonators

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This article investigates the mechanism of electronic signal phase noise degradation induced by laser phase noise in optical links comprising optical resonators. Through theoretical derivation, we find that the phase noise of the output electronic signal has the same spectral shape of optical intensity noise as the output of the optical resonator. We propose that the optical resonator transfers laser phase noise to light intensity fluctuation and then the intensity fluctuation is converted to electric phase noise through AM-PM conversion mechanism in the photodiode. An optical link comprising a Fabry-Perot resonator was constructed to verify the proposed mechanism. The experimental results agree with our theoretical prediction verifying that the supposition is correct.

Keywords : Phase noise, Fabry-Perot resonators, Optical resonators, AM-PM conversion
OCIS codes : (060.0060) Fiber optics and optical communications; (140.0140) Lasers and laser optics

I. INTRODUCTION

Phase noise is very important to the performance of coherent optical data links and microwave photonic links [1, 2]. In coherent optical data links, phase noise may deteriorate bit error rate. In microwave photonic links, phase noise may deteriorate coherence, which is very important in phase array antennas. Optical resonators which are usually used for wavelength demultiplexing in data links and spectrum filtering in microwave photonic links [3, 4] may increase the intensity noise of the optical signal. It has already been found that an optical resonator can convert the phase noise of an optical carrier to intensity fluctuation in cavity lasers [5-7]. Some groups have found that the nonlinear optical scattering mechanisms inside the resonator, especially the stimulated Brillouin scattering, can deteriorate electronic 1/f phase noise [8], while in our studies, we found that the significant electronic phase noise in the optical link, including both 1/f phase noise and white phase noise, was converted from laser phase noise through the optical resonator. Consequently, it is necessary to study the influence of optical resonators in an optical link.

In this paper, we revealed the mechanism of laser phase noise to electrical phase noise conversion in an optical link with an optical resonator inside. Through theoretical derivation, we found that the phase noise power spectral density (PSD) expression of the electrical signal transferred through an optical link matched very well the PSD of the unmodulated optical intensity noise. Therefore, there may exist an optical phase noise to electrical phase noise conversion mechanism, which comprised the conversion of the optical phase to intensity fluctuation through the optical resonator [9] firstly and the conversion of optical intensity fluctuation to electrical phase noise through amplitude modulation-phase modulation (AM-PM) conversion in the photodiode (PD) [10] secondly. To verify this hypothesis, we built an optical link with a Fabry-Perot resonator, and compared the PSD of electrical phase noise in a modulated optical link and the PSD of relative intensity noise (RIN) in an unmodulated optical link.

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II. THEORETICAL DERIVATION

In the following section, we build a theoretical model to describe the electrical phase noise induced by laser phase noise in the optical link with an optical resonator inside. Figure 1 shows the schematic diagram of a microwave photonic link with optical resonator.

An optical carrier with amplitude E_o, angular frequency ω_o and phase $\phi_o(t)$ can be expressed as:

$$E_{oc} = E_o \exp\left[j\left(\omega_o t + \phi_o(t)\right)\right] \tag{1}$$

$P_0 = E_0^2 / (2\eta)$ is the average optical power, with η the wave impedance of the optical fiber. Given that the electrical signal driving the Mach-Zehnder Modulator (MZM) is a sinusoidal wave with angular frequency ω_e, amplitude V_A and phase $\phi_e(t)$, then

$$V_e = V_A \cos\left[\omega_e t + \phi_e(t)\right] \tag{2}$$

We assume that the E/O modulator employed in the following derivation is a dual-electrode MZM, while we also got a similar result with a single-drive MZM.

The MZM has a half-wave voltage V_π and is biased at a voltage of V_b. The output optical signal of the MZM can be expressed as:

$$\begin{aligned} E_{\mathrm{MO}} &= \frac{1}{2} E_0 \exp\left[j\left(\omega_o t + \phi_o(t)\right)\right] \cdot \\ &\left\{ \exp\left[j\left(\frac{\pi}{V_\pi} V_e + \pi V_b / V_\pi\right)\right] + \exp\left[-j\frac{\pi}{V_\pi} V_e\right]\right\} \end{aligned} \tag{3}$$

Assume that R is the reflectivity of the cavity mirror, α_l is the round-trip loss coefficient, τ is the round-trip delay and ρ is the responsivity of the PD. Given $\tau\omega_e << 1$, after the signal passes through the optical resonator and is incident on the PD, the current output from the PD without considering the AM-PM conversion mechanism can be approximated as (detailed derivation is shown in Appendix A):

$$\begin{aligned} I_{RO,1st} &= -\rho(1-R)^2 P_0 J_1\left(2\frac{\pi}{V_\pi} V_A\right) \cos\left[\omega_e t + \phi_e(t)\right] \\ &\cdot \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} (\alpha_l R)^{n+m} \cos\left[\phi_o(t-n\tau) - \phi_o(t-m\tau)\right] \end{aligned} \tag{4}$$

Eq. (4) can also be expressed as:

$$I_{RO,1st} = -\rho B F(t) P_0 \cos\left[\omega_e t + \phi_e(t)\right] \tag{5}$$

where

$$\begin{aligned} F(t) &= (1-R)^2 \cdot \\ &\left\{ \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (\alpha_l R)^{m+n} \cos[\phi_o(t-n\tau) - \phi_o(t-m\tau)]\right\} \end{aligned} \tag{6}$$

$$\mathrm{B} = J_1\left(2\frac{\pi}{V_\pi} V_A\right) \tag{7}$$

As shown in Eq. (6), the amplitude fluctuation F(t) of the electronic signal is influenced by the phase perturbation $f_0(t)$ of the optical carrier, while the phase of the electrical signal is not influenced by it. However, if we take into account the AM-PM conversion mechanism in the PD, the phase noise of the electric signal will deteriorate. The AM-PM conversion coefficient denotes that the phase change is proportional to the normalized optical power fluctuation and can be expressed as:

$$\alpha_{AM-PM} = \frac{\Delta\phi}{\Delta P / P_0} \tag{8}$$

In the case of small amplitude modulation where the optical power injected into the PD changes little over time, α_{AM-PM} can be treated as a constant. The phase variation of the output electrical signal can be evaluated as:

$$\phi_{n,MPL}(t) = \phi_e(t) + \Delta\phi = \phi_e(t) + \alpha_{AM-PM} \cdot B\rho F(t) \tag{9}$$

As can be seen from Eq. (9), the total phase noise of the output electrical signal comprises its original phase noise and the additive phase noise generated from optical phase noise. Although this expression is difficult to be simplified to a closed form, it can serve to reveal the additive phase noise mechanism. Firstly, the laser phase noise is converted into intensity noise of the optical signal by the optical resonator. Secondly, the optical intensity noise is converted to electric phase noise by AM-PM conversion during PD detection.

In order to prove this mechanism, we derive the expression for the intensity PSD of an unmodulated photonic link (UMPL) with an optical resonator inside and compare it with the electric phase noise of the modulated photonic link (MPL), as shown in Fig. 1.

The output current of the PD is proportional to optical intensity (detailed derivation is shown in Appendix B):

$$I_{PD,UMPL} = \rho F(t) P_0 \tag{10}$$

Thus, the RIN induced by the optical resonator can be evaluated as:

$$I_{n,UMPL} = \rho F(t) \tag{11}$$

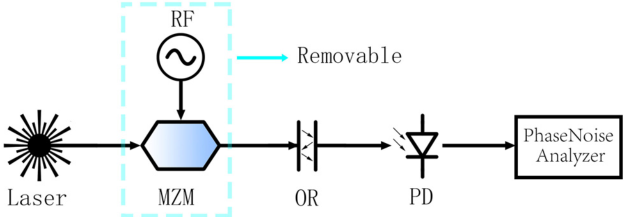

FIG. 1. Schematic diagram of the microwave photonic link with an optical resonator. MZM: Mach-Zehnder Modulator. OR: Optical Resonator. PD: photodiode. MZM and RF are removable to measure the optical intensity noise induced by optical phase noise through OR conversion.

Comparing Eq. (11) with Eq. (9), if the phase noise of the original electric signal $\phi_e(t)$ is insignificant, they are consistent except for a constant coefficient $B\alpha_{AM-PM}$. Consequently, the measurement of phase noise PSD induced by the optical resonator in Fig. 1 with the MZM should be proportional to the RIN PSD brought about by the optical resonator in Fig. 1 without the MZM when $\phi_e(t)$ is small.

For an optical resonator, the relationship between Q factor and R can be expressed as [11]:

$$Q = \frac{2\pi L\sqrt{\alpha_l R}}{(1-\alpha_l R)\lambda} \tag{12}$$

where L is the cavity length of the optical resonator and λ is the optical wavelength in vacuum. As can be seen from Eq. (12), the higher Q in the optical resonator, the closer the $\alpha_l R$ to 1, so the F(t) in Eq. (6) accumulates more, resulting in the laser phase noise having higher influence on electronic phase noise.

III. EXPERIMENT AND RESULTS

The experimental setup for measuring FP induced optical RIN, which is well above the RIN of laser source (-163 dBc/Hz) is illustrated in Fig. 2. The result in Fig. 4 is obvious that the FP can cause significant optical intensity noise.

In Fig. 3, an MPL for measuring the electric phase noise is illustrated. The phase noise floor was measured by removing the FP. The MPL comprises a DFB laser, an optical isolator, a Dual-Drive Mach-Zehnder Modulator (DD-MZM, EOSpace) with a -3 dB bandwidth of 20 GHz, a polarization controller (PC), a 50 GHz PD and a tunable fiber Fabry-Perot (FP) resonator (Micron Optics FFP-TF2) with a free spectral range (FSR) tuned to around 25 GHz, corresponding to a round trip delay τ of about 40 ps. The -3 dB bandwidth of the FP is 1.1 GHz which corresponds to the reflectivity R of 0.966. An 80 MHz low phase noise electronic signal is modulated on the optical carrier

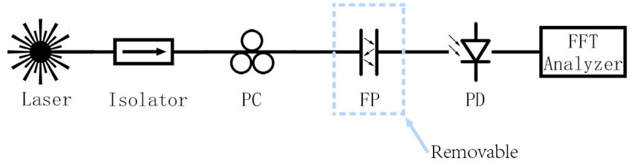

FIG. 2. Measurement of FP induced optical intensity fluctuation.

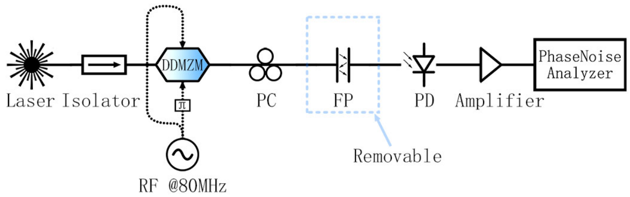

FIG. 3. Schematic diagram of FP phase noise measurement. FP is removable to measure the phase noise floor of this system.

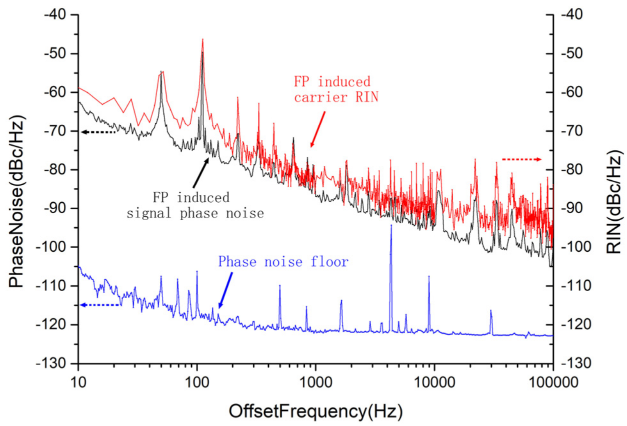

FIG. 4. Phase noise measurement result. The laser phase noise, optic link with FP and its phase noise floor of 80 MHz carrier are measured separately utilizing a phase noise analyzer (Agilent 5052A). The optical carrier RIN brought about by FP near DC is measured by FFT analyzer (HP35670A).

by the DDMZM, with a phase difference of π on two arms. The transmission peak of the FP was tuned to align with the optical carrier frequency. Meanwhile, the temperature of the FP was actively stabilized. The output of the PD is amplified through a low phase noise amplifier and then sent to the phase noise analyzer.

In Fig. 4, the log-log plot of measured RIN and phase noise with respect to the offset frequency are plotted. It can be seen that the phase noise floor of the optical link without the FP is low enough to estimate the influence of the FP. Usually, the phase noise of the laser is large enough to produce a strong electronic phase fluctuation when optical resonator is inside the optical link. As predicted in Eq. (9), the phase noise of RF signal increases a lot, approximately 20~40 dB. In order to measure the optical carrier RIN at the output of the FP, the DDMZM and RF signals are removed from the optical link, and the result is shown in Fig. 4. The results indicate that the FP induced RIN is a little bit higher than the FP induced phase noise on average, they are of the same spectral shape and therefore a strong correlation between FP induced carrier RIN and PF induced electronic signal phase noise exists. Moreover, the phase noise peaks with respect to the frequencies of 111 Hz, 222 Hz, 333 Hz and 444 Hz are generated from the control voltage which was applied to the FP for tuning the transmission peak wavelength. Meanwhile, the peak corresponding to 50 Hz is due to AC power interference in the laser source (DFB laser) for it changes to 32.5 Hz when the optical source is switched to an InGaAsP Fabry-Perot laser (HP 8168F). Except for these spurs, it is shown that the PSD of the electric phase noise and PSD of the optical carrier RIN at the output of the FP match very well, proving that the electric phase noise is generated from laser phase noise. The laser phase noise is converted to intensity noise through FP and then to phase noise through AM-PM conversion in PD.

IV. CONCLUSION

This article has presented a theoretical model of the optical phase noise to electronic phase noise conversion mechanism induced by optical resonators in optical links. The laser phase noise is converted to optical intensity noise through the optical resonator first, and then converted to electronic phase noise through AM-PM conversion in the PD. Higher Q factor of an optical resonator may make this conversion more significant. This mechanism has been verified experimentally where a good consistency between the PSD of electronic phase noise and the PSD of the RIN at the output of the optical resonator was found.

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APPENDIX

A.

The optical signal output from the optical resonator is the summation of signals that experience different times of circulation:

$$
\begin{aligned}
E_{\mathrm{RO}} &= \sum_{0}^{\infty}(\alpha_l R)^n (1-R) E_{MO} \\
&= (1-R)\frac{E_0}{2}\sum_{n=0}^{\infty}(\alpha_l R)^n \cdot \\
&\left\{\exp\left[j\left(\omega_o(t-n\tau)+\phi_o(t-n\tau)+\frac{\pi V_e}{V_\pi}+\frac{\pi V_b}{V_\pi}\right)\right]\right. \\
&\left.+\exp\left[j\left(\omega_o(t-n\tau)+\phi_o(t-n\tau)-\frac{\pi V_e}{V_\pi}\right)\right]\right\}
\end{aligned} \tag{A1}
$$

Given that ω_o matches with the resonance frequency of the optical resonator, $\omega_o \tau = 2k\pi,\ k \in N$. Eq. (A1) can be simplified as:

$$
\begin{aligned}
E_{\mathrm{RO}} &= (1-R)\frac{E_0}{2}e^{j\omega_o t}\sum_{n=0}^{\infty}(\alpha_l R)^n \cdot \\
&\left\{\exp\left[j\left(\phi_o(t-n\tau)+\frac{\pi V_e}{V_\pi}+\frac{\pi V_b}{V_\pi}\right)\right]\right. \\
&\left.+\exp\left[j\left(\phi_o(t-n\tau)-\frac{\pi V_e}{V_\pi}\right)\right]\right\}
\end{aligned} \tag{A2}
$$

The output signal of the resonator is detected by a PD, which generates an output current of $I_{PD,MPL} = \rho\frac{E_{\mathrm{RO}}E_{RO}^*}{2\eta}$:

$$
\begin{aligned}
I_{PD,MPL} &= \frac{1}{2}\rho(1-R)^2 P_0\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(\alpha_l R)^{n+m} \cdot \\
&\exp\left[j\left(\phi_o(t-n\tau)-\phi_o(t-m\tau)\right)\right]\cdot \\
&\cdot\left\{\cos\left[\frac{\pi}{V_\pi}\left[V_e(t-n\tau)-V_e(t-m\tau)\right]\right]\right. \\
&\left.+\cos\left[\frac{\pi}{V_\pi}\left[V_e(t-n\tau)+V_e(t-m\tau)\right]+\frac{\pi V_b}{V_\pi}\right]\right\}
\end{aligned} \tag{A3}
$$

where we pair every n = u, m = v with its conjugation n = v, m = u. For large m and n, the exponential component $(\alpha_l R)^{n+m}$ decreases rapidly, making these terms insignificant. Furthermore, in the case of electrical modulation signal $V_e = V_A\cos\left[\omega_e t + \phi_e(t)\right]$ and short cavity condition $\tau\omega_e << 1$ while m and n are small, $V_e(t-n\tau)-V_e(t-m\tau)\approx 0$ and $V_e(t+n\tau)+V_e(t+m\tau)\approx 2V_e(t)$. Consequently, Eq. (A3) can be expressed as:

$$
\begin{aligned}
I_{PD,MPL} &= \frac{\rho}{2}(1-R)^2 P_0\left\{1+\cos\left[\frac{2\pi}{V_\pi}V_e(t)+\pi V_b/V_\pi\right]\right\} \\
&\cdot\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(\alpha_l R)^{n+m}\cos\left[\phi_o(t-n\tau)-\phi_o(t-m\tau)\right]
\end{aligned} \tag{A4}
$$

If the modulator works at the quadrature bias point $\pi\frac{V_b}{V_\pi}=\frac{1}{2}\pi$, Eq. (A4) can be expressed as:

$$I_{PD,MPL}=\frac{1}{2}\rho\left(1-R\right)^2 P_0 \cdot\left[1+\cos\left(2\frac{\pi}{V_\pi}V_A\cos\left[\omega_e t+\phi_e\left(t\right)\right]+\pi/2\right)\right] \cdot\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left(\alpha_l R\right)^{n+m}\cos\left[\phi_o\left(t-n\tau\right)-\phi_o\left(t-m\tau\right)\right] \tag{A5}$$

Utilizing Bessel expansion [12], Eq. (A5) can be expressed by the sum of infinite order terms, while only the first-harmonic wave, which accounts for the transmitted microwave signal, needs to be considered. Thus, Eq. (A5) can be written as:

$$\begin{aligned}I_{RO,1st}&=\frac{1}{2}\rho\left(1-R\right)^2 P_0\left[1-\sin\left(2\frac{\pi}{V_\pi}V_A\cos\left[\omega_e t+\phi_e\left(t\right)\right]\right)\right]\\&\cdot\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left(\alpha_l R\right)^{n+m}\cos\left[\phi_o\left(t-n\tau\right)-\phi_o\left(t-m\tau\right)\right]\\&=\frac{1}{2}\rho\left(1-R\right)^2 P_0\\&\cdot\left[1+2\sum_{p=1}^{\infty}(-1)^p J_{2p-1}(2\frac{\pi}{V_\pi}V_A)\cos\left[(2p-1)\left(\omega_e t+\phi_e\left(t\right)\right)\right]\right]\\&\cdot\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left(\alpha_l R\right)^{n+m}\cos\left[\phi_o\left(t-n\tau\right)-\phi_o\left(t-m\tau\right)\right]\\&\approx-\rho\left(1-R\right)^2 P_0 J_1(2\frac{\pi}{V_\pi}V_A)\cos\left[\omega_e t+\phi_e\left(t\right)\right]\\&\cdot\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left(\alpha_l R\right)^{n+m}\cos\left[\phi_o\left(t-n\tau\right)-\phi_o\left(t-m\tau\right)\right]\end{aligned} \tag{A6}$$

B.

When the optical carrier is still expressed as Eq. (1), the optical signal output from the optical resonator can be expressed as:

$$E_{UMOL}=E_o\left(1-R\right)\sum_{n=0}^{\infty}\left(\alpha_l R\right)^n\cdot \exp\left[j\left(\omega_o(t-n\tau)+\phi_o(t-n\tau)\right)\right] \tag{B1}$$

The output current of PD is proportional to optical power:

$$\begin{aligned}I_{PD,UMOL}&=\rho\frac{E_{UMOL}E^*_{UMOL}}{2\eta}\\&=\frac{\rho}{2\eta}E_o^{\,2}(1-R)^2\\&\cdot\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left(\alpha_l R\right)^{m+n}\cos\left[\phi_o(t-m\tau)-\phi_o(t-n\tau)\right]\\&=\rho F(t)P_0\end{aligned} \tag{B2}$$

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