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A Strength Analysis of Gear Train for Hydro-Mechanical Continuously Variable Transmission

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Abstract

The power train of hydro-mechanical continuously variable transmission(HMCVT) for the middle class forklift makes use of an hydro-static unit, hydraulic multi-wet disc brake & clutches and complex helical & planetary gears. The complex helical & planetary gears are a very important part of the transmission because of strength problems. The helical & planetary gears belong to the very important part of the HMCVT's power train where strength problems are the main concerns including the gear bending stress, the gear compressive stress and scoring failures. The present study, calculates specifications of the complex helical & planetary gear train and analyzes the gear bending and compressive stresses of the gears. It is necessary to analyze gear bending and compressive stresses confidently for an optimal design of the complex helical & planetary gears in respect of cost and reliability. This paper not only analyzes actual gear bending and compressive stresses of complex helical & planetary gears using Lewes & Hertz equation, but also verifies the calculated specifications of the complex helical & planetary gears by evaluating the results with the data of allowable bending and compressive stress from the Stress - No. of cycles curves of gears. In addition, this paper explains actual gear scoring and evaluates the possibility of scoring failure of complex helical & planetary gear train of hydro-mechanical continuously variable transmission for the forklift.

Keywords: Differential Planetary Gears, Gear Bending Stress, Gear Compressive Stress, Gear Scoring Factor, HMCVT

1. Introduction

The hydro-mechanical continuously variable transmission(HMCVT) is a type transmission device, which consists of a mechanical transmission(MT) combined in parallel with a hydrostatic unit(HSU) featuring a pair of hydraulic units. The HMCVT has a continuously variable shifting ratio by the combination of HSU and MT and achieves high efficiency as MT. Hydrostatic drives are widely recognized as an excellent means of power transmission when variable output speed is required. Typically outperforming mechanical and electrical variable-speed drives and gear-type transmissions, they offer fast responses, maintain a precise speed under varying loads, and allow infinitely variable speed control from zero to maximum. Unlike gear transmissions, hydrostatics have a continuous power curve without peaks and valleys, and they can increase available torque without shifting gears. The HMCVT uses a planetary gear system to provide a combination of hydraulic and mechanical power for a vehicle or stationary equipment. It is based on hydro-mechanical continuously variable

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transmission gear train which can transmit 73.6 kW. The revolution speed of the engine, gears and clutches of the HMCVT were calibrated by using results obtained by theoretical calculation. The needed power and torques of mechanical power input and hydropower output torque were calculated by a simulation.

This study, calculates specifications of the complex helical & planetary gear train and analyzes the gear bending and compressive stresses of the gears. It is necessary to analyze gear bending and compressive stresses confidently for optimal design of the complex helical & planetary gears in respect of cost and reliability. The paper not only analyzes actual gear bending and compressive stresses of complex helical & planetary gears using Lewes & Hertz equation, but also verifies the calculated specifications of the complex helical & planetary gears by evaluating the results with the data of allowable bending and compressive stress from the Stress - No. of cycles curves of gears. In addition, this paper also analyzes actual gear scoring and evaluates the possibility of scoring failure of complex helical & planetary gear train of hydro-mechanical continuously variable transmission for forklift. Here, a schematic structure is shown in Figure 1 of 8 ton class capacity forklift with HMCVT.



Figure 1. Schematic structure of HMCVT for 8 ton class forklift

The schematic working mechanism of an analytical model for HMCVT is shown in Figure 2.

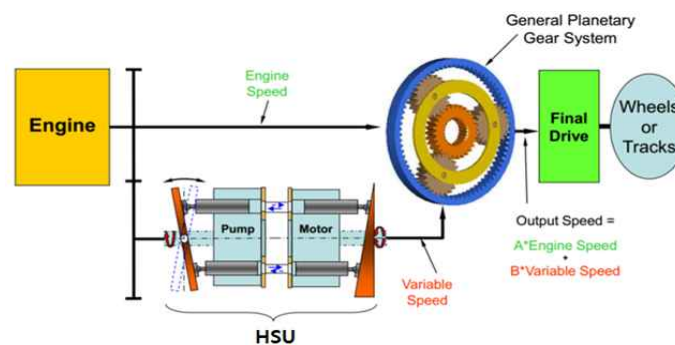


Figure 2. Working mechanism for middle class forklift

Table 1 indicates the specifications of HMCVT for 8 ton class forklift.

Table 1. Specifications of the transmission

Max. engine torque /speed(N·m/rpm)	Max. engine power /speed(kW/rpm)	Gear ratio range	T/M, max. output torque(N·m)	Final gear ratio	H/R ratio	Tie radius (m)	Speed (km/h)
362.6/1600	73.6/2200	0.689~5.00	3,694	3.10	4.0	0.491	30

Gear teeth are damaged due to the lack of fatigue strength, compound planetary gears and severe operating conditions of a transmission that become a problem. Several investigations have been reported, as cited by M.H. Bae et al[1]. In the paper the bending and compressive stress distribution of planetary gear system of mixer reducer for concrete mixer truck is calculated. Imwalle[2] cited load equalization in planetary gear systems. Seager[3] established load distribution calculation of the planetary gears. Cunliffe et al.[4], dynamic tooth loads in epicyclic gears for planetary gears. Castellani and Castelli[5] also cited the gear strength analysis method. Coy et al.[6] further emphasized the dynamic capacity and surface pressure durability life of spur and helical gears. Oda and Tsubokura[7] similarly stressed the effect of bending endurance strength for addendum modification of spur gears and was likewise investigated. There is also an inclusion of typical bending strength calculation of planetary gears AGMA 218.01[8] and Gear Handbook by D. W. Dudley[9] shows the bending strength calculation method of planetary gears. Moreover, it also developed the stress analysis program of differential planetary gear system by Lewis[10] & Hertz equation and analyzed the safety factor of gear bending and compressive stresses consider required life time of transmission and the S/N curve presented in the Gear Handbook by Dudley. This study developed the gear specifications calculation program and produced detailed specifications of the differential planetary gear system for HMCVT based on Gear Handbook by D. W. Dudley. It also verified the predictive validity with respect to the development and estimate of the scoring failure of differential planetary gears by scoring factor analysis. Figure 3 shows the equation system solving with gear specifications calculation and strength analysis of the gear system for HMCVT.

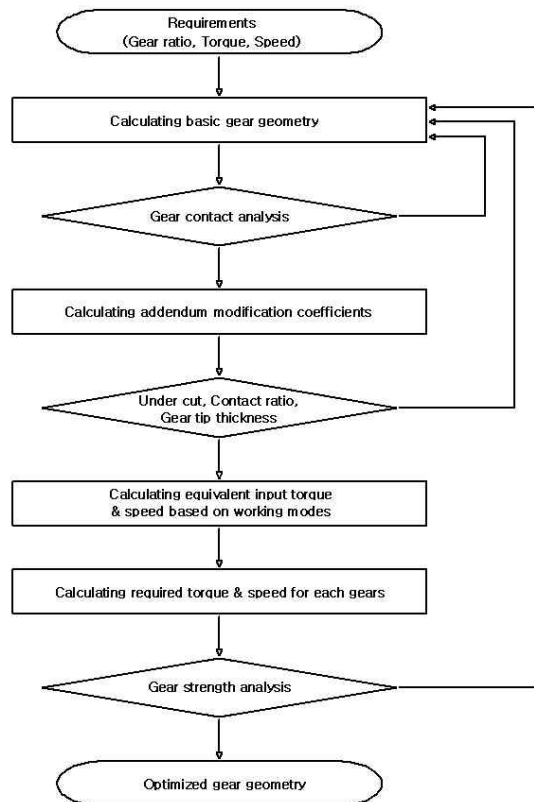


Figure 3. Equation system solving with gear specifications and strength analysis

2. Material and analytical method

2.1 Calculation of gear specifications

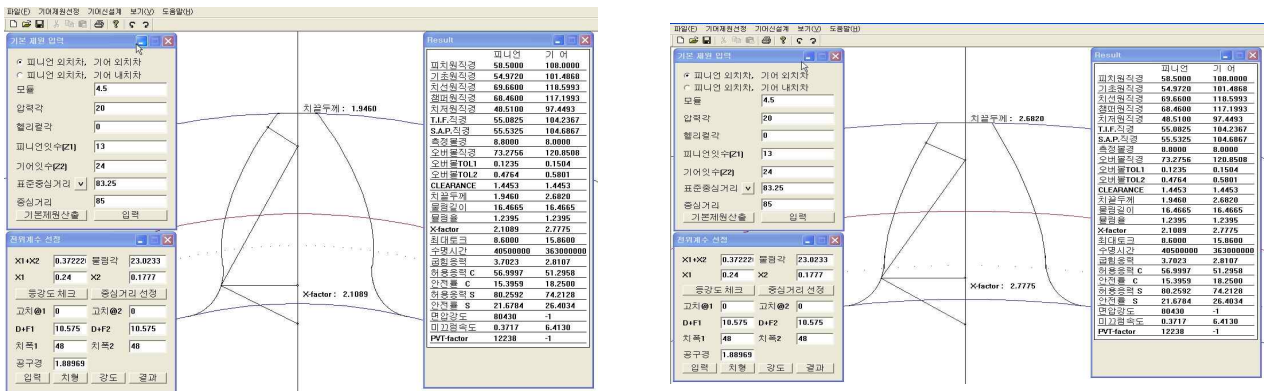
Table 2 and table 3 show the calculated specifications of the helical gear and specifications of planetary gears for transmission. Figure 4 indicates the results of the gear specifications calculation program.

Table 2. Specifications of helical gear

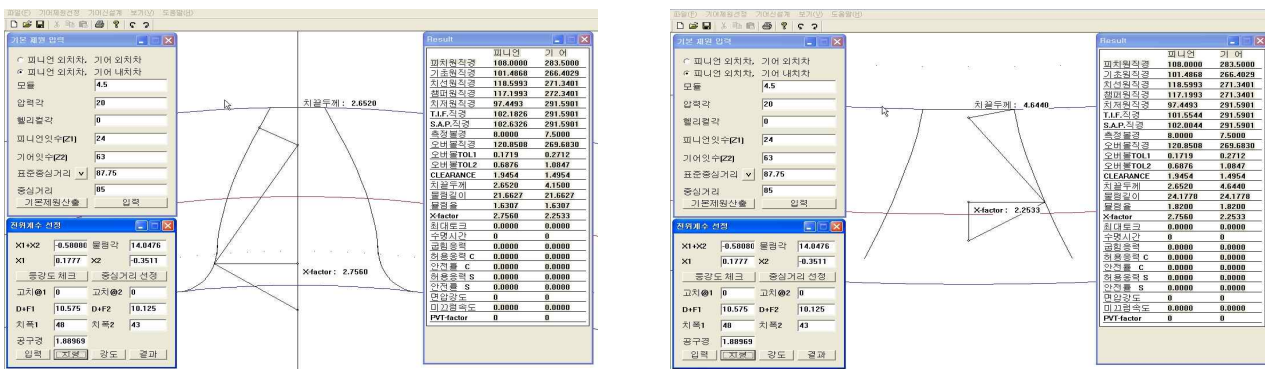
Items	Z1	Z2	Z3	Z4	Z5	Z6	Z7a	Z7b	Z8	Z9	Z10	Z11	Z13(Z15)	Z13(Z16)	Z12	Z17
Module	4	←	4	←	4	←	4	←	4	←	3	←	3	←	4	←
Press Angle	25°	←	25°	←	25°	←	25°	←	25°	←	25°	←	25°	←	25°	←
Helix Angle	10°	←	10°	←	10°	←	10°	←	10°	←	10°	←	10°	←	10°	←
No. of Gear Teeth	52	26	38	61	56	28	32	32	61	21	59	20	49	62	69	69
Tooth modification factor	0	0.4070	0.3966	0.1	-0.15	0.4825	0	0	0	0.4789	-0.19	0.4167	0.1448	0	-0.032	-0.032
Outside Dia.	219.2	116.86	165.51	256.56	234.25	125.58	137.97	137.97	255.76	97.12	184.59	69.42	156.13	194.86	288	288
Over Pin Measurement	221.206 ± 0.137	121.287 ± 0.114	163.678 ± 0.118	253.447 ± 0.137	236.311 ± 0.131	123.466 ± 0.118	138.022 ± 0.113	138.022 ± 0.113	257.68 ± 0.138	100.066 ± 0.111	136.306 ± 0.118	72.668 ± 0.109	153.284 ± 0.118	197.137 ± 0.137	280.067 ± 0.118	280.067 ± 0.118
Face Width	12	12	20	20	25	25	25	25	32	35	22	22	21(25)	21(25)	15	15
Backlash	0.13-0.54		0.14-0.58		0.16-0.64		0.14-0.54		0.16-0.64		0.13-0.54		0.14-0.58		0.14-0.58	
Center Distance	160		203		171.9		129.975		168.4		121		169.508		280	
Contact Ratio	1.468		1.618		1.651		1.697		1.713		1.811		1.740		1.634	

Table 3. Specifications of the planetary gear

Items	No1 S/G	No1 P/G	No1 R/G	No2 S/G	No2 P/G	No2 R/G
Module	4.5	←	←	4	←	←
Press angle	20°	←	←	20°	←	←
Helix angle	0°	←	←	0°	←	←
No. of gear teeth	13	24	63	13	24	63
Tooth modification factor	0.24	0.1777	-0.3511	0.24	0.1618	-0.3767
Outside dia.	67.5	120.76	273.5	60	107.21	242.9
Over pin measurement	73.278 ± 0.128	120.880 ± 0.118	269.632 ± 0.132	65.070 ± 0.121	106.955 ± 0.118	240.222 ± 0.128
Face width	48	48	43	34	34	29
Backlash	0.14-0.54		0.16-0.64		0.14-0.54	
Center distance	85		←	75.5		←
Contact ratio	1.354		1.546		1.345	



(a) No.1 Sun Gear + No.1 Pinion Gear



(b) No.1 Pinion Gear + No.1 Ring Gear

Figure 4. The results of gear specifications calculation program

2.2 Calculation of input equivalent torque and rotational speed

Table 4 shows the operation modes of the HMCVT for forklift according to the standard of KIMM[11]. The required service period of life for a HMCVT is 10,000h.

Table 4. Operating mode

HMCVT's working mode	Present A/T's shifting speed	Frequency (%)	Required life(Hrs)
Max. output torque (1st speed , Fwd./Rev.)	1st speed(4.71), Fwd./Rev.	21.7	2170
Discontinuity shifting (2nd Speed Fwd./Rev.)	2nd speed(2.34), Fwd./Rev.	58.3	5830
Max. efficiency (2nd Speed, Fwd./Rev.)	3rd speed(0.97), Fwd.	20	2000
Max. speed (2nd Speed Fwd./Rev.)			
Total		100	10,000

Equivalent torque for the average equivalent load of HMCVT, T_m is as follows:

$$T_{mi} = \left[\frac{\sum N_i t_i T_i^n}{\sum N_i t_i} \right]^{\frac{1}{n}} \quad (1)$$

T_i is working torque, N_i is rotating speed, t_i is working time and n is power index ($n=20.8$)

Equivalent rotating speed for the average equivalent rotating speed of HMCVT, N_{mi} is as follows:

$$N_{mi} = \left[\frac{\sum N_i t_i}{\sum t_i} \right] \quad (2)$$

N_{mi} is equivalent rotating speed for the average equivalent rotating speed, N_i is rotating speed and t_i is working time.

2.3 Gear bending stress analysis

The actual gear bending stress equation by Lewes formula is as follows:

$$S = \frac{29,400\pi T}{N_a F X Z} \quad (3)$$

S is actual gear bending stress(N/mm^2), T is torque on gears($N\cdot m$), N_a is contact length of action(mm), F is face width of gear(mm), X is Lewes bending factor(mm) and Z is the number of teeth in gear.

Allowable gear bending stress equation by Gear Handbook of D. W. Dudley and AGMA Standard 218.01⁸⁾ including gear bending S/N curve is as follows:

$$S_{ab} = \frac{C_1}{N_F^{20.5}} \quad (4)$$

S_{ab} is allowable gear bending stress(N/mm^2), N_F is No. of cycles and C_1 is coefficient.

2.4 Gear compressive stress analysis

The actual gear compressive stress, $P(N/mm^2)$ applied at the tip of the planetary gears based on contact formula of Hertz is as follows:

In the case of the external gear contact for helical gear, the actual gear compressive stresses of sun gear and pinion gear are,

$$P_s = 19.43 \sqrt{\frac{2\pi T_s \times CD \sin \Phi}{A_s (CD \sin \Phi - A_s) \times F_c \times N_a \times Z_s}} \quad (5)$$

$$P_p = 19.43 \sqrt{\frac{2\pi T_s \times CD \sin \Phi}{A_p (CD \sin \Phi - A_p) \times F_c \times N_a \times Z_s}} \quad (6)$$

In the case of the internal gear contact, the actual gear compressive stresses of pinion gear and ring gear are,

(7)

$$P_R = 19.43 \sqrt{\frac{2\pi T_P \times CD \sin \alpha}{A_R (A_R - CD \sin \Phi) \times F_c \times N_c \times Z_P}} \quad (8)$$

α is normal pressure angle, Φ is transverse pressure angle, T is torque on driving gear(N · m), F_c is active face width in contact(mm), Z is No. of gear teeth, CD is operating center distance, N_a is contact length of action(mm), $A = \sqrt{OR^2 - BR^2}$, OR is outside radius of gear and BR is base radius of gear.

Allowable gear compressive stress equation by Gear Handbook of D. W. Dudley and AGMA Standard 218.01 including gear compressive S/N curve is as follows:

$$S_{ac} = \frac{C_2}{N_F^{0.5433}} \quad (9)$$

S_{ac} is allowable gear compressive stress(N/mm²), N_F is No. of cycles, C_2 is coefficient. N_F is No. of cycles and C_2 is coefficient.

2.5 Gear Scoring factor analysis

Gear scoring factor is based on Gear Handbook of D. W. Dudley. To predict coring failure of differential planetary gears, PVT_s (sun gear), PVT_p (pinion gear) are as follow:

$$PVT_s = \frac{\pi N_s}{0.08525} \left(\frac{Z_p + Z_s}{Z_p} \right) (A_s - PR_s \times S \sin \Phi)^2 \quad (10)$$

$$PVT_p = \frac{\pi N_p}{0.08525} \left(\frac{Z_p + Z_s}{Z_s} \right) (A_p - PR_p \times S \sin \Phi)^2 \quad (11)$$

In the case of internal gear contact, the actual gear compressive stresses of pinion gear and ring gear are as follow:

$$PVT_p = \frac{\pi N_p}{0.08525} \left(\frac{Z_r - Z_p}{Z_r} \right) (A_p - PR_p \times S \sin \Phi)^2 \quad (12)$$

$$PVT_r = \frac{\pi N_p}{0.08525} \left(\frac{Z_r - Z_p}{Z_r} \right) (PR_r \times S \sin \Phi - A_r)^2 \quad (13)$$

P is actual compressive stress (N/mm²), V is sliding velocity(m/sec), T is contact length from pitch point to contact point(mm), N is speed of gears(rpm), Z is number of gear teeth and Φ is operating pressure angle.

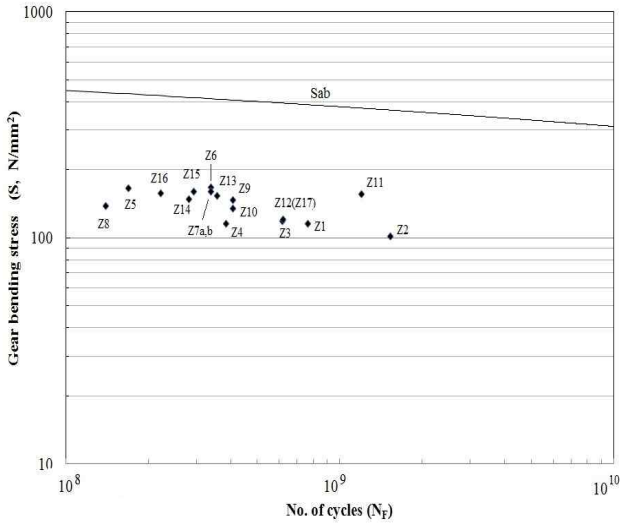
The Gear scoring factor, PVT must not exceed 38,860(N/sec · mm) to prevent a scoring failure under HRC 60, carburized gears in mineral oil condition.

2.6 The results of gear bending and compressive stress analysis

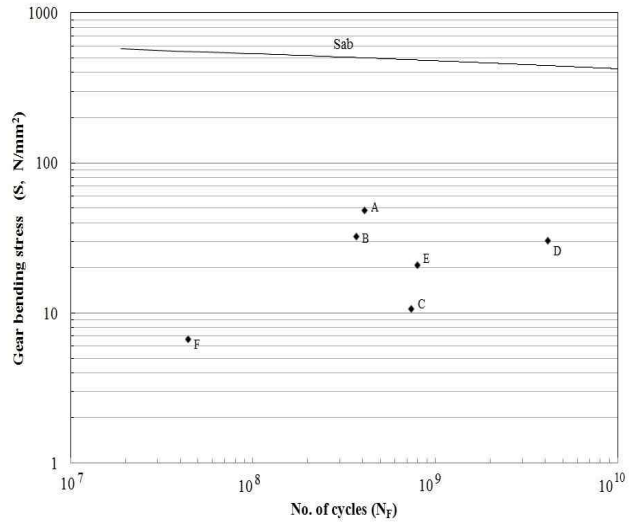
Calculating actual gear bending and compressive stresses of planetary gear system for gears and considering allowable gear bending and compressive stresses, produce safety factors and verify the problems of gear strength for the calculated specifications of the planetary gear system in HMCVT.

Figure 5 shows the results of gear bending stress analysis and Figure 6 the results of gear compressive stress

analysis of planetary gear system. It can be shown that actual gear bending & compressive stresses of the differential planetary gears are under the allowable gear bending and compressive stresses in these S/N curves. Thus, calculation results are set safely and have been verified as valid.

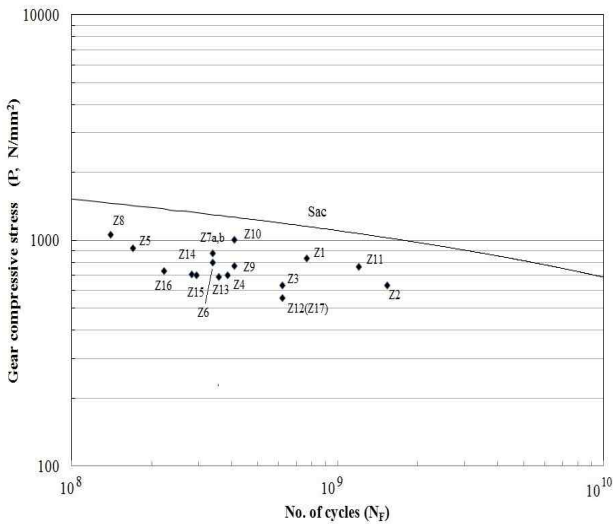


(a) Helical gears

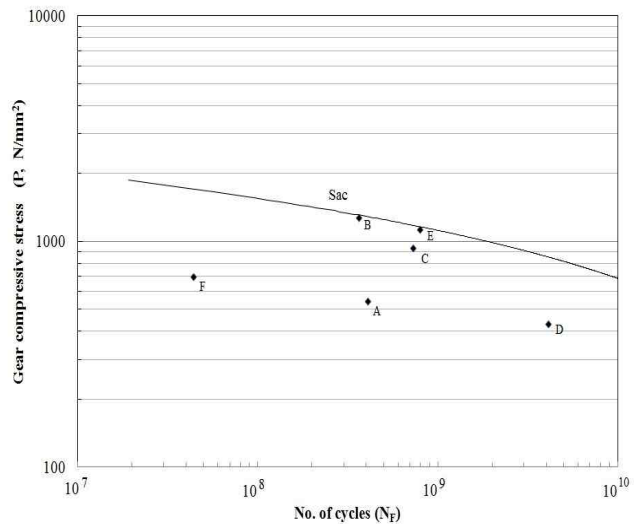


(b) Planetary gears

Figure 5. The results of the gear bending stress analysis



(a) Helical gears



(b) Planetary gears

Figure 6. The results of the gear compressive stress analysis

2.7 The results of Gear Scoring factor analysis

This calculates the actual scoring factor, *PVT* and judge the safety of gear scoring failure considering the limit value. Table 5 shows the results of calculated actual gear scoring factor .

Table 5. Calculated actual gear scoring factor

Items	Z1	Z2	Z3	Z4	Z5	Z6	Z7a,b	Z8
Actual scoring factor, $PVT < 38,860 (N \cdot m / \text{sec} \cdot \text{mm})$	8,487	15,876	8,771	4,841	2,636	8,957	7,379	5,801
Items	Z9	Z10	Z11	Z12	Z13	Z14	Z15	Z16
Actual scoring factor, $PVT < 38,860 (N \cdot m / \text{sec} \cdot \text{mm})$	13,298	7,516	20,932	4,606	4,312	3,273	3,626	2,665
(a) Helical gears								
Items	No1 Sun gear	No1 Pinion gear		No1 Ring gear				
Actual scoring factor, $PVT < 38,860 (N \cdot m / \text{sec} \cdot \text{mm})$	2,577	8,996	2,665	901				
(b) Sun gear(A) + Pinion gear(B) + Ring gear(C)								
Items	No2 Sun gear	No2 Pinion gear		No1 Ring gear				
Actual scoring factor, $PVT < 38,860 (N \cdot m / \text{sec} \cdot \text{mm})$	16	13	3,714	1,156				
(c) Sun gear(D) + Pinion gear(E) + Ring gear(F)								

3. Conclusion

This study shows actual gear bending and compressive stresses of the differential planetary gears using Lewes & Hertz equation by making use of the calculated specifications of differential planetary gears of HMCVT for 8 ton grade forklift and evaluate the results with the data of allowable bending and compressive stress from the Stress-No. of cycles curves of gears, based on Gear Handbook of D. W. Dudley and AGMA Standard 218.01.

- (1) In respect of the result of gear bending and compressive stress analysis of calculated specifications of differential planetary gears of HMCVT, the strength of the differential planetary gears and the developed programs have been verified as valid.
- (2) And in respect of the result of actual scoring factor analysis of calculated specifications of the differential planetary gears, scoring failure is not predicted.
- (3) The developed programs calculating the specifications and analyzing the gear bending and compressive stresses and gear scoring factor of the planetary gear system are expected to be effectively utilized. And future researches on more excellent planetary gear system of the various reducers for construction machines are expected to be still performed.

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