International Journal of Advanced Smart Convergence Vol.7 No.3 37-43 (2018)
http://dx.doi.org/10.7236/IJASC.2018.7.3.37

IJASC 18-3-5

Performance Analysis of LR-aided ZF Receiver for MIMO Systems

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Abstract

Lattice-reduction (LR) techniques have been developed for signal detection in spatial multiplexing multiple input multiple output (MIMO) systems to obtain the largest diversity gain. Thus, an LR-assisted zero-forcing (ZF) receiver can achieve the maximum diversity gain in spatial multiplexing MIMO systems. In this paper, a simplified analysis of the achievable diversity gain is presented by fitting the channel coefficients lattice-reduced by a complex Lenstra-Lenstra-Lováz (LLL) algorithm into approximated Gaussian random variables. It will be shown that the maximum diversity gain corresponding to two times the number of receive antennas can be achieved by the LR-based ZF detector. In addition, the approximated bit error rate (BER) expression is also derived. Finally, the analytical BER performance is comparatively studied with the simulated results.

Keywords: *Lattice Reduction (LR), Zero-forcing (ZF) receiver, Spatial multiplexing, Multiple input multiple output (MIMO), Diversity gain*

1. Introduction

Lattice-reduction (LR) techniques have been employed to enhance the performance of linear receivers over multiple input multiple output (MIMO) systems [1]. An LR-aided linear receiver can achieve the largest diversity gain with an average polynomial complexity. Although prior works [2]-[5] on the analysis of diversity gain collected by the LR-based linear receiver have been based on a bound on δ, the orthogonality defect of the Lenstra-Lenstra-Lov*á*z (LLL) reduction [6], direct comparison between the resulting theoretical bit error rate (BER) performances and the simulated ones was missing. In this work, analyzing the diversity gain and deriving BER performance are simplified and thus the theoretical BER results are directly compared with the simulated ones.

This paper focuses on a zero-forcing (ZF) receiver as a linear receiver, which can be applied to separate the spatially multiplexed data in spatial multiplexing MIMO systems [7, 8]. The rest of this par is organized as follows. Section 2 describes the system model based on LR. In Section 3, the performance analysis for the

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Manuscript Received: July. 1, 2018 / Revised: July. 11, 2018 / Accepted: July. 16, 2018
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LR-assisted ZF receiver is included. Section 4 contains simulation results and conclusions are drawn in the last section.

2. System Model

Consider a MIMO system with N_T transmit antennas and N_R receive antennas. Then the received signal, the transmitted signal, and the noise signal vectors are given by, respectively,

$$\begin{aligned}\mathbf{y} &= \left[y_1\ y_2 \cdots y_{N_R}\right]^T, \\ \mathbf{s} &= \left[s_1\ s_2 \cdots s_{N_T}\right]^T, \\ \mathbf{w} &= \left[w_1\ w_2 \cdots w_{N_R}\right]^T,\end{aligned} \tag{1}$$

where y_r and w_r, $r = 1,2,\cdots,N_R$, respectively, are the received signal and noise signal at the *r*th received antenna and s_t, $t = 1,2,\cdots,N_T$, is the signal transmitted at the *t*th transmit antenna. With the $N_R \times N_T$ matrix $\mathbf{H}$ is the channel matrix, the system model can be represented as $\mathbf{y} = \mathbf{Hs} + \mathbf{w}$. This work concentrates on a spatial multiplexing system with a binary phase shift keying modulation scheme. The average bit energy is assumed to be unity. The elements of $\mathbf{H}$ are independent and identically distributed (i.i.d.) with the zero-mean unit-variance complex Gaussian distribution and the noise is i.i.d. complex additive white Gaussian with zero-mean and variance σ_w^2. The LR-assisted ZF detector is considered to separate the transmitted data streams. The received signal can be rewritten as

$$\mathbf{y} = \mathbf{Hs} + \mathbf{w} = \tilde{\mathbf{H}}\mathbf{z} + \mathbf{w} \tag{2}$$

where $\tilde{\mathbf{H}} = \mathbf{HT}$ and $\mathbf{z} = \mathbf{T}^{-1}\mathbf{s}$. The new channel matrix $\tilde{\mathbf{H}}$ is a lattice basis reduced matrix generated by the complex LLL algorithm [4] and $\mathbf{T}$ is an complex integer unimodular transformation matrix. At the receiver, the LR-based ZF detection can be performed to detect the following.

$$\hat{\mathbf{z}} = \left\lceil \tilde{\mathbf{H}}^{\dagger}\mathbf{y} \right\rfloor = \left\lceil \mathbf{z} + \tilde{\mathbf{H}}^{\dagger}\mathbf{w} \right\rfloor \tag{3}$$

where $\lceil \cdot \rfloor$ is the rounding operation and $\tilde{\mathbf{H}}^{\dagger}$ is the pseudo-inverse of $\tilde{\mathbf{H}}$. Then the final detection results are obtained by mapping $\mathbf{T}\hat{\mathbf{z}}$ to the signal constellation and $\mathbf{s}$ can be recovered.

3. Performance Analysis

From the decision variable for $\mathbf{z}$ prior to quantization, the effective postprocessing signal-to-noise ratio (SNR) of the *k*th data stream is given by

$$\rho_k = \frac{1}{\sigma_w^2 \left[\tilde{\mathbf{H}}^H \tilde{\mathbf{H}}\right]_{kk}^{-1}} = \frac{1}{\sigma_w^2} \tilde{\mathbf{h}}_k^H \mathbf{F} \tilde{\mathbf{h}}_k \tag{4}$$

where $\tilde{\mathbf{h}}_k$ is the *k*th column vector of $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{F}}$ is an $N_R \times N_R$ non-negative Hermitian matrix constructed from $\tilde{\mathbf{h}}_1, \cdots, \tilde{\mathbf{h}}_{k-1}, \tilde{\mathbf{h}}_{k+1}, \cdots, \tilde{\mathbf{h}}_{N_T}$. To simplify the analysis of diversity gain in this work, we assume that the LR matrix $\tilde{\mathbf{H}}$ consists of perfectly orthogonal column vectors. In this case, $\tilde{\mathbf{h}}_k$ and its projection on the subspace of $\tilde{\mathbf{F}}$ are orthogonal. Then the SNR expression of (4) can be simplified as

$$\rho_k = \frac{1}{\sigma_w^2} \left\| \tilde{\mathbf{h}}_k \right\|^2 = \frac{1}{\sigma_w^2} \sum_{i=1}^{N_R} \left| \tilde{h}_{k,i} \right|^2 \tag{5}$$

where $\tilde{h}_{k,i}$ is the *i*th element of the vector $\tilde{\mathbf{h}}_k$. As an additional simplification, $\tilde{h}_{k,i}$ will be fitted into an approximated Gaussian random variable with zero-mean and variance $\sigma_{k,i}^2$ for the purpose of theoretical analysis in respect of diversity gain and BER performance. Thus it is shown in Figures 1, 2, and 3 that the simulated histograms of real parts of $\tilde{h}_{1,1}$, $\tilde{h}_{2,1}$, and $\tilde{h}_{3,1}$ for the $(N_T, N_R) = (3,3)$ MIMO system are approximated as probability density functions (pdfs) of Gaussian distribution with a variance of $0.5\sigma_{1,1}^2 = 0.2058$, $0.5\sigma_{2,1}^2 = 0.2939$, and $0.5\sigma_{3,1}^2 = 0.4079$, respectively. The value of each $\sigma_{k,i}^2$ is affected by the LR operation trying to yield reduced bases with shorter basis vectors and thus depends on the values of N_R, N_T, and δ, which is a parameter selected in the complex LLL algorithm. Here the complex LLL algorithm with $\delta = 1$ is used.

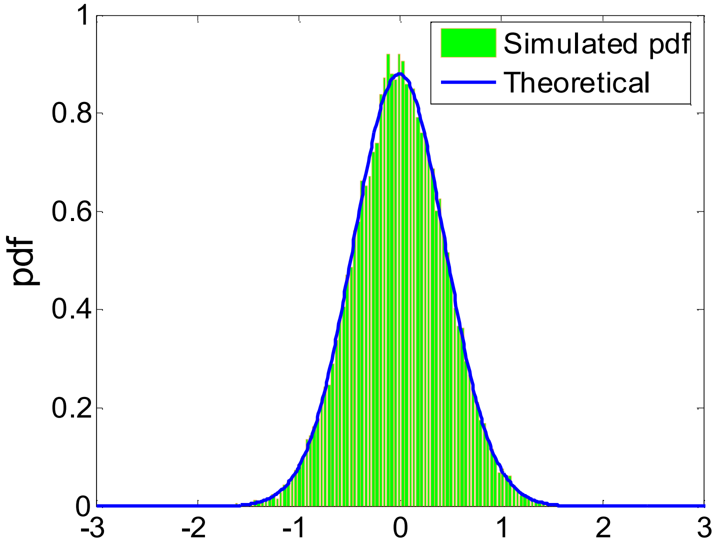

Figure 1. Approximated pdf of $\mathrm{Re}\left\{ \tilde{h}_{1,1} \right\}$

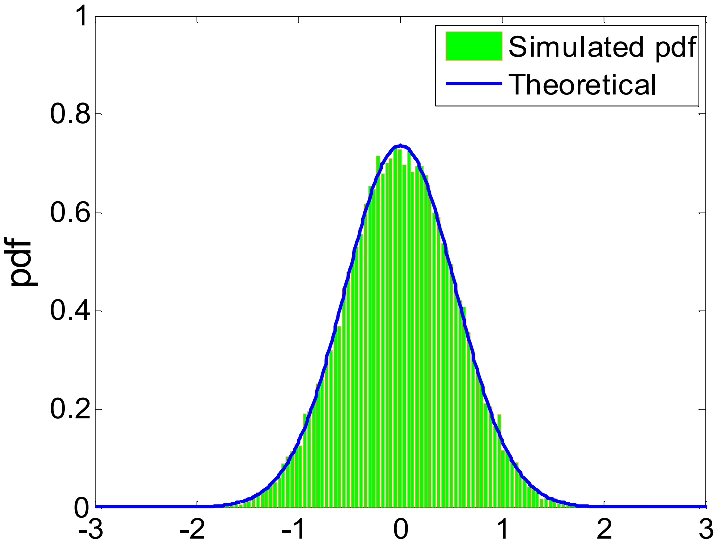

Figure 2. Approximated pdf of $\mathrm{Re}\left\{\tilde{h}_{2,1}\right\}$

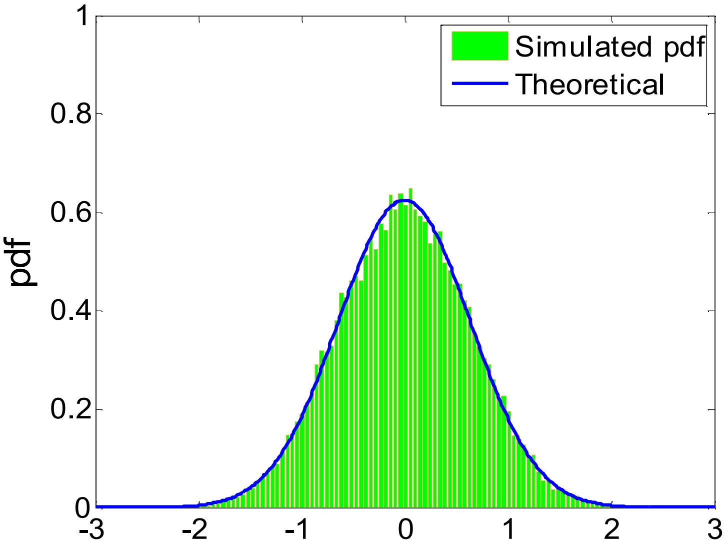

Figure 3. Approximated pdf of $\mathrm{Re}\left\{\tilde{h}_{3,1}\right\}$

With an assumption that $\tilde{x}_{k,i}$ is a complex Gaussian random variable with zero-mean and unit-variance, $\tilde{h}_{k,i}$ can be expressed as

$$\tilde{h}_{k,i} = E\left[\sigma_{k,i}\right]\tilde{x}_{k,i} = \sqrt{c_k^{\delta}}\,\tilde{x}_{k,i} \qquad (6)$$

where c_k^δ is the average variance of the approximated Gaussian pdfs. Thus, $E\left[\tilde{\mathbf{h}}_k \tilde{\mathbf{h}}_k^H\right] = c_k^\delta \mathbf{I}_{N_R}$. Then the SNR expression of (5) can be described as

$$\rho_k = \frac{c_k^\delta \rho_{k,0}}{\sigma_w^2} \tag{7}$$

where $\rho_{k,0} = \sum_{i=1}^{N_R} \left| \tilde{x}_{k,i} \right|^2$ can be assumed to be central Chi-square distributed with $D = 2N_R$ degrees of freedom and its pdf is given by

$$f_{\rho_{k,0}}(t) = \frac{1}{2\Gamma(0.5D)} \left(0.5t\right)^{0.5D-1} e^{-0.5t} \tag{8}$$

where $\Gamma(\cdot)$ is the gamma function. By averaging the conditional BER expression given by

$$P_{b|\rho_k}(t) = 0.5\,erfc\left(\sqrt{\frac{c_k^\delta}{2\sigma_w^2} t}\right) \tag{9}$$

over $f_{\rho_{k,0}}(t)$, the average BER of the LR-based ZF detection scheme can be computed as

$$P_b = \int_0^\infty P_{b|\rho_k}(t) f_{\rho_{k,0}}(t)\,dt \tag{10}$$

Note that in order to compute the theoretical BER performances from the expression (10), the value of c_k^δ should be required. In this work, the variance, $\sigma_{k,i}^2$, is obtained through simulations approximating as Gaussian pdf as in Figure 1. Note that the variance of the approximated Gaussian pdf could be varying according to the size of the $N_R \times N_T$ channel matrix $\tilde{\mathbf{H}}$ and the value of δ in the LLL algorithm. Eventually it could affect the output SNR value. As the value of δ increases up to 1 from 0.5, the variance c_k^δ of the approximated Gaussian pdf gets smaller. Although the perfect orthogonality is assumed in the process of developing an approximated analytic BER expression, the variance c_k^δ is calculated using the parameter $\delta = 1$ in the complex LLL algorithm and then the computed variance c_k^δ is applied to the SNR expression to obtain the theoretical BER results.

4. Simulation Results

Figures 4 and 5 show the analytical and simulated BER results of LR-aided ZF receiver for the $(N_T, N_R) = (4,4)$ and $(6,6)$ MIMO systems, respectively. Here the SNR per bit in decibel is defined as

$$SNR = 1/\sigma_w^2(dB) + 10\log_{10} D \tag{11}$$

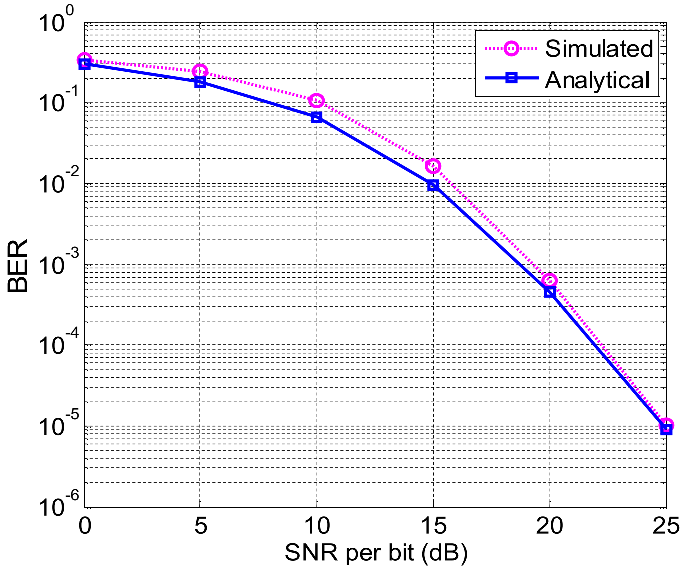

Figure 4. Analytical and simulated BER comparison for $(N_T, N_R) = (4,4)$ **with** $c_1^\delta = 0.4049$, $c_2^\delta = 0.5310$, $c_3^\delta = 0.6369$, $c_4^\delta = 0.7911$

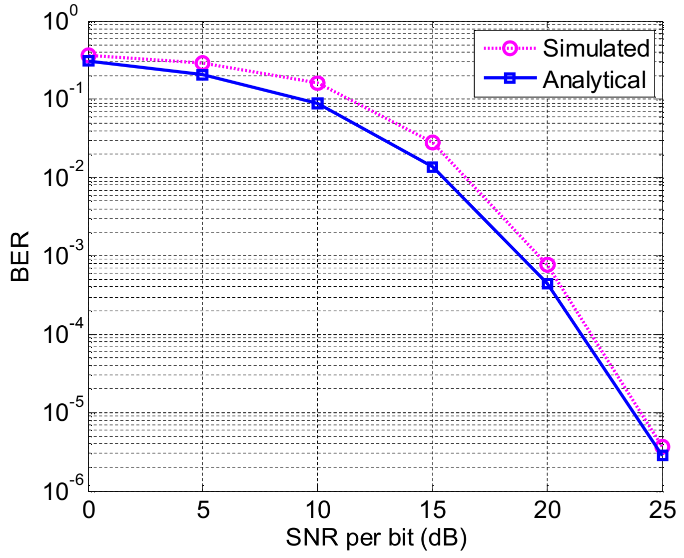

Figure 5. Analytical and simulated BER comparison for $(N_T, N_R) = (6,6)$ **with** $c_1^\delta = 0.4431$, $c_2^\delta = 0.5442$, $c_3^\delta = 0.6042$, $c_4^\delta = 0.6652$, $c_5^\delta = 0.7180$, $c_6^\delta = 0.8120$

It is found that the LR-aided ZF receiver can achieve the full diversity gain of $2N_R$. The plots display the variance c_k^δ obtained through simulations, which is used to compute the analytical BER value. It is observed that although the analytical BER results are slightly different from the simulated ones, the diversity gain of the one is similar to the other because their slopes are almost identical for the given system parameters with $N_T = N_R$.

5. Conclusions

This paper presents the simplified analysis of diversity gain and BER performance offered by an LR-aided ZF detection method for spatial multiplexing MIMO systems on Rayleigh fading channels. It is analytically shown that it can obtain the maximum achievable diversity gain of $2N_R$. The analytical BER performance is compared with the simulated results. Simulations are conducted to confirm the analytical results of diversity gain and BER performance.

Acknowledgement

This study was supported by research funds from Dong-A University.

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