

## LATTICE ORDERED SOFT NEAR RINGS

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ABSTRACT. Keeping in view the expediency of soft sets in algebraic structures and as a mathematical approach to vagueness, in this paper the concept of lattice ordered soft near rings is introduced. Different properties of lattice ordered soft near rings by using some operations of soft sets are investigated. The concept of idealistic soft near rings with respect to lattice ordered soft near ring homomorphisms is deliberated.

### 1. Introduction

In daily life human beings face different kinds of uncertainties in fields like environmental sciences, economics, social sciences, engineering and medicine etc. To tackle such uncertainties different kinds of tools are used, such as fuzzy sets [27] and rough sets [17] etc. Although these tools are very affective [2, 12] but have their own limitations. In 1999, Molodtsov [16] came to front and introduced the concept of soft sets, which solved these uncertainties more effectively.

Molodtsov [16] when proposed the idea of soft set theory, he pointed out some basic results, and also predicted its future work. After that Maji et al [14] defined some basic operations on soft sets and after that applied the concept of soft sets in decision making problems [15]. But

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later on Ali et al [3] proved that the operations defined in [14] have some shortcomings. In algebraic structures Aktas and Cagman [1] were the first who introduced the concept of soft groups. Aslam and Qurashi [5] also contributed in soft groups. Feng et al [8] initiated the concept of soft semirings and Sezgin et al [19] worked on soft near rings. Jun [11] initiated the idea of soft BCK/BCI-algebras. Ali et al [4] worked on lattice ordered soft sets and applied this concept in daily life problems. More recently Ma et al [13] applied the concept of soft sets in decision making methods based on certain hybrid soft set models. Zhan and others did a spade work in algebraic structures, decision making methods, fuzzy sets and rough sets by using the concept of soft sets [24–26]. For more applications of soft sets one can also see [6, 9, 20, 21, 23].

In this paper the concept of lattice ordered soft near rings is introduced. Different properties of lattice ordered soft near rings by using the binary operations are investigated. The concept of idealistic soft near rings with respect to lattice ordered soft near ring homomorphisms is discussed.

An algebraic system  $(\tilde{N}, +, \cdot)$  is known to be right near ring ( $NR$ ) if it satisfies the following axioms:

1.  $(\tilde{N}, +)$  forms a group (not necessarily abelian),
2.  $(\tilde{N}, \cdot)$  forms a semigroup,
3.  $(a + b) \cdot c = a \cdot c + b \cdot c, \forall a, b, c \in \tilde{N}$ .

Note that  $a \cdot 0 = 0$  &  $a \cdot (-b) = -a \cdot b$ , but  $0 \cdot b \neq 0$ , for  $a, b \in \tilde{N}$ , where  $\tilde{N}$  will always denote a  $NR$ , if otherwise stated. From now onward instead of writing  $a \cdot b$  we will write  $ab$ .

An element  $d \in \tilde{N}$  is called distributive if  $\forall n, n_1 \in \tilde{N}, d(n + n_1) = dn + dn_1$ . We denote and define  $\tilde{N}d = \{d \in \tilde{N} : d \text{ is distributive}\}$ . Let  $(M, +) \leq (\tilde{N}, +)$  for  $\tilde{N}$  with for all  $a, b \in M \Rightarrow ab \in M$ . Then  $M$  is known as subnear ring ( $subNR$ ) of  $\tilde{N}$ . A normal subgroup  $I$  of  $(\tilde{N}, +)$  is called an ideal of  $\tilde{N}$  if  $I\tilde{N}$  is subset of  $\tilde{N}$ , we write it as  $I\tilde{N} \subseteq I$ , and  $\forall n, n_1 \in \tilde{N}, i \in I, n(n_1 + i) - nn_1 \in I$ . Then we write  $I \triangleleft \tilde{N}$ . Let  $\tilde{N}_1$  and  $\tilde{N}_2$  be two  $NR$ s,  $g : \tilde{N}_1 \rightarrow \tilde{N}_2$  is called a  $NR$  homomorphism if  $\forall n_1, n_2 \in \tilde{N} g(n_1 + n_2) = g(n_1) + g(n_2)$  and  $g(n_1n_2) = g(n_1)g(n_2)$  [18]. For all undefined concepts about prime  $N$ -ideals, we refer to [10].

**DEFINITION 1.1.** [16] Let  $\tilde{N}$  be a preliminary universe set and the parameter set be  $E$ . Let  $P(\tilde{N})$  represent the power set of  $\tilde{N}$  and  $A \subseteq E$ .

Then a pair  $(F, A)$  is known as soft set ( $SS$ ) over  $\tilde{N}$ , where  $F$  is a mapping  $F : A \rightarrow P(\tilde{N})$ .

In other sense a  $SS$  over  $\tilde{N}$  is a parametrized family of subsets of the universe  $\tilde{N}$ , for  $e \in A$ ,  $F(e)$  may be considered as the set of  $e$ -approximate elements of the  $SS (F, A)$ .

Throughout in this section collection of all soft sets over  $\tilde{N}$  will be represented by  $SS(\tilde{N})$ ,  $E_N$  will denote a parameter set and  $A_N, B_N \subseteq E_N$ , if otherwise stated.

DEFINITION 1.2. [14] Let  $(F, A_N), (G, B_N) \in SS(\tilde{N})$ .  $(F, A_N)$  is known a soft subset of  $(G, B_N)$  if

- 1)  $A_N \subseteq B_N$  &
- 2)  $\forall e \in A_N, F(e)$  and  $G(e)$  are the same approximations.

This relation is represented by  $(F, A_N) \subseteq (G, B_N)$ . Similarly  $(F, A_N)$  is known a soft super set of  $(G, B_N)$  if  $(G, B_N)$  is a soft subset of  $(F, A_N)$ . This relationship is represented by  $(F, A_N) \supseteq (G, B_N)$ .

DEFINITION 1.3. [14] If  $(F, A_N)$  is a soft subset of  $(G, B_N)$  and  $(G, B_N)$  is soft subset of  $(F, A_N)$ , then  $(F, A_N)$  and  $(G, B_N)$  are said to be equal. It is represented by  $(F, A_N) = (G, B_N)$ .

DEFINITION 1.4. [14]  $(F, A_N) \in SS(\tilde{N})$  is known to be a null  $SS$  represented by  $\phi$  if  $\forall e \in A_N \Rightarrow F(e) = \emptyset$  (null-set).

DEFINITION 1.5. [14]  $(F, A_N) \in SS(\tilde{N})$  is known to be an absolute  $SS$  represented by  $\acute{A}_N$  if  $\forall e \in A_N \Rightarrow F(e) = \tilde{N}$ .

DEFINITION 1.6. [8] Let  $(F, A_N) \in SS(\tilde{N})$ . Support of  $(F, A_N)$  is denoted and defined by  $Supp(F, A_N) = \{e \in A_N : F(e) \neq \emptyset\}$ .

DEFINITION 1.7. [14] Let  $(F, A_N), (G, B_N) \in SS(\tilde{N})$ . The binary operation  $AND$  of  $(F, A_N)$  and  $(G, B_N)$  is represented by  $(F, A_N) \tilde{\wedge} (G, B_N) = (H, A_N \times B_N)$  and defined by  $H(\alpha, b) = F(\alpha) \cap G(b) \forall (\alpha, b) \in A_N \times B_N$ .

DEFINITION 1.8. [14] Let  $(F, A_N), (G, B_N) \in SS(\tilde{N})$ . The binary operation  $OR$  of  $(F, A_N)$  and  $(G, B_N)$  is represented by  $(F, A_N) \tilde{\vee} (G, B_N) = (H, A_N \times B_N)$  and defined as  $H(\alpha, b) = F(\alpha) \cup G(b) \forall (\alpha, b) \in A_N \times B_N$ .

DEFINITION 1.9. [14] Let  $(F, A_N), (G, B_N) \in SS(\tilde{N})$ . Then union of  $(F, A_N)$  and  $(G, B_N)$  is represented by  $(F, A_N) \tilde{\cup}_B (G, B_N) = (H, C_N)$ , where  $C_N = A_N \cup B_N$  and  $\forall \varsigma \in C_N$

$$H(\varsigma) = \begin{cases} F(\varsigma) & \text{if } \varsigma \in A_N - B_N \\ G(\varsigma) & \text{if } \varsigma \in B_N - A_N \\ F(\varsigma) \cup G(\varsigma) & \text{if } \varsigma \in A_N \cap B_N \end{cases}$$

DEFINITION 1.10. [14] Let  $(F, A_N), (G, B_N) \in SS(\tilde{N})$  such that  $C_N = A_N \cap B_N \neq \emptyset$ . Intersection of  $(F, A_N)$  and  $(G, B_N)$  is represented by  $(F, A_N) \widetilde{\cap} (G, B_N) = (H, C_N)$  and defined by  $H(\varsigma) = F(\varsigma)$  or  $G(\varsigma) \forall \varsigma \in C_N$ .

DEFINITION 1.11. [3] Let  $(F, A_N), (G, B_N) \in SS(\tilde{N})$  such that  $C_N = A_N \cap B_N \neq \emptyset$ . Restricted union of  $(F, A_N)$  and  $(G, B_N)$  is represented by  $(F, A_N) \widetilde{\cup}_R (G, B_N) = (L, C_N)$  and defined by  $L(\varsigma) = F(\varsigma) \cup G(\varsigma) \forall \varsigma \in C_N$ .

DEFINITION 1.12. [3] Let  $(F, A_N), (G, B_N) \in SS(\tilde{N})$  such that  $C_N = A_N \cap B_N \neq \emptyset$ . Restricted intersection of  $(F, A_N)$  and  $(G, B_N)$  is represented by  $(F, A_N) \widetilde{\cap}_R (G, B_N) = (H, C_N)$  and defined by  $H(\varsigma) = F(\varsigma) \cap G(\varsigma) \forall \varsigma \in C_N$ .

DEFINITION 1.13. [3] Let  $(F, A_N), (G, B_N) \in SS(\tilde{N})$ . Extended intersection of  $(F, A_N)$  and  $(G, B_N)$  is represented by  $(F, A_N) \widetilde{\cap}_E (G, B_N) = (H, C_N)$ , where  $C_N = A_N \cup B_N$  and  $\forall \varsigma \in C_N$  and defined as

$$H(\varsigma) = \begin{cases} F(\varsigma) & \text{if } \varsigma \in A_N - B_N \\ G(\varsigma) & \text{if } \varsigma \in B_N - A_N \\ F(\varsigma) \cap G(\varsigma) & \text{if } \varsigma \in A_N \cap B_N \end{cases}$$

DEFINITION 1.14. [7] A partially ordered set (POS)  $\mathcal{L}$  is said to be a lattice iff for every  $\alpha, b \in \mathcal{L}$ ,  $sup \{\alpha, b\}, inf \{\alpha, b\} \in \mathcal{L}$ .

In our next study  $E_N$  will always denote a lattice, if otherwise stated.

DEFINITION 1.15. [4] A  $SS (F, A_N)$  is known as lattice ordered soft set (LOSS) over  $\tilde{N}$  (anti-lattice ordered soft sets (ALOSS) over  $\tilde{N}$ ) if  $a_1 \preceq a_2 \Rightarrow F(\alpha_1) \subseteq F(\alpha_2) (F(\alpha_1) \supseteq F(\alpha_2)), \forall \alpha_1, \alpha_2 \in A_N$ .

DEFINITION 1.16. [19] Let  $(F, A_N)$  be a non-null  $SS$  over  $\tilde{N}$ . Then  $(F, A_N)$  is known as soft near ring (SNR) over  $\tilde{N}$  if  $F(a)$  is the *subNR* of  $\tilde{N}$ ,  $\forall \alpha \in Supp(F, A_N)$ .

From now onward, if stated otherwise, collection of all soft near rings over  $\tilde{N}$  will be denoted by  $SNR(\tilde{N})$ .

DEFINITION 1.17. [19] Let  $(F, A_N), (G, B_N) \in SNR(\tilde{N})$ . Then  $(F, A_N)$  is known sub soft near ring (*subSNR*) of  $(G, B_N)$  if  $A_N \subseteq B_N$  and  $F(\alpha)$  is a *subNR* of  $G(\alpha)$ ,  $\forall \alpha \in Supp(F, A_N)$ .

## 2. Lattice Ordered Soft Near Rings

In this section we define lattice ordered soft near rings, anti-lattice ordered soft near rings and discuss the associated results.

DEFINITION 2.1. A soft set  $(F, A_N)$  over  $\tilde{N}$  is said to be lattice (anti-lattice) ordered soft near ring over  $\tilde{N}$  iff

- (1)  $F(\alpha)$  is a *subNR* of  $\tilde{N} \forall \alpha \in A_N$ ,
- (2) If  $\alpha_1 \preceq \alpha_2$  then  $F(\alpha_1) \subseteq F(\alpha_2)$ , ( $F(\alpha_1) \supseteq F(\alpha_2)$ ), for any  $\alpha_1, \alpha_2 \in A_N$ .

DEFINITION 2.2. From now onward  $LOSNR(\tilde{N})$  will denote the collection of all lattice ordered soft near rings over  $\tilde{N}$  and  $ALOSNR(\tilde{N})$  will denote the collection of all anti-lattice ordered soft near rings over  $\tilde{N}$ .

DEFINITION 2.3. Let  $(F, A_N), (G, B_N) \in LOSNR(\tilde{N})$ .  $(F, A_N)$  is said to be *LOS subNR* of  $(G, B_N)$  if it satisfies the following conditions:

- (1)  $A_N \subseteq B_N$
- (2)  $F(\alpha)$  is *subNR* of  $G(b)$  for  $\alpha \in Supp(F, A_N)$ .

EXAMPLE 2.4. Consider  $\tilde{N} = \mathbb{Z}_8 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}\}$  with  $\bar{0} \preceq \bar{1} \preceq \bar{2} \preceq \bar{3} \preceq \bar{4} \preceq \bar{5} \preceq \bar{6} \preceq \bar{7}$ . Take  $A_N = \{\bar{1}, \bar{2}, \bar{4}\}$  define  $F : A_N \rightarrow P(\tilde{N})$  by

$F(\alpha) = \{n \in \tilde{N} : \alpha n = \bar{0}\}$ , then  $(F, A_N) = \{F(\bar{1}) = \{\bar{0}\}, F(\bar{2}) = \{\bar{0}, \bar{4}\}, F(\bar{4}) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}\}$  is *LOS NR*.

Now let  $B_N = \{\bar{1}, \bar{2}, \bar{4}\}$  and let define  $G : B_N \rightarrow P(\tilde{N})$  by

$G(b) = \{n \in \tilde{N} : bn \in \{\bar{0}, \bar{4}\}\}$ , then  $G(\bar{1}) = \{\bar{0}, \bar{4}\}$ ,  $G(\bar{2}) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}$ ,  $G(\bar{4}) = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}\}$ ,  $\implies (G, B_N) = \{G(\bar{1}) = \{\bar{0}, \bar{4}\}, G(\bar{2}) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}, G(\bar{4}) = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}\}\}$  is *LOS NR*.

Now for  $\bar{1} \in Supp(F, A_N)$ ,  $F(\bar{1}) = \{\bar{0}\}$  is *subNR* of  $G(\bar{1}) = \{\bar{0}, \bar{4}\}$ , for  $\bar{2} \in Supp(F, A_N)$ ,  $F(\bar{2}) = \{\bar{0}, \bar{4}\}$  is *subNR* of  $G(\bar{2}) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}$ , for  $\bar{4} \in Supp(F, A_N)$ ,  $F(\bar{4}) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}$  is *subNR* of  $G(\bar{4}) = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}\}$  with  $A_N \subseteq B_N$ . So  $(F, A_N)$  is a *LOS subNR* of  $(G, B_N)$ .

THEOREM 2.5. Let  $(F, A_N), (G, B_N) \in LOSNR(\tilde{N})$ . Then,  $(F, A_N) \tilde{\wedge} (G, B_N) \in LOSNR(\tilde{N})$ , whenever  $(F, A_N) \tilde{\wedge} (G, B_N)$  is non-null *SS*( $\tilde{N}$ ).

*Proof.* As by Definition 1.7 we know that  $(F, A_N) \tilde{\wedge} (G, B_N) = (H, C_N)$  with  $C_N = A_N \times B_N$ . Then for any  $\alpha \in A_N, b \in B_N$  and for  $H(\alpha, b) = F(\alpha) \cap G(b)$ , where  $F(\alpha)$  and  $G(b)$  are *subNR* over  $\tilde{N}$ . As  $(H, A_N \cap B_N)$  is

non-empty then  $F(\alpha) \cap G(b) \neq \emptyset$ . Also we know that intersection of any number of *subNRs* of  $\tilde{N}$  is *subNR* over  $\tilde{N}$ . So  $H(\alpha, b) = F(\alpha) \cap G(b)$  is *subNR* over  $\tilde{N}$ . Hence  $(H, A_N \tilde{\cap} B_N)$  is *SNR* over  $\tilde{N}$ .

Now we have to show that  $(H, A_N \tilde{\cap} B_N)$  contains lattice order. Since  $A_N, B_N \subseteq E_N$  so partial order is promoted into  $A_N$  and  $B_N$  from  $E_N$ . Now for  $\alpha_1 \preceq_{A_N} \alpha_2$  we have  $F(\alpha_1) \subseteq F(\alpha_2) \forall \alpha_1, \alpha_2 \in A_N$ , for  $b_1 \preceq_{B_N} b_2$  we have  $G(b_1) \subseteq G(b_2) \forall b_1, b_2 \in B_N$ . Now  $\preceq$  is the partial order on  $C_N$  which is transferred by partial orders on  $A_N$  and  $B_N$ , therefore for any  $(\alpha_1, b_1), (\alpha_2, b_2) \in C_N$  if  $(\alpha_1, b_1) \preceq (\alpha_2, b_2)$  then  $F(\alpha_1) \subseteq F(\alpha_2)$  and  $G(b_1) \subseteq G(b_2)$  implies  $F(\alpha_1) \cap G(b_1) \subseteq F(\alpha_2) \cap G(b_2) \Rightarrow H(\alpha_1, b_1) \subseteq H(\alpha_2, b_2)$ .

Hence  $(F, A_N) \tilde{\wedge} (G, B_N) \in LOSNR(\tilde{N})$ .  $\square$

REMARK 2.6. The binary operation *OR* of two *LOSNRs* may or may not be a *LOSNR*.

EXAMPLE 2.7. Consider the nearring  $\tilde{N} = \mathbb{Z}_8 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}\}$  with  $\bar{0} \preceq \bar{1} \preceq \bar{2} \preceq \bar{3} \preceq \bar{4} \preceq \bar{5} \preceq \bar{6} \preceq \bar{7}$ . Let  $A_N = \{\bar{2}, \bar{4}\}$  and define  $F : A_N \rightarrow P(\tilde{N})$  by

$H(\alpha) = \{n \in \tilde{N} : \alpha n = \bar{0}\}$ , then  $(F, A_N) = \{F(\bar{2}) = \{\bar{0}, \bar{4}\}, F(\bar{4}) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}\}$ .

Now take  $B_N = \{\bar{1}, \bar{2}, \bar{4}\}$  and define  $G : B_N \rightarrow P(\tilde{N})$  by

$G(b) = \{n \in \tilde{N} : bn \in \{\bar{0}, \bar{4}\}\}$ , then  $(G, B_N) = \{G(\bar{1}) = \{\bar{0}, \bar{4}\}, G(\bar{2}) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}, G(\bar{4}) = \tilde{N}\}$ . Here  $(F, A_N), (G, B_N) \in LOSNR(\tilde{N})$ .

Now  $A_N \times B_N = \{(\bar{2}, \bar{1}), (\bar{2}, \bar{2}), (\bar{2}, \bar{4}), (\bar{4}, \bar{1}), (\bar{4}, \bar{2}), (\bar{4}, \bar{4})\}$ , then  $(L, C_N) = \{L(\bar{2}, \bar{1}) = \{\bar{0}, \bar{4}\}, L(\bar{2}, \bar{2}) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}, L(\bar{2}, \bar{4}) = \{\tilde{N}\}, L(\bar{4}, \bar{1}) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}, L(\bar{4}, \bar{2}) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}, L(\bar{4}, \bar{4}) = \tilde{N} \notin LOSNR(\tilde{N})\}$ .

THEOREM 2.8. Let  $(F, A_N), (G, B_N) \in LOSNR(\tilde{N})$ . Then,  $(F, A_N) \tilde{\vee} (G, B_N) \in LOSNR(\tilde{N})$ , whenever  $(F, A_N) \tilde{\vee} (G, B_N)$  is non-null *SS*( $\tilde{N}$ ) and either  $F(\alpha) \subseteq G(b)$  or  $G(b) \subseteq F(\alpha)$ .

*Proof.* For any  $(\alpha, b) \in A_N \times B_N$  we consider that  $F(\alpha) \subseteq G(b)$ . By Definition 1.8 we know that  $(F, A_N) \tilde{\vee} (G, B_N) = (L, C_N)$ ,  $C_N = A_N \times B_N$ , then for any  $(\alpha, b) \in A_N \times B_N$  we have  $L(\alpha, b) = F(\alpha) \cup G(b)$ , where  $F(\alpha)$  and  $G(b)$  are *subNR* over  $\tilde{N}$ .

Now as  $A_N \subseteq B_N$ , then  $A_N \cap B_N = A_N \Rightarrow (L, A_N \times B_N) = (L, A_N) \Rightarrow F(x) \Rightarrow (F, A_N) \tilde{\vee} (G, B_N) = (F, A_N)$ , and  $(F, A_N) \in LOSNR(\tilde{N})$ .

Hence  $(F, A_N) \tilde{\vee} (G, B_N) \in LOSNR(\tilde{N})$ .  $\square$

**THEOREM 2.9.** *Let  $(F, A_N), (G, B_N) \in LOSNR(\tilde{N})$ . Then  $(F, A_N)\widetilde{\cap}_R(G, B_N) \in LOSNR(\tilde{N})$ , whenever  $(F, A_N)\widetilde{\cap}_R(G, B_N)$  is non-null  $SS(\tilde{N})$ .*

*Proof.* As by Definition 1.12 we know that  $(F, A_N)\widetilde{\cap}_R(G, B_N) = (H, C_N)$ , where  $C_N = A_N \cap B_N$  with  $A_N \cap B_N \neq \emptyset$ , then for  $\varsigma \in C_N$  we have  $H(\varsigma) = F(\varsigma) \cap G(\varsigma)$ , where  $F(a)$  and  $G(b)$  are *subNR* over  $\tilde{N}$ . As  $(H, A_N \cap B_N)$  is non-empty then  $F(\alpha) \cap G(b) \neq \emptyset$ , Also we know that intersection of any number of *subNRs* over  $\tilde{N}$  is *subNR* over  $\tilde{N}$ . So  $H(\varsigma) = F(\alpha) \cap G(b)$  is *subNR* over  $\tilde{N}$ . Hence  $(H, A_N \cap B_N)$  is *SNR* over  $\tilde{N}$ .

Now we have to show that  $(H, A_N \cap B_N)$  contains lattice ordered. Since  $A_N, B_N \subseteq E_N$ , so partial order is promoted into  $A_N$  and  $B_N$  from  $E_N$ . Now for  $\alpha_1 \preceq_{A_N} \alpha_2$  we have  $F(\alpha_1) \subseteq F(\alpha_2) \forall \alpha_1, \alpha_2 \in A_N$ , for  $b_1 \preceq_{B_N} b_2$  we have  $G(b_1) \subseteq G(b_2) \forall b_1, b_2 \in B_N$ . And for  $\varsigma_1 \preceq_{C_N} \varsigma_2$  we have  $F(\varsigma_1) \subseteq F(\varsigma_2)$  and  $G(\varsigma_1) \subseteq G(\varsigma_2) \forall \varsigma_1, \varsigma_2 \in C_N$ . Also  $F(\varsigma_1) \cap G(\varsigma_1) \subseteq F(\varsigma_2) \cap G(\varsigma_2) \Rightarrow H(\varsigma_1) \subseteq H(\varsigma_2)$ .

Hence  $(F, A_N)\widetilde{\cap}_R(G, B_N) = (H, A_N \cap B_N) \in LOSNR(\tilde{N})$ . □

**REMARK 2.10.** The restricted union of two *LOSNRs* may or may not be a *LOSNR*.

**EXAMPLE 2.11.** Consider  $\tilde{N} = \mathbb{Z}_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$  with  $\bar{0} \preceq \bar{1} \preceq \bar{2} \preceq \bar{3} \preceq \bar{4} \preceq \bar{5}$ . Take  $A_N = \{\bar{1}, \bar{2}\}$  define  $F : A_N \rightarrow P(\tilde{N})$  by

$F(\alpha) = \{n \in \tilde{N} : \alpha n = \bar{0}\}$ , then  $(F, A_N) = \{F(\bar{1}) = \{\bar{0}\}, F(\bar{2}) = \{\bar{0}, \bar{3}\}\}$  is *LOSNR*.

Now  $B_N = \{\bar{2}, \bar{3}\}$  define  $G : B_N \rightarrow P(\tilde{N})$  by

$G(b) = \{n \in \tilde{N} : bn \in \{\bar{0}, \bar{2}, \bar{4}\}\}$ , then  $(G, B_N) = \{G(\bar{2}) = \{\bar{0}, \bar{2}, \bar{4}\}, G(\bar{3}) = \{\bar{0}, \bar{2}, \bar{4}\}\}$  is *LOSNR*.

Now let us consider  $(F, A_N)\widetilde{\cup}_R(G, B_N)$ . Then  $(F, A_N)\widetilde{\cup}_R(G, B_N) = (L, A_N \cap B_N)$ , where  $L(\varsigma) = F(\varsigma) \cup G(\varsigma)$  for  $\varsigma \in A_N \cap B_N$ .

Now for  $2 \in A_N \cap B_N$  we have  $L(2) = F(2) \cup G(2) = \{\bar{0}, \bar{2}, \bar{3}, \bar{4}\}$  is not *subNR* So  $(F, A_N)\widetilde{\cup}_R(G, B_N)$  is not *LOSNR*.

**THEOREM 2.12.** *Let  $(F, A_N), (G, B_N) \in LOSNR(\tilde{N})$ . Then  $(F, A_N)\widetilde{\cup}_R(G, B_N) \in LOSNR(\tilde{N})$ , whenever  $(F, A_N)\widetilde{\cup}_R(G, B_N)$  is non-null  $SS$  over  $\tilde{N}$  and one of them is soft subset of the other.*

*Proof.* Let  $(F, A_N) \subseteq (G, B_N)$  with  $A_N \subseteq B_N$ . As by Definition 1.11 we know that  $(F, A_N)\widetilde{\cup}_R(G, B_N) = (H, C_N)$ ,  $C_N = A_N \cap B_N$  with

$A_N \cap B_N \neq \emptyset$ . Then for any  $\varsigma \in C_N$  we have  $H(\varsigma) = F(\alpha) \cup G(b)$ , where  $F(\alpha)$  and  $G(b)$  are *subNR* over  $\tilde{N}$ .

Now as  $A_N \subseteq B_N$ , then  $A_N \cap B_N = A_N \Rightarrow (H, A_N \cap B_N) = (H, A_N) \Rightarrow F(x) \Rightarrow (F, A_N) \cup_R (G, B_N) = (F, A_N)$ , but  $(F, A_N) \in \text{LOSNR}(\tilde{N})$ .

Hence  $(F, A_N) \widetilde{\cup}_R (G, B_N) \in \text{LOSNR}(\tilde{N})$ .  $\square$

**THEOREM 2.13.** *Let  $(F, A_N), (G, B_N) \in \text{LOSNR}(\tilde{N})$ . Then  $(F, A_N) \widetilde{\cap}_E (G, B_N) \in \text{LOSNR}(\tilde{N})$ , whenever  $(F, A_N) \widetilde{\cap}_E (G, B_N)$  be non-null  $SS(\tilde{N})$ .*

**THEOREM 2.14.** *Let  $(F, A_N), (G, B_N) \in \text{LOSNR}(\tilde{N})$ . Then  $(F, A_N) \widetilde{\cap}_B (G, B_N) \in \text{LOSNR}(\tilde{N})$ , provided  $A_N \cap B_N \neq \emptyset$ .*

**REMARK 2.15.** The union of two *LOSNRs* may or may not be a *LOSNR*.

**EXAMPLE 2.16.** Consider  $\tilde{N} = \mathbb{Z}_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$  with  $\bar{0} \preceq \bar{1} \preceq \bar{2} \preceq \bar{3} \preceq \bar{4} \preceq \bar{5}$ . Take  $A_N = \{\bar{1}, \bar{2}\}$  define  $F : A_N \rightarrow P(\tilde{N})$  by

$F(\alpha) = \{n \in \tilde{N} : \alpha n = \bar{0}\}$ , then  $(F, A_N) = \{F(\bar{1}) = \{\bar{0}\}, F(\bar{2}) = \{\bar{0}, \bar{3}\}\}$  is *LOSNR*.

Now  $B_N = \{\bar{2}, \bar{3}\}$  define  $G : B_N \rightarrow P(\tilde{N})$  by

$G(b) = \{n \in \tilde{N} : bn \in \{\bar{0}, \bar{2}, \bar{4}\}\}$ , then  $(G, B_N) = \{G(\bar{2}) = \{\bar{0}, \bar{2}, \bar{4}\}, G(\bar{3}) = \{\bar{0}, \bar{2}, \bar{4}\}\}$  is *LOSNR*.

Now let us consider  $(F, A_N) \widetilde{\cup}_B (G, B_N)$ . Then  $(F, A_N) \widetilde{\cup}_B (G, B_N) = (L, A_N \cup B_N)$  and define by

$$L(\varsigma) = \begin{cases} F(\varsigma) & \text{if } \varsigma \in A_N - B_N \\ G(\varsigma) & \text{if } \varsigma \in B_N - A_N \\ F(\varsigma) \cap G(\varsigma) & \text{if } \varsigma \in A_N \cap B_N \end{cases} \quad \forall \varsigma \in A_N \cup B_N$$

Now for  $\bar{2} \in A_N \cap B_N$  we have  $L(\bar{2}) = F(\bar{2}) \cup G(\bar{2}) = \{\bar{0}, \bar{2}, \bar{3}, \bar{4}\}$  is not *subNR*. So  $(F, A_N) \widetilde{\cup}_B (G, B_N)$  is not *LOSNR*.

**THEOREM 2.17.** *Let  $(F, A_N), (G, B_N) \in \text{LOSNR}(\tilde{N})$ . Then  $(F, A_N) \widetilde{\cup}_B (G, B_N) \in \text{LOSNR}(\tilde{N})$ , whenever  $(F, A_N) \widetilde{\cup}_B (G, B_N)$  is non-null  $SS(\tilde{N})$  and  $A_N \cap B_N = \emptyset$  or one of them is soft subset of other.*

*Proof.* As by Definition 1.9  $(F, A_N) \widetilde{\cup}_B (G, B_N) = (L, C_N)$ , where  $C_N = A_N \cup B_N$ . Now for any  $\varsigma \in C_N$

$$L(\varsigma) = \begin{cases} F(\varsigma) & \text{if } \varsigma \in A_N - B_N \\ G(\varsigma) & \text{if } \varsigma \in B_N - A_N \\ F(\varsigma) \cup G(\varsigma) & \text{if } \varsigma \in A_N \cap B_N \end{cases}, \text{ where } F(\varsigma) \text{ and } G(\varsigma)$$

are *subNR* of  $\tilde{N}$ . As  $A_N \cap B_N = \emptyset$  so we have



$$L(\varsigma) = \begin{cases} F(\varsigma) & \text{if } \varsigma \in A_N - B_N \\ G(v) & \text{if } \varsigma \in B_N - A_N \end{cases}$$

When  $\varsigma \in A_N - B_N$ , then  $\Rightarrow L(\varsigma) = F(\varsigma) \Rightarrow (L, C_N) = (F, C_N)$ ,  $(F, A_N) \in LOSNR(\tilde{N})$ . So  $(L, C_N) \in LOSNR(\tilde{N})$ .

When  $\varsigma \in B_N - A_N$ , then  $L(\varsigma) = G(\varsigma) \Rightarrow (L, C_N) = (G, B_N)$ ,  $(G, B_N) \in LOSNR(\tilde{N})$ . So  $(L, C_N) \in LOSNR(\tilde{N})$ .

Now consider  $(F, A_N) \subseteq (G, B_N)$  with  $A_N \subseteq B_N \Rightarrow L(\varsigma) = G(v) \Rightarrow (L, C_N) = (G, B_N)$ .  $(G, B_N) \in LOSNR(\tilde{N}) \Rightarrow (L, C_N) \in LOSNR(\tilde{N})$ , hence  $(F, A_N) \widetilde{\cup}_B (G, B_N) \in LOSNR(\tilde{N})$ .  $\square$

### 3. Lattice Ordered Idealistic Soft Near Rings

DEFINITION 3.1. Let  $(F, A_N) \in LOSNR(\tilde{N})$ . A non-empty  $SS(M, I)$  over  $\tilde{N}$  is known the  $LOISNR$  over  $\tilde{N}$  represented by  $(M, I) \triangleleft (F, A_N)$  if the following conditions are satisfied:

- (1)  $I \subseteq A_N$ ,
- (2)  $\forall \alpha \in Supp(M, I)$ ,  $M(\alpha) \triangleleft F(\alpha)$  and  $\forall \alpha_1, \alpha_2 \in \alpha_N$  with  $\alpha_1 \preceq \alpha_2$ ,  $M(\alpha_1) \preceq M(\alpha_2)$ .

From now onward collection of all lattice ordered idealistic soft near rings over  $\tilde{N}$  will be represented by  $LOISNR(\tilde{N})$ .

EXAMPLE 3.2. Let  $\tilde{N} = \mathbb{Z}_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$  with  $\bar{0} \preceq \bar{1} \preceq \bar{2} \preceq \bar{3} \preceq \bar{4} \preceq \bar{5}$ . Now  $I = \{\bar{1}, \bar{2}\}$  and define  $M : I \rightarrow P(\tilde{N})$  by  $M(I) = \{n \in \tilde{N} : in = \bar{0}\}$ , then  $(M, I) = \{M(\bar{1}) = \{\bar{0}\}, M(\bar{2}) = \{\bar{0}, \bar{3}\}\} \Rightarrow M(\bar{1}) \subseteq M(\bar{2})$ . Hence  $(M, I) \in LOISNR(\tilde{N})$ .

THEOREM 3.3. Let  $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$ . Then  $(M_1, I_1) \widetilde{\wedge} (M_2, I_2) \in LOISNR(\tilde{N})$ , whenever  $(M_1, I_1) \widetilde{\wedge} (M_2, I_2)$  is non-null  $SS(\tilde{N})$ .

*Proof.* As by Definition 1.7 we know that  $(M_1, I_1) \widetilde{\wedge} (M_2, I_2) = (M, I)$  where  $I = I_1 \times I_2$  with  $I_1 \cap I_2 \neq \emptyset$ . Then for  $\alpha_1 \in I_1, \alpha_2 \in I_2$  and for  $(\alpha_1, \alpha_2) \in I_1 \times I_2$  we have  $M(\alpha_1, \alpha_2) = M_1(\alpha_1) \cap M_2(\alpha_2)$ , where  $M_1(\alpha_1)$  and  $M_2(\alpha_2)$  are both ideals of  $\tilde{N}$ .

As  $(M, I)$  is non-empty, then  $M_1(\alpha_1) \cap M_2(\alpha_2) \neq \emptyset$ . Since the intersection of any number of ideals is a ideal  $\tilde{N}$ ,  $M(\alpha_1, \alpha_2) = M_1(\alpha_1) \cap M_2(\alpha_2)$  is ideal over  $\tilde{N}$ . Hence  $(M, I)$  is ideal  $SS(\tilde{N})$ .

Now we have to show that  $(M, I)$  contains lattice order. Since  $I_1, I_2 \subseteq E_N$ , so partial order is promoted into  $I_1$  and  $I_2$  from  $E_N$ .

Now for  $\alpha_1 \preceq_{I_1} \alpha_2$ , we have  $M_1(\alpha_1) \subseteq M_1(\alpha_2)$ ,  $\forall b_1, b_2 \in I_2$ , for  $b_1 \preceq_{I_2} b_2$  we have  $M_2(b_1) \subseteq M_2(b_2)$ ,  $\forall b_1, b_2 \in I_2$ . Now  $\preceq$  is the partial order on  $I$  which is promoted by partial order on  $I_1$  and  $I_2$ , therefore for any  $(\alpha_1, b_1), (\alpha_2, b_2) \in I_1 \times I_2$  we have  $M_1(\alpha_1) \subseteq M_1(\alpha_2)$  and  $M_2(b_1) \subseteq M_2(b_2)$  this implies  $M_1(\alpha_1) \cap M_2(b_1) \subseteq M_1(\alpha_2) \cap M_2(b_2) \Rightarrow M(\alpha_1, b_1) \subseteq M(\alpha_2, b_2)$ .

Hence  $(M_1, I_1) \widetilde{\wedge} (M_2, I_2) \in LOISNR(\tilde{N})$ . □

**THEOREM 3.4.** *Let  $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$ . Then  $(M_1, I_1) \widetilde{\vee} (M_2, I_2) \in LOISNR(\tilde{N})$ , whenever  $(M_1, I_1) \widetilde{\vee} (M_2, I_2)$  is non-null  $SS(\tilde{N})$  and either  $M_1(\alpha_1) \subseteq M_2(\alpha_2)$  or  $M_2(\alpha_2) \subseteq M_1(\alpha_1)$ .*

*Proof.* For any  $(\alpha_1, \alpha_2) \in I_1 \times I_2$  we consider that  $M_1(\alpha_1) \subseteq M_2(\alpha_2)$ . By Definition 1.8  $(M_1, I_1) \widetilde{\vee} (M_2, I_2) = (M, I)$ , where  $I = I_1 \times I_2$  and for any  $(\alpha_1, \alpha_2) \in I_1 \times I_2$  we have  $M(\alpha_1, \alpha_2) = M_1(\alpha_1) \cup M_2(\alpha_2)$ . As  $M_1(\alpha_1) \subseteq M_2(\alpha_2)$ ,  $M_1(\alpha_1) \cup M_2(\alpha_2) = M_2(\alpha_2) \Rightarrow (M_2, I_2) = (M, I)$ ,  $(M_2, I_2) \in LOISNR(\tilde{N})$ . So  $(M, I) \in LOISNR(\tilde{N})$ .

It follows that  $(M_1, I_1) \widetilde{\vee} (M_2, I_2) = (M, I) \in LOISNR(\tilde{N})$ . □

**THEOREM 3.5.** *Let  $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$ . Then  $(M_1, I_1) \widetilde{\cap}_R (M_2, I_2) \in LOISNR(\tilde{N})$ , whenever  $(M_1, I_1) \widetilde{\cap}_R (M_2, I_2)$  is a non-null  $SS$  over  $\tilde{N}$ .*

*Proof.* By Definition 1.12  $(M_1, I_1) \widetilde{\cap}_R (M_2, I_2) = (M, I)$ , as  $M(\alpha) = M_1(\alpha) \cap M_2(\alpha)$ , where  $I = I_1 \cap I_2$ ,  $\forall \alpha \in I$ . Since  $I_1 \subseteq A_N$  and  $I_2 \subseteq A_N$ , it is obvious that  $I \subseteq A_N$ . Then for  $\alpha \in I$  we have  $M(\alpha) = M_1(\alpha) \cap M_2(\alpha)$ . We deduce that the non-empty sets  $M_1(\alpha)$  and  $M_2(\alpha)$  are both ideals of  $F(\alpha)$ . Since the intersection of any number of ideals of  $\tilde{N}$  is an ideal of  $\tilde{N}$ ,  $M(\alpha) = M_1(\alpha) \cap M_2(\alpha)$  is an ideal of  $\tilde{N}$ , therefore  $(M_1, I_1) \widetilde{\cap}_R (M_2, I_2)$  is an ideal  $SS$  over  $\tilde{N}$ .

Now we have to show that  $(M, I)$  contains lattice ordered. Since  $I_1, I_2 \subseteq E_N$  so partial order is promoted into  $I_1$  and  $I_2$  from  $E_N$ . Now for  $\alpha_1 \preceq_{I_1} \alpha_2$  we have  $M_1(\alpha_1) \subseteq M_1(\alpha_2)$   $\forall b_1, b_2 \in I_1$ , for  $b_1 \preceq_{I_2} b_2$  we have  $M_2(b_1) \subseteq M_2(b_2)$   $\forall b_1, b_2 \in I_2$  and for  $\varsigma_1 \preceq_I \varsigma_2$  we have  $M_1(\varsigma_1) \subseteq M_1(\varsigma_2)$  and  $M_2(\varsigma_1) \subseteq M_2(\varsigma_2)$   $\forall \varsigma_1, \varsigma_2 \in I$  also  $M_1(\varsigma_1) \cap M_2(\varsigma_1) \subseteq M_1(\varsigma_2) \cap M_2(\varsigma_2) \Rightarrow M(\varsigma_1) \subseteq M(\varsigma_2)$ .

Hence  $(M_1, I_1) \widetilde{\cap}_R (M_2, I_2) = (M, I) \in LOISNR(\tilde{N})$ . □

**THEOREM 3.6.** *Let  $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$ . Then  $(M_1, I_1) \widetilde{\cup}_R (M_2, I_2) \in LOISNR(\tilde{N})$ , whenever  $(M_1, I_1) \widetilde{\cup}_R (M_2, I_2)$  is non-null  $SS(\tilde{N})$  and one of them is soft subset of other.*

*Proof.* By Definition 1.11 we know that  $(M_1, I_1) \widetilde{\cup}_R (M_2, I_2) = (M, I)$ , where  $I = I_1 \cap I_2$  with  $I_1 \cap I_2 = \emptyset$ , for  $\alpha \in I$ .  $M(\alpha) = M_1(\alpha) \cup M_2(\alpha)$ , where  $M_1(\alpha)$  and  $M_2(\alpha)$  are both ideals of  $\tilde{N}$ .

Since  $I_1 \subseteq I_2$ , then  $I_1 = I_1 \cap I_2 \Rightarrow (M_1, I_1) = (M, I) \Rightarrow (M_1, I_1) \cup_R (M_2, I_2) = (M_1, I_1)$ ,  $(M_1, I_1) \in LOISNR(\tilde{N})$ . So  $(M, I) \in LOISNR(\tilde{N})$ . Hence,  $(M_1, I_1) \cup_R (M_2, I_2) \in LOISNR(\tilde{N})$ .  $\square$

**THEOREM 3.7.** *Let  $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$ . Then  $(M_1, I_1) \cap_B (M_2, I_2) \in LOISNR(\tilde{N})$ , provided  $I_1 \cap I_2 \neq \emptyset$ .*

**THEOREM 3.8.** *Let  $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$ . Then  $(M_1, I_1) \widetilde{\cup}_B (M_2, I_2) \in LOISNR(\tilde{N})$ , whenever  $(M_1, I_1) \widetilde{\cup}_B (M_2, I_2)$  is non-null  $SS(\tilde{N})$  and  $I_1 \cap I_2 = \emptyset$  or if  $(M_1, I_1) \subseteq (M_2, I_2)$  or  $(M_2, I_2) \subseteq (M_1, I_1)$ .*

*Proof.* By Definition 1.9 we know that  $(M_1, I_1) \widetilde{\cup}_B (M_2, I_2) = (M, I)$ , where  $I = I_1 \cup I_2, \forall \alpha \in I$

$$M(\alpha) = \begin{cases} M_1(\alpha) & \text{if } \alpha \in I_1 - I_2 \\ M_2(\alpha) & \text{if } \alpha \in I_2 - I_1 \\ M_1(\alpha) \cup M_2(\alpha) & \text{if } \alpha \in I_1 \cap I_2 \end{cases}, \text{ where } M_1(\alpha) \text{ and}$$

$M_2(\alpha)$  are ideals of  $\tilde{N}$ . As  $I_1 \cap I_2 = \emptyset$  so we have

$$M(\alpha) = \begin{cases} M_1(\alpha) & \text{if } \alpha \in I_1 - I_2 \\ M_2(\alpha) & \text{if } \alpha \in I_2 - I_1 \end{cases}$$

When  $\alpha \in I_1 - I_2 \Rightarrow M(\alpha) = M_1(\alpha) \Rightarrow (M_1, I_1) = (M, I), (M_1, I_1) \in LOISNR(\tilde{N})$ . So  $(M, I) \in LOISNR(\tilde{N})$ .

When  $\alpha \in I_2 - I_1 \Rightarrow M(\alpha) = M_2(\alpha) \Rightarrow (M_2, I_2) = (M, I), (M_2, I_2) \in LOISNR(\tilde{N})$ . So  $(M, I) \in LOISNR(\tilde{N})$ .

Now consider  $(M_1, I_1) \cup_E (M_2, I_2)$  with  $I_1 \subseteq I_2 \Rightarrow M(\alpha) = M_1(\alpha) \Rightarrow (M_1, I_1) = (M, I), (M_1, I_1) \in LOISNR(\tilde{N}) \Rightarrow (M, I) \in LOISNR(\tilde{N})$ .

Hence  $(M_1, I_1) \widetilde{\cup}_B (M_2, I_2) \in LOISNR(\tilde{N})$ .  $\square$

**THEOREM 3.9.** *Let  $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$ . Then  $(M_1, I_1) \widetilde{\cap}_E (M_2, I_2) \in LOISNR(\tilde{N})$ , whenever  $(M_1, I_1) \widetilde{\cap}_E (M_2, I_2)$  is non-null  $SS(\tilde{N})$  and  $I_1 \cap I_2 = \emptyset$ .*

*Proof.* By Definition 1.13 we know that  $(M_1, I_1) \widetilde{\cap}_E (M_2, I_2) = (M, I)$ , where  $I = I_1 \cup I_2, \forall \alpha \in I$

$$M(\alpha) = \begin{cases} M_1(\alpha) & \text{if } \alpha \in I_1 - I_2 \\ M_2(\alpha) & \text{if } \alpha \in I_2 - I_1 \\ M_1(\alpha) \cap M_2(\alpha) & \text{if } \alpha \in I_1 \cap I_2 \end{cases}, \text{ where } M_1(\alpha) \text{ and}$$

$M_2(\alpha)$  are ideals of  $\tilde{N}$ . As  $I_1 \cap I_2 = \emptyset$  so we have

$$M(\alpha) = \begin{cases} M_1(\alpha) & \text{if } \alpha \in I_1 - I_2 \\ M_2(\alpha) & \text{if } \alpha \in I_2 - I_1 \end{cases}$$

When  $\alpha \in I_1 - I_2 \Rightarrow M(\alpha) = M_1(\alpha) \Rightarrow (M_1, I_1) = (M, I)$ ,  $(M_1, I_1) \in \text{LOISNR}(\tilde{N})$ . So  $(M, I) \in \text{LOISNR}(\tilde{N})$ .

Since  $\alpha \in I_2 - I_1 \Rightarrow M(\alpha) = M_2(\alpha) \Rightarrow (M_2, I_2) = (M, I)$ ,  $(M_2, I_2) \in \text{LOISNR}(\tilde{N})$ . So is  $(M, I) \in \text{LOISNR}(\tilde{N})$ .

It follows that  $(M_1, I_1) \widetilde{\cap}_E (M_2, I_2)$  is *LOISNR* over  $(F, A_N)$ .  $\square$

#### 4. Lattice Ordered Soft Near Ring Homomorphism

**DEFINITION 4.1.** Let  $(F, A_N)$  and  $(G, B_N)$  be two *LOSNR* over  $\tilde{N}_1$  and  $\tilde{N}_2$  respectively and  $f : \tilde{N}_1 \rightarrow \tilde{N}_2$  and  $h : A_N \rightarrow B_N$  be two functions. We say that  $(f, h)$  is lattice ordered soft near ring homomorphism (*LOSNRH*) from  $(F, A_N)$  to  $(G, B_N)$  if it satisfies the following conditions

- (1)  $f : \tilde{N}_1 \rightarrow \tilde{N}_2$  is a near ring homomorphism (*NRH*).
- (2)  $h : A_N \rightarrow B_N$  is an onto mapping.
- (3)  $f(F(\alpha)) = G(h(\alpha)) \forall \alpha \in A_N$ .
- (4)  $\forall \alpha_1, \alpha_2 \in A_N$  with  $\alpha_1 \preceq \alpha_2$  we have  $f(F(\alpha_1)) \preceq f(F(\alpha_2))$ .

**EXAMPLE 4.2.** Let  $\tilde{N}_1 = 3Z_6, \tilde{N}_2 = Z_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$ ,  $A_N = \{\alpha_1, \alpha_2, \alpha_3\}$  with  $\alpha_1 \preceq \alpha_2 \preceq \alpha_3$  and  $B_N = \{\beta_1, \beta_2, \beta_3\}$  with  $\beta_1 \preceq \beta_2 \preceq \beta_3$ .

Now define  $f : \tilde{N}_1 \rightarrow \tilde{N}_2$  by  $f(n) = 3n$ ,

$h : A_N \rightarrow B_N$  by  $h(\alpha_i) = \beta_i$  for  $1 \leq i \leq 3$ ,

$F : A \rightarrow P(\tilde{N}_1)$  by  $F(\alpha_1) = \{\bar{0}\}, F(\alpha_2) = F(\alpha_3) = \tilde{N}_1$  and

$G : A \rightarrow P(\tilde{N}_2)$  by  $G(\beta_1) = \{\bar{0}\}, G(\beta_2) = G(\beta_3) = 3\tilde{N}_2$ .

Then  $(f, h)$  is *LOSNRH* from  $(F, A_N)$  to  $(G, B_N)$ .

**THEOREM 4.3.** Let  $(F, A_N), (G, B_N) \in \text{LOSNR}(\tilde{N}_1)$ . If  $f : \tilde{N}_1 \rightarrow \tilde{N}_2$  be a *NRH*, then  $(f(F), A_N), (f(G), B_N) \in \text{LOSNR}(\tilde{N}_2)$  and if  $(G, B_N)$  is *LOS subNR* of  $(F, A_N)$ , then  $(f(G), B_N)$  is *LOS subNR* of  $(f(F), A_N)$ .

*Proof.* Given that  $(F, A_N), (G, B_N) \in \text{LOSNR}(\tilde{N}_1)$ . Then we have  $\forall \alpha_1, \alpha_2 \in \text{Supp}(F, A_N)$ ,  $\alpha_1 \preceq \alpha_2$  we have  $F(\alpha_1) \subseteq F(\alpha_2)$ . Similarly  $\forall b_1, b_2 \in \text{Supp}(G, B_N)$ ,  $b_1 \preceq b_2$  we have  $G(b_1) \subseteq G(b_2)$ , where we have  $G(b_1), G(b_2), F(\alpha_1)$  and  $F(\alpha_2)$  are all *subNR* over  $\tilde{N}_1$ .

Now  $f(F) : A_N \rightarrow P(\tilde{N}_2)$  is defined by  $f(F)(\alpha) = g(F(\alpha)) \forall \alpha \in A_N$  and  $f(G) : B_N \rightarrow P(\tilde{N}_2)$  is defined by  $f(G)(b) = f(G(b)) \forall b \in B_N$ .

Now consider  $m_1, m_2 \in F(\alpha_1)$ , as we have  $F(\alpha_1)$  is subNR of  $\tilde{N}_1$ , then we have  $m_1 - m_2 \in F(\alpha_1), m_1 m_2 \in F(\alpha_1)$ . Let  $F(m_1), F(m_2) \in g((\alpha_1))$ .

Now consider  $m_1 - m_2 \in F(\alpha_1) \Rightarrow f(m_1 - m_2) \in f(F(\alpha_1)) \Rightarrow f(m_1) - f(m_2) \in f(F(\alpha_1))$  and  $m_1 m_2 \in F(\alpha_1) \Rightarrow f(m_1 m_2) \in f(F(\alpha_1)) \Rightarrow f(m_1) f(m_2) \in f(F(\alpha_1))$ .

As  $f(m_1) - f(m_2) \in f(F(\alpha_1))$  and  $f(m_1) f(m_2) \in f(F(\alpha_1))$ . So  $f(F(\alpha_1))$  is subNR of  $\tilde{N}_2$ , in similar way we have  $f(F(\alpha_1))$  is subNR of  $\tilde{N}_2$ .

Hence  $\forall \alpha \in A_N, f(F(\alpha))$  is subNR of  $\tilde{N}_2$ . So we have  $(f(F), A_N)$  is SNR over  $\tilde{N}_2$ . Similarly we have  $(f(G), B_N)$  is SNR over  $\tilde{N}_2$ , as we have  $(G, B_N)$  is LOS subNR of  $(F, A_N)$ , so  $\forall b \in B_N, G(b)$  is subNR of  $F(b)$ , therefore  $f(G)(b)$  is subNR of  $f(F(b))$  and It follows that  $(f(G), B_N)$  is soft subNR of  $(f(F), A_N)$ .

As it is given that  $(F, A_N), (G, B_N) \in LOSNR(\tilde{N}_1)$ , then by Definition we have  $\forall \alpha_1, \alpha_2 \in A_N, \alpha_1 \preceq \alpha_2$  we have  $F(\alpha_1) \subseteq F(\alpha_2)$ . Similarly  $\forall b_1, b_2 \in B_N, b_1 \preceq b_2$  we have  $G(b_1) \subseteq G(b_2)$ .

Now let  $m_1 \in F(\alpha_1) \Rightarrow g(m_1) \in g(F(\alpha_1))$ . Now as  $F(\alpha_1) \subseteq F(\alpha_2)$  and  $m_1 \in F(\alpha_1) \Rightarrow m_1 \in F(\alpha_2) \Rightarrow f(m_1) \in f(F(\alpha_2))$ . As for  $f(m_1) \in f(F(\alpha_1))$  we have  $f(m_1) \in f(F(\alpha_2)) \Rightarrow f(F(\alpha_1)) \subseteq f(F(\alpha_2))$ .

This implies that  $(f(F), A_N)$  is LOSNR( $\tilde{N}_2$ ) and similarly we have  $(f(G), B_N) \in LOSNR(\tilde{N}_2)$  such that  $(f(F), A_N)$  is LOS subNR of  $(f(G), B_N)$ .  $\square$

**THEOREM 4.4.** Let  $(F, A_N)$  and  $(G, B_N)$  is LOSNR over  $\tilde{N}_1$  and  $\tilde{N}_2$  respectively, where  $\tilde{N}_1$  and  $\tilde{N}_2$  are two NRs and  $(f, h)$  is a LOSNRH from  $(F, A_N)$  to  $(G, B_N)$ . If  $(F, A_N)$  is LOISNR over  $\tilde{N}_1$ , then  $(G, B_N)$  is LOISNR over  $\tilde{N}_2$ .

*Proof.* Given that  $(F, A_N) \in LOISNR(\tilde{N}_1)$ , then by we have  $\forall, \alpha_1, \alpha_2 \in A_N, \alpha_1 \preceq_{A_N} \alpha_2$  we have  $F(\alpha_1) \subseteq F(\alpha_2)$ , where we have  $F(\alpha_1)$  and  $F(\alpha_2)$  are ideal of  $\tilde{N}_1$ . Also as it is given that  $(G, B_N) \in LOSNR(\tilde{N}_2)$ , then  $\forall b_1, b_2 \in B_N, b_1 \preceq_{B_N} b_2$  we have  $G(b_1) \subseteq G(b_2)$ , where we have  $G(b_1)$  and  $G(b_2)$  are ideal of  $\tilde{N}_2$ . Also as  $(f, h)$  is a LOSNRH from  $(F, A_N)$  to  $(G, B_N)$ , then for any  $b_1 \in B_N$  there exists an element  $\alpha_1 \in A_N$  for which we have  $b_1 = h(\alpha_1) \Rightarrow G(b_1) = G(h(\alpha_1)) = f(F(\alpha_1))$ . As  $F(\alpha_1)$  is ideal of  $\tilde{N}_1$ , so  $f(F(\alpha_1))$  is a ideal NR of  $G(h(b_1))$ . Similarly

for any  $b_2 \in B_N$ , there exists an element  $\alpha_2 \in A_N$  for which we have  $b_2 = h(\alpha_2)$  and  $f(F(\alpha_2))$  is an ideal of  $G(h(b_2))$ . Thus,  $(G, B_N) \in LOISNR(\tilde{N}_2)$ .  $\square$

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