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Power System Enhanced Monitoring through Strategic PMU Placement Considering Degree of Criticality of Buses

Ajeet Kumar Singh[†] **and Manoj Fozdar***

Abstract – This paper proposes a method for optimal placement of Phasor Measurement Units (PMUs) considering system configuration and its attributes during the planning phase of PMU deployment. Each bus of the system is assessed on four diverse attributes; namely, redundancy of measurements, rotor angle and frequency monitoring of generator buses, reactive power deficiency, and maximum loading limit under transmission line outage contingency, and a consolidated 'degree of criticality' is determined using Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). The major contribution of the proposed work is the development of modified objective function which incorporates values of the degree of criticality of buses. The problem is formulated as maximization of the aggregate degree of criticality of the system. The resultant PMU configuration extends complete observability of the system and majority of the PMUs are located on critical buses. As budgetary restrictions on utilities may not allow installation PMUs even at optimal locations in a single phase, multi-horizon deployment of PMUs is also addressed. The proposed approach is tested on IEEE 14-bus, IEEE 30-bus, New England (NE) 39-bus, IEEE 57-bus and IEEE 118-bus systems and compared with some existing methods.

Keywords: Critical buses, Optimal PMU placement, Topological observability, TOPSIS, Wide Area Monitoring Systems (WAMS).

1. Introduction

The ever-increasing demand for electricity has forced the nations to interconnect regional grids together and amalgamate renewable energy resources to the central grids. Thus, modern power systems are large dynamic systems having many complicated interconnections with adjoining networks. This has made the operation and control of the power system more complex. Intermittency of renewable resources and uncertainties in operating conditions directly affect the dynamics of the power system. Environmental restrictions on power system expansion, market pressure for the cost-efficient supply, enlarging urban infrastructure, and increased power demand; drives the operators to run the power system under stressed conditions leaving narrow margins of stability. Any remote disturbance under such circumstances can prove to be fatal and cause the system to lose equilibrium. This has necessitated developing more efficient and robust monitoring schemes to capture even the small deviation of the system from the normal operating state. Thus, an efficient plan for monitoring the power system is indispensable for its appropriate operation and control.

The outset of Phasor Measurement Units (PMUs) was a breakthrough in the surveillance of electrical power systems. A PMU installed at a bus measures its voltage phasor, current phasors of all transmission lines connected to it, frequency and the rate of change of frequency (ROCOF) [3]. Since voltage phasor at one end of the line and current phasor flowing through it are known; the voltage at another end of the line can be estimated by utilizing line parameters. Thus, a PMU observes the bus where it is installed and all adjacent buses. Apparently, PMUs located optimally in the system can make the system completely observable. The Optimal PMU Placement (OPP) problem is concerned about minimizing the number of PMUs required while preserving the complete observability of the system.

The OPP is an NP-complete problem per se with 2^N different viable combinations available in solution space for an N-bus power system, by taking into consideration two probable events of availability and unavailability of a PMU at each bus [5]. Various techniques devoted to solve the OPP include bisecting search [6], non-dominated sorting genetic algorithm [7], integer linear programming (ILP) [8-10], tree search [11], integer quadratic programming [12], exhaustive binary search [1], iterated local search [13], binary particle swarm optimization [14], Tabu search [15], fuzzified artificial bee colony algorithm [16], etc. However, all these methods focused only on minimizing the number of PMUs and do not consider the attributes and configuration of the power system, which resulted in the dispersion of PMUs at random buses within the system. However, few buses or lines in the system are more crucial from security and stability point of view. A failure to monitor

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[†] Corresponding Author: Dept. of Electrical Engineering, Malaviya National Institute of Technology Jaipur, India. (ajeetksingh@yahoo.co.in)

* Dept. of Electrical Engineering, Malaviya National Institute of Technology Jaipur, India. (mfozdar.ee@mnit.ac.in)

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such buses can lead to catastrophic results. Therefore, such buses should be kept under the constant supervision of a PMU directly rather than being observed indirectly by a PMU installed at adjoining bus. Observability of that bus may be lost in case of connecting line outage contingency. Sodhi *et al.* [17] proposed an ILP approach including voltage stability based contingency ranking of buses. Thukaram *et al.* [18] suggested critical buses on the basis of transient stability analysis for PMU placement. Gomez *et al.* [19] identified critical buses/area based on inter-area and intra-area oscillations. Placement of more PMUs at the bus with higher sensitivity of node voltage with respect to line parameters to effectively monitor disturbances is proposed in [20]. A multi-criteria selection on critical buses is also available in [21-23]. Furthermore, OPP has a non-unique solution, i.e., it offers multiple solutions with the identical number of PMUs. Hitherto, most of the existing methods rank these solutions in order of redundancy and select the solution with maximum redundancy [24].

This paper recommends a strategic planning before PMU deployment. A comprehensive investigation of system topology is conducted, and each placement site is evaluated by quantifying four attributes; namely, redundancy of measurements, rotor angle and frequency monitoring of generator buses, reactive power deficiency, and maximum loading limit under line outage contingency. An overall score of criticality of the bus and, subsequently its rank is evaluated through TOPSIS. In this method, Euclidean Distance of all the alternatives from the theoretical ideal and the theoretical negative ideal solution is computed, and relative proximity of a particular alternative to the ideal solution is taken as a measure of criticality of a bus. Buses with higher ‘*degree of criticality*’ are provided greater weight in the problem formulation. The problem is expressed in binary integer programming model as the maximization of aggregate criticality of the entire network. The resultant PMU configuration covers more critical buses with the same number of PMUs as proposed in the available literature [1, 2, 4]. This further eliminates the issue of the non-unique solution generated by conventional ILP.

2. Quantifying Degree of Criticality of Buses

Any plausible contingency, including, but not limited to, line outage, generator outage can lead to the unstable operation of the system. The voltage, frequency of the buses and rotor angles of machines can be severely disturbed if not taken care of timely. Under such conditions, an adequate monitoring scheme must be available to track down the cause of the stemming instability and report it to the operator for timely initiation of the corrective measures if the need arises. The PMUs installed at critical locations can provide vital information of the system health at the remote control center. In the present work, four attributes are utilized for quantifying the severity of each candidate bus in the network. The attributes considered are a redundancy of measurements, rotor angle and frequency monitoring of generator buses, reactive power deficiency, and maximum loading limit under line outage contingency. Each bus is scored on these attributes and a consolidated ‘degree of criticality’ is computed employing TOPSIS. Finally, buses are prioritized on the basis of the criticality for potential locations for PMU installation.

2.1 Evaluation of the attributes at each bus

The four attributes considered in the present work are evaluated for each bus, and a composite score is calculated to adjudge its suitability for PMU placement. The buses having better scores for all the attributes are considered to be better locations for PMU installation. The four attributes and their assessment are demonstrated below:

2.1.1 Increased redundancy of measurements

In any power system, several buses have a higher number of connections to other adjacent buses. Such buses are a better choice for PMU installation as they increase redundancy which is essential for the determination of the unique system state and bad data detection [4]. For instance, a PMU installed at a radial bus observes two buses exclusively. However, a PMU placed at a bus directly connected to a radial bus, monitors more than two buses and, consequently, is a better site for PMU installation. In this work, connectivity of the buses to their adjacent buses is used to prioritize them, and each bus is scored equal to the number of lines emerging from that bus. The Redundancy Criteria Score $(C_{R,i})$ is calculated for the i^{th} bus in an N-bus system as follows:

$$C_{R,i} = \sum_{j=1}^{N} A_{ij} - 1 \tag{1}$$

where A is a graph theory based binary connectivity matrix as explained in [8].

2.1.2 Rotor angle and frequency monitoring of generator buses

Angular stability and frequency stability issues can stem from any generator outage followed by the loss of synchronism among generators resulting in a cascading blackout. Moreover, the reactances of the generator change from their effective values during the transient state following a disturbance, which introduces an inconsistency between the rotor angle of the alternator and its terminal voltage angle. A PMU installed at a generator bus produces fast and accurate measurements which facilitate the estimation of rotor angle of the generator [25]. Thus considering these issues, this paper suggests that all generator buses are suitable locations for keeping under constant supervision of a PMU. Therefore, a Generation Bus Criteria Score $(C_{G,i})$ is calculated for the i^{th} bus in an N-bus system as follows:

$$C_{G,i} = \begin{cases} 1 & \textit{if } i^{th} \textit{ bus is a generator bus} \\ 0 & \textit{otherwise} \end{cases} \tag{2}$$

All generator buses have been scored equally to keep the computation simple. However, generator buses may also be scored in accordance with their generation capacity, the inertia of connected generator, load demand served, participation in damping low-frequency oscillations and so forth.

2.1.3 Maximum loading limit under line outage contingency

A transmission line outage can push the system towards instability. For a secure operation of the system, the operator must ensure that system can supply the present load for each possible line outage contingency; a condition acknowledged as *N-1* contingency stability criterion. In the proposed work, Continuation Power Flow (CPF) [26] is utilized to assess maximum loading limit $\lambda_{\max}$ under the impact of each line outage contingency. If contingency evaluation results in $\lambda_{\max} < 1$, where 1 signifies the current loading level, then stable operation under such a line outage is not possible. Hence, installation of PMU on buses connecting such lines is paramount. Therefore, a Line Contingency Criteria Score $(C_{L,i})$ is expressed for the i^{th} bus in an N-bus system as follows:

$$C_{L,i} = \min\left[\lambda_{\max,i-j}\right]_{j=1,j\neq i}^{N} \tag{3}$$

where $\lambda_{\max,i-j}$ is the maximum loadability limit in case of the outage of *i-j* transmission line.

2.1.4 Reactive power deficiency

Although voltage stability is correlated to load dynamics [27], transmission and generation systems play significant roles. Voltage instability originates when loads attempt to withdraw power beyond the combined transfer capability of transmission and generation [28]. In a modern power system, it mostly results from an insufficient generation of reactive power, the incompetence of transmission lines to deliver required reactive power to the load bus, or both. Hence, buses with reactive power deficiency are a viable option for monitoring using a PMU. The method proposed in [29] is used to determine deficit or surplus reactive power available at all load buses. Every bus is assigned score equal to reactive power deficiency at that bus, the procedure for determining the reactive power deficiency at each bus is summarized as follows:

Step 1: For a power system with G generator and L load buses, arrange steady state generator bus current injection I_G and load bus current injection I_L equations as follows:

$$[I_G] = [Y_{GG}][V_G] + [Y_{GL}][V_L] \tag{4}$$

$$[I_L] = [Y_{LG}][V_G] + [Y_{LL}][V_L] \tag{5}$$

where $[Y_{GG}]$, $[Y_{GL}]$, $[Y_{LG}]$ and $[Y_{LL}]$ are submatrices of reshuffled bus admittance matrix Y_{bus}.

Step 2: Each generator is modeled as equivalent shunt admittance Y_{Gj} as follows:

$$Y_{G,j} = \frac{1}{V_{G,j}}\left(-\frac{S_{G,j}}{V_{G,j}}\right)^* \tag{6}$$

where S_{Gj} and V_{Gj} are apparent power and bus voltage of j^{th} generator respectively. Equivalent shunt admittances of each generator are added to corresponding diagonal elements of bus admittance matrix which is again repartitioned as in step 1 to obtain $[Y'_{GG}]$, $[Y'_{GL}]$, $[Y'_{LG}]$ and $[Y'_{LL}]$.

Step 3: Following (4), voltages of generator buses are formulated as a linear function of voltages of load buses as follows:

$$[V_G] = -[Y'_{GG}]^{-1}[Y_{GL}][V_L] = [Y^B_{GL}][V_L] \tag{7}$$

Step 4: Eliminating V_L from (4) using (5) and substituting V_G from (7), generator bus injection current can be expressed as:

$$[I_G] = -\left([Y_{GG}] - \frac{[Y_{GL}][Y_{LG}]}{[Y_{LL}]}\right)[Y^B_{GL}][V_L] + \frac{[Y_{GL}]}{[Y_{LL}]}[I_L] \tag{8}$$

Step 5: V_G, V_L, and I_L are modified to form diagonal matrices. Reactive Power contribution of all generators can be formulated as:

$$\begin{aligned} [Q_{gen}]_{GXL} &= \mathrm{Im}([V_G]_{GXG}[I_G]^*_{GXL}) \\ &= \mathrm{Im}\left([V_G]_{GXG}\begin{pmatrix} -\left([Y_{GG}] - \dfrac{[Y_{GL}][Y_{LG}]}{[Y_{LL}]}\right)[Y^B_{GL}][V_L]_{LXL} \\ +\dfrac{[Y_{GL}]}{[Y_{LL}]}[I_L]_{LXL} \end{pmatrix}^*\right) \end{aligned} \tag{9}$$

Step 6: The Reactive Power Loss assigned to each load bus i can be estimated as

$$Q_{loss,i} = \sum_{j=1}^{G} Q_{gen(j,i)} - Q_{L,i} \tag{10}$$

where $Q_{L,i}$ is the reactive power load demand at i^{th} bus. Finally, the Reactive Power Loss Criterion Score $(C_{Q,i})$ can be computed for the i^{th} bus as follows:

$$C_{Q,i} = \begin{cases} Q_{loss,i} & \textit{if } Q_{loss,i} > 0 \\ 0 & \textit{otherwise} \end{cases} \tag{11}$$

Buses having surplus reactive power are not considered for PMU installation and, hence, assigned zero score.

2.2 Computing consolidated 'degree of criticality' using TOPSIS

Multiple criteria scores evaluated in preceding section are incommensurable. In this work, TOPSIS is used to process these numerical values. TOPSIS is a Multi-Criteria Decision Making (MCDM) approach applied to select the best alternative when dealing with manifold and, usually, contradictory criteria. It is based on the notion that the accepted alternative should have the minimum Euclidean distance to the ideal solution, and the maximum Euclidean distance to the negative ideal solution [30]. The steps for implementing TOPSIS are summarized as follows:

Step 1: A Decision Matrix is constructed comprising of the scores of four attributes of each bus as shown in Table 1.

Table 1. Decision matrix

Bus No. I	Criterion Score of i^{th} bus			
	$C_{R,i}$	$C_{G,i}$	$C_{L,i}$	$C_{Q,i}$
1	M_{11}	M_{12}	M_{13}	M_{14}
2	M_{21}	M_{22}	M_{23}	M_{24}
:	:	:	:	:
N	M_{N1}	M_{N2}	M_{N3}	M_{N4}

Step 2: The Decision Matrix is normalized to transform diverse attribute dimensions into dimensionless numerals. Each element r_{ij} of the Normalized Decision Matrix (NDM) is calculated by dividing individual score in the Decision Matrix by l^2-norm of all the scores in a particular attribute, as stated below:

$$r_{ij} = \frac{M_{ij}}{\sqrt{\sum_{i=1}^{N} M_{ij}^2}} \tag{12}$$

where $i \in \{1,2...N\}$ and $j \in \{1,2,3,4\}$ for an N-bus power system.

Step 3: As utilities can have varied hierarchy of priorities among the four attributes, a set of weights $W = (w_1, w_2, w_3, w_4)$ as decided by the utility is incorporated in the decision matrix to form the weighted normalized matrix V. Each element of V is defined as follows:

$$v_{ij} = w_j * r_{ij} \tag{13}$$

A 'weighted' normalized decision matrix substantiates the preferences of the decision maker on various criteria. This paper considers $w_1 = w_2 = w_3 = w_4 = 1$. However, other suitable values may also be selected without the loss of generality.

Step 4: Next, ideal (A^*) and negative-ideal (A^-) solutions are decided. The ideal solution is a hypothetical solution containing the best value for each attribute while the negative-ideal is a hypothetical solution comprising of worst value of each attribute among all alternatives.

$$A^* = \left\{ \left(\max v_{ij} \mid j \in J \right), \left(\min v_{ij} \mid j \in J' \right), i \in 1,2...N \right\} \tag{14}$$

$$A^- = \left\{ \left(\min v_{ij} \mid j \in J \right), \left(\max v_{ij} \mid j \in J' \right), i \in 1,2...N \right\} \tag{15}$$

where J is related to criteria to be maximized and J' is related with criteria to be minimized. In a power system, buses with higher redundancy score $(C_{R,i})$, availability of generator $(C_{G,i})$, higher reactive power loss $(C_{Q,i})$, but lower maximum loading limit under N-1 contingency $(C_{L,i})$ should be preferred for installing PMUs. Accordingly,

$$\begin{aligned} A^+ &= \{\max(v_{i1}), \max(v_{i2}), \min(v_{i3}), \max(v_{i4})\}, i \in 1,2...N \\ &= \{v_1^*, v_2^*, v_3^*, v_4^*\} \end{aligned} \tag{16}$$

and

$$\begin{aligned} A^- &= \{\min(v_{i1}), \min(v_{i2}), \max(v_{i3}), \min(v_{i4})\}, i \in 1,2...N \\ &= \{v_1^-, v_2^-, v_3^-, v_4^-\} \end{aligned} \tag{17}$$

Step 5: In this step, Euclidean Distance from each alternative to the ideal solution and the negative ideal solution is determined.

$$S_i^* = \sqrt{\sum_{j=1}^{4} (v_{ij} - v_j^+)^2}, \qquad i = 1,2,3..N \tag{18}$$

$$S_i^- = \sqrt{\sum_{j=1}^{4} (v_{ij} - v_j^-)^2}, \qquad i = 1,2,3..N \tag{19}$$

Step 6: Finally, the relative proximity of an alternative A_i to the ideal solution A^* is determined as follows:

$$\zeta_i^* = \frac{S_i^-}{S_i^* + S_i^-}, \qquad i = 1,2,3...N \tag{20}$$

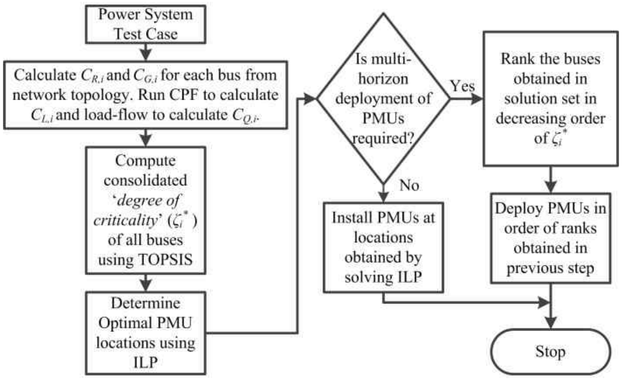

Fig. 1. Flowchart of the proposed PMU placement methodology

where $0 \le \zeta_i^* \le 1, \forall i$ is a consolidated degree of criticality of the i^{th} bus. The bus with the highest value of ζ_i^* has minimum Euclidean distance to the ideal solution and maximum Euclidean distance to the non-ideal solution. Buses arranged in decreasing order of ζ_i^* indicate the most suitable and least suitable alternative, respectively, for PMU installation. The flowchart of the complete proposed methodology is shown in Fig. 1.

3. Optimal PMU Placement Problem Formulation

The objective of the proposed scheme is to minimize the total number of PMUs location in the system while ensuring complete power system observability such that most of the PMUs remain concentrated at critical buses. For an N-bus power system, the problem is formulated as integer linear programming with binary decision variables as follows:

$$\min \sum_{i=1}^{N} (1-\zeta_i^*)\, x_i \tag{21}$$

subject to the following constraints of connectivity of buses:

$$A.[x_1\, x_2 \quad . \quad . \quad x_N]^T \ge 1 \tag{22}$$

where A is a binary connectivity matrix as defined in sec. 2.1.1 and $x_i \in \{0,1\}, \forall i$ is the PMU placement variable defined as:

$$x_i = \begin{cases} 1 & \text{if PMU is installed at } i^{th} \text{ bus} \\ 0 & \text{if PMU is not installed at } i^{th} \text{ bus} \end{cases} \tag{23}$$

Minimizing $\sum(1-\zeta_i^*)$ is equivalent to maximizing the total sum of the degree of criticality in the network. Connectivity constraints in (22) guarantee that at least one PMU observes every bus in the network. A higher redundancy also can be achieved, such that at least 2 PMUs observe each bus, by substituting 2 in the right-hand side of (22).

4. Simulation and Results

The effectiveness of the proposed methodology is examined on IEEE 14-bus, 30-bus, NE 39-bus, 57-bus and 118-bus test systems. The single line diagrams and elaborate system data for each of these networks is available in [31, 32]. For more realistic results a distributed slack bus model [33] was utilized, and all the generators contributed equally to the system power losses. The reactive power loss ($Q_{loss,i}$) at each load bus are determined at peak loaded condition retaining a spinning reserve of 10%.

4.1 Test results of the proposed method on IEEE-14 test systems

IEEE 14-bus is a small network consisting of 20 transmission lines, 2 alternators connected to buses 1, 2, whereas 3 synchronous condensers are installed at buses 3, 6, 8, and a shunt capacitor connected to bus 9. The decision matrix obtained for this system is presented in Table 2. The transmission line connecting buses 1-2 transfers 63.79% of the total load demand under peak loading condition. *N-1* Contingency analysis using CPF shows that maximum loading limit $(\lambda_{\max})$ drops below 1 in the event of line 1-2 outage, indicating instability of the system under the given contingency. Hence, both buses 1 and 2 have $C_{L,i} = \lambda_{\max,1-2} = 0.9955$. An ample number of generators in this system provide abundant reactive power reserves and allow sufficient voltage control capability. As buses 11, 12, 13 are lightly loaded; there is surplus reactive power available at these buses additionally. Hence, buses 1, 2, 3, 6, 8, 11, 12, and 13 have $C_{Q,i} = 0$ as given in (11). It can be observed from the table; the buses numbered 1, 2, 4, 5, 6, 7 and 9 have a high consolidated score of criticality as compared to other buses indicating potential PMU placement sites. Fig. 2 shows the consolidated degree of criticality at each bus in IEEE 14 bus system.

Table 3 shows the optimal location and minimum number of PMUs required for complete observability of the system. PMU installation percentage which denotes ratio between the number of PMUs and the number of system buses is also calculated. As evident from the table, 4 PMUs located at buses 2, 6, 7 and 9 are required for complete observability of the system. However, as utilities prefer the installation of PMUs in a multi-phase manner in a large system due to financial limitations, therefore the

Table 2. Decision matrix of IEEE 14-bus system

Bus No. i	Criterion Score of i^{th} bus				Degree of Criticality
	$C_{R,i}$	$C_{G,i}$	$C_{L,i}$	$C_{Q,i}$	ζ_i^*
1	2	1	0.9955	0	0.4212
2	4	1	0.9955	0	0.4423
3	2	0	1.3024	0	0.0885
4	5	0	1.5980	0.2020	0.2820
5	4	0	1.3081	0.9255	0.5832
6	4	0	1.3081	0	0.1856
7	3	0	1.5038	0.1587	0.1838
8	1	0	1.6929	0	0.0092
9	4	0	1.5038	0.0487	0.1887
10	2	0	1.7507	0.0163	0.0680
11	2	0	1.7507	0	0.0661
12	2	0	1.7582	0	0.0661
13	3	0	1.6758	0	0.1259
14	2	0	1.6632	0.0306	0.0726

Table 3. Optimal Location of PMUs for IEEE 14 bus system

No. of PMUs Required	PMU Location	PMU Installation %
4	2, 6, 7, 9	28.57 %

bus in the obtained solution set with a highest degree of criticality (ζ_i^*) in the decision matrix must be equipped with the PMU first and so on. As observed from Table , for multiphase PMU placement, bus 2 is to be installed with PMU first, followed by bus 9, 6 and 7.

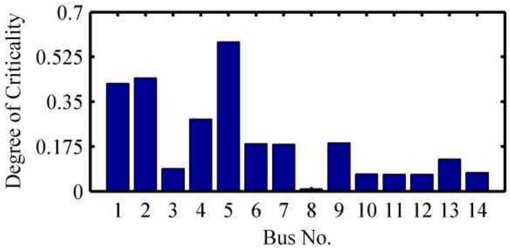

Fig. 2. Degree of Criticality of all buses (IEEE 14 bus system)

Table 4. Decision Matrix for IEEE-30 Bus System

Bus No.i	Criterion Score of i^{th} bus				Degree of Criticality
	$C_{R,i}$	$C_{G,i}$	$C_{L,i}$	$C_{Q,i}$	ζ_i^*
1	2	1	0.9211	0	0.4403
2	4	1	0.9211	0	0.4558
3	2	0	1.2358	0.8560	0.5275
4	4	0	1.2438	0.4949	0.3926
5	2	0	1.1407	0	0.0649
6	7	0	1.3915	0	0.2429
7	2	0	1.3925	0	0.0516
8	2	0	1.5018	0	0.0492
9	3	0	1.4099	0.1114	0.1342
10	6	0	1.4099	0.0027	0.2110
11	1	0	1.4810	0	0.0068
12	5	0	1.2932	0.0066	0.1774
13	1	0	1.4740	0	0.0076
14	2	0	1.5381	0.0123	0.0504
15	4	0	1.5036	0.0175	0.1384
16	2	0	1.5367	0.0072	0.0495
17	2	0	1.5416	0.0223	0.0529
18	2	0	1.5371	0.0082	0.0497
19	2	0	1.5320	0.0267	0.0543
20	2	0	1.5144	0.0058	0.0495
21	2	0	1.5233	0.0488	0.0644
22	3	0	1.5307	0	0.0944
23	2	0	1.5334	0.0089	0.0498
24	3	0	1.5307	0.0262	0.0980
25	3	0	0.6845	0	0.1267
26	1	0	0.6845	0.0135	0.0903
27	4	0	1.2639	0.0000	0.1392
28	3	0	1.2639	0	0.0987
29	2	0	1.4913	0.0084	0.0500
30	2	0	1.4567	0.0403	0.0608

4.2 Test results of the proposed method on IEEE-30 bus test systems

IEEE 30-bus consists of 41 transmission lines, 2 alternators connected to buses 1, 2 whereas 4 synchronous condensers are installed at buses 5, 8, 11 and 13. The shunt capacitors are connected to bus 10 and 14. Decision matrix obtained for this system is presented in Table 4. The transmission line connecting buses 1-2 is a bulk power carrier line, individually transferring 64.73% of the total load demand under peak loading condition. *N-1* Contingency analysis using CPF reveals that maximum loading limit $(\lambda_{\max})$ drops below 1 in the event of line 1-2 outage, indicating that stable operation under this contingency is not possible. Hence, both buses 1 and 2 have $C_{L,i} = \lambda_{\max,1-2} = 0.9211$. The buses 6, 9, 11, 13, 22, 25, 27 and 28 do not have any load connected to them while bus 7 has surplus reactive power available. Therefore, buses 1, 2, 5, 6, 7, 8, 9, 11, 13, 22, 25, 27 and 28 have $C_{Q,i} = 0$ as given in (11). shows the degree of criticality of each bus in IEEE 30 bus system. Table 6 shows the minimum number of PMUs required for complete observability and their optimal locations. It can be observed that only 30% of the total buses require PMUs installation and PMUs located at buses 2, 3, 6, 9, 10, 12, 15, 19, 25 and 27 completely observe the system.

4.3 Test results of the proposed method on NE 39-bus, IEEE 57-bus, and IEEE 118-bus test systems

The proposed methodology is tested further on NE 39-

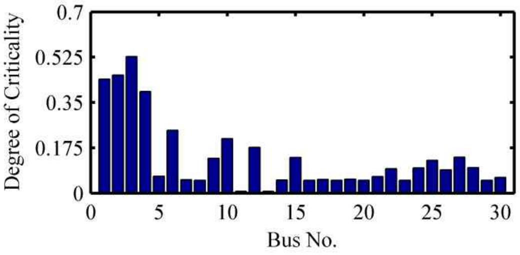

Fig. 3. Degree of Criticality of all buses (IEEE 30 bus system)

Table 5. Comparison of optimal PMU locations obtained by proposed methodology with available techniques

Test System	No. of PMUs required for complete Observability	PMU Installation %	Optimal PMU Locations				
			Common PMU Location	Proposed Methodology	Chakrabarti and Kyriakides [1]	Alvarez *et al.* [2]	Roy *et al.* [4]
NE 39-bus	13	33.33 %	2, 6, 9, 14, 17, 20, 22, 23, 29	11, 32, 33, 37	10, 11, 19, 25	NA	10, 12, 19, 25
IEEE 57-bus	17	29.82 %	1, 4, 9, 32, 36, 38, 41	7, 15, 20, 24, 25, 28, 47, 50, 53, 57	NA	7, 15, 19, 22, 25, 27, 47, 51, 53, 57	20, 24, 27, 29, 30, 39, 45, 46, 51, 54
IEEE 118-bus	32	27.12 %	5, 12, 17, 21, 23, 49, 56, 62, 68, 71, 75, 77, 80, 85, 94, 105, 110	3, 10, 15, 28, 30, 36, 40, 44, 47, 52, 64, 87, 91, 101, 115	NA	3, 9, 15, 25, 29, 34, 37, 40, 45, 52, 64, 86, 90, 102, 115	1, 9, 13, 26, 28, 34, 37, 41, 45, 53, 63, 86, 90, 101, 114

bus, IEEE 57-bus, and IEEE 118-bus networks to validate its efficacy. However, decision matrix associated with these systems is not provided due to limitations of space. Fig. -6 show the degree of criticality of different buses in NE 39-bus, IEEE 57-bus, IEEE 118-bus test systems. Simulation results for the minimum number of PMUs and their optimal locations are summarized in Table 6 and compared with 3 existing techniques proposed in [1, 2] and [4]. Table 5 shows the number of PMUs reported by the proposed method is consistent with other methods; however, a difference in optimal locations can be observed. Large networks can have a significant number of alternative PMU configurations which allow complete observability with the same number of PMUs. Most of the methods proposed earlier rely on the selection of PMU configuration which maximizes the redundancy in the system. The proposed method selects the configuration which has more PMUs on critical buses. It can be observed from the table, that inclusion of the degree of criticality in the objective function by the proposed technique causes more critical buses to be directly observed by the PMUs. Table 5 shows that certain PMU locations are common among the proposed method and other methods [1,2,4]. However, the advantage of the proposed method can be observed in dissimilar PMU locations. The proposed methodology proposes buses 11, 32, 33, 37 as suitable buses for PMU placement in NE-39 bus system. Bus 11 offers high redundancy $C_{R,11}$=3 (only two other buses 6 and 16 have greater redundancy ($C_{R,6}$=$C_{R,16}$=4) than bus 11). Similarly, buses 32, 33 and 37 are generator buses and monitoring such buses directly is essential as loss of generator at any of these buses will instigate power imbalance and risk the stability of the system. Similarly in IEEE 118 bus system, buses 10, 36, 87 and 91 in the solution set are generator buses, bus 30 is a high redundancy bus ($C_{R,30}=4$) and buses 44 and 47 are reactive power deficit and undergo rapid voltage decline on load increase. Thus, the solution set obtained using the proposed approach encompasses more critical buses than other methods. As the number of PMUs is increased to achieve higher redundancy of measurements, the proposed approach concentrates even more PMUs on critical buses, which shows the effectiveness of the presented scheme.

Table 6. **Optimal Location of PMUs for IEEE 30 bus system**

No. of PMUs Required	PMU Location	PMU Installation %
10	2, 3, 6, 9, 10, 12, 15, 19, 25, 27	30 %

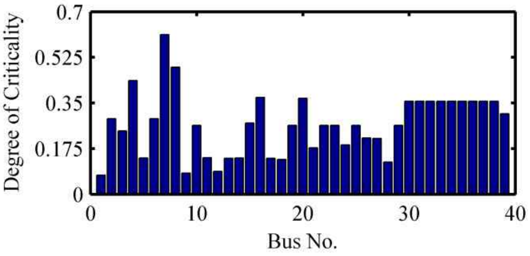

Fig. 4. Degree of Criticality of all buses (NE 39 bus system)

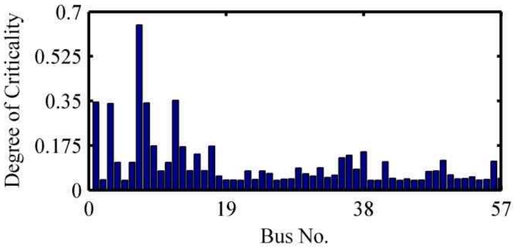

Fig. 5. Degree of Criticality of all buses (IEEE 57 bus system)

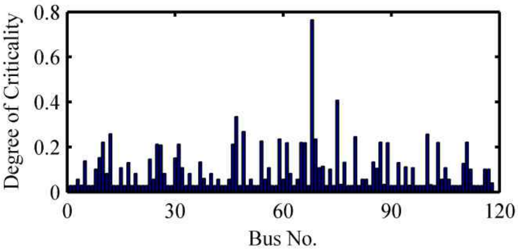

Fig. 6. Degree of Criticality of all buses (IEEE 118 bus system)

5. Conclusion

This paper presented a methodology for determining the minimum number of PMUs and their optimal locations for complete topological observability of the system considering critical buses. Four different attributes; namely redundancy of measurements, rotor angle, and frequency monitoring of generator buses, reactive power deficiency, and maximum loading limit under transmission line outage contingency are considered for selecting a bus for PMU installation. Each attribute is evaluated for all the buses in the system and a consolidated score, referred as the degree of criticality, is determined using TOPSIS. The optimal PMU placement problem is formulated as a binary integer linear programming problem. The bus with the high degree of criticality is assigned more weight for PMU installation in the optimization problem. The results verify that the majority of PMU locations are at the buses having a high degree of criticality. The proposed scheme is also compared with the three existing methods for PMU placement. The resultant PMU configuration has an equal number of PMUs as reported by other methods but different PMU locations. The multi-horizon deployment of PMUs is also addressed for utilities with limited financial resources.

This paper accounts four attributes for accessing the criticality of each bus. Nevertheless, the selection of critical buses is highly subjective and system-specific. Several additional buses including, but not limited to, substantial power transfer buses, buses with high DG penetration, buses in high voltage corridor, buses contributing in damping inter-area oscillations, buses with FACTS and small-signal controller support, HVDC terminal buses, may also be considered and assessed in the similar manner.

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Ajeet Kumar Singh received his B. Tech. in Electrical Engineering from GBTU, Lucknow in 2010 and M. Tech. in Control System from National Institute of Technology, Kurukshetra in 2012. Currently, he is a doctoral candidate in Department of Electrical Engineering, Malaviya National Institute of Technology, Jaipur. His areas of research interest are power system stability and machine learning applications.

Manoj Fozdar received his B.Sc. (Engg.) in Electrical Engineering and M.Sc.(Engg.) in Power System and Drives from Aligarh Muslim University, Aligarh in 1989 and 1992 respectively, and Ph.D. in Electrical Engineering from Rajasthan University, Jaipur in 2007. Presently he is working as Professor in Electrical Engineering Department at Malaviya National Institute of Technology, Jaipur (Rajasthan), India. His areas of research interest are power system operation and control & FACTS.