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A Search for the Meaning of Constructivism: Constructivism Revisited and Reviewed**

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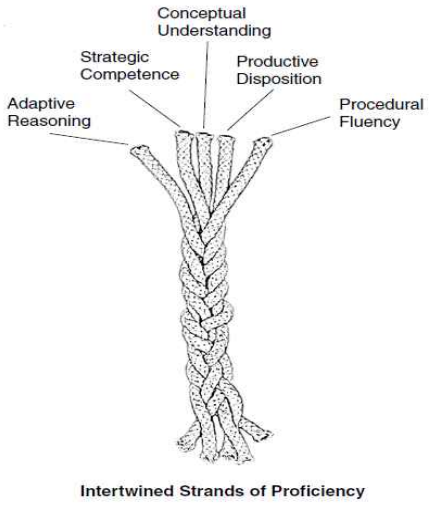
In the current era of mathematics education, constructivism is a core theory of learning. For teachers, understanding and applying constructivism to their teaching practices are crucial for student centered teaching. However, some mathematics educators understand Constructivism in a different way. For example, some future teachers view Constructivism as making mathematics 'fun' by creating game without considering conceptual understanding. In this paper, the original articles of Constructivism were revisited and investigated to understand and to search for their meanings. Also several types and sources of Constructivism were identified; Radical Constructivism, Vygotsky's social-cultural theory of development, Social Constructionism, and Social Constructivism. This paper investigated arguments of the several types of Constructivism and discussed their implications for mathematics teaching.

I. Introduction

Recent mathematics education documents emphasize achievement of 'both of' conceptual understanding and procedural fluency (CCSSI, 2010; Kilpatrick, Swafford, & Findel, 2001). For example, as shown in [Fig 1], Kilpatrick et al (2001) suggested five strands of mathematical proficiency. Not only conceptual understanding but also procedural fluency are the components of the strands. In the strands,

> • *conceptual understanding* – comprehension of mathematical concepts, operations, and relations
> • *procedural fluency* – skill in carrying out procedures flexibly, accuratedly, efficiently, and appropriately
>
> Kilpatrick et al (2001) p.116



[Fig. 1] Strands of Mathematical Proficiency, Kilpatrick et al (2001) p.117

However, some mathematics educators overemphasize only one aspect, conceptual understanding. And sometimes they overlook the importance of procedural fluency in mathematics learning, regarding that this is what constructivism suggests. Even some teacher candidates view

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conceptual understanding and procedural fluency as dichotomous aspects of mathematics teaching and learning and believe that if a teacher pursuit one of them then the other cannot be achieved.

To resolve the situation, we need to consider the view of knowledge. No one would disagree the view of knowledge is crucial in education. The way we define the concept of knowledge can affect all of the educational practices such as learning and also teaching. There have been skeptics of traditional knowledge and the discussion is still an ongoing process, however, it seems that in recent mathematics education the effects of Constructivism is significant. Several different types of constructivism exist and they emphasize common and different aspects. In this paper, various sources of constructivism were identified, discussed and compared to help understand Constructivism appropriately. Also some misunderstandings of Constructivism were identified and discussed.

II. Various Sources of Constructivism

1. Radical Constructivism

A. An overview

Radical constructivism emerged from Jean Piaget's cognitive-development theory and Giambattista Vico. And it was introduced in his book "The Construction of Reality in the Child". At the beginning of the 18th century, Vico was the first genuine constructivist. Ernst Von Glasersfeld said the interpretation of Piaget's neo-Kantian theory of knowledge as "radical constructivism". In mathematics education field, Leslie Steffe has been introducing and developing radical constructivism (Ernest, 1996; Von Glasersfeld, 1995, 1996). One thing that we might want to keep in mind is that Constructivism is interested in 'conceptual understanding only'; the construction of 'conceptual knowledge' (Von Glasersfeld, 1995).

Constructivism is 'radical' because it is different radically from the traditional Western epistemology. The fundamental difference concerns the relation of knowledge and reality. Whereas in the traditional epistemology, as well as of cognitive psychology, the relation between knowledge and reality is always regarded as a more or less picture like correspondence or match. Radical constructivism considers the relation as an 'adaptation'. Human knowledge is nothing but the endeavor to make things correspond to one another gracefully (Von Glasersfeld, 1984). We can also consider radical constructivism as "post-epistemological" approach as Nel Noddings mentioned (as cited in Von Glasersfeld, 1996). Radical constructivism is indeed proposes 'a theory of knowing'. Thus, it is requires efforts of analysis of how the thinking subject comes to have others in his or her construction of the experiential world (Von Glasersfeld, 1995) and this is what Radical Constructivists are interested.

B. Model of knowing

In radical constructivism, knowing is an adaptive activity as Piaget mentioned. Radical constructivists suggest that we need to change the traditional concept of knowledge that the cognizing activity should lead to ontologically "true" representation of a world which was what skeptics argued. Also it is an attempt to develop a theory of knowing (epistemology) that the cognizing activity is instrumental and neither does nor can concern anything but the experiential world of the knower; radical constructivism does not question the existence of ontological reality. Because whether it exists or not, there is no way to cognize it because we can only know that we can feel and sense. This experiential world is constituted and structured by the knower's own ways and means of perceiving and conceiving. Thus it is always and certainly subjective. In this sense, there is no exit from

subjectivity. Thus sharing meaning, ideas, and knowledge is somewhat like sharing an apple pie or a bottle of wine which indicates that none of the participants can taste the share another is having (Von Glasersfeld, 1996).

C. The notion of viability

Then the question is how come our reality is so stable despite of no exit from subjectivity? Here 'viability' takes the important role to answer the question. This was from Piaget's idea of "adaptation" but Von Glasersfeld preferred the term "viability" (Von Glasersfeld, 1980). The notion of 'viability' replaces the concept of traditional 'truth' as the ontological reality in radical constructivism. For example, one of the goal of science is constructing a consistently reasonable model as possible of the experiential world and it is way beyond just solving specific problems (Von Glasersfeld, 1995).

Also as Vico said, the world that we experience and get to know is necessarily constructed by ourselves, it is not surprising that it seems relatively stable. Vico also anticipated the basic principle of viability in the constructivist theory of knowledge (as cited in Von Glasersfeld, 1984). Thus all that radical constructivism requires for knowledge is that it should be viable so that it fits into the world of the knower's experience which is the only reality accessible to human sense (Von Glasersfeld, 1996).

Viability allows any description that is related to the observer from whose experience it is derived. Thus,, there will always be more than one way of solving a problem or achieving a goal (Von Glasersfeld, 1995). Consequently radical constructivism admits the idea that no knowledge can insist singularity. In other words, no matter how viable and satisfactory the solution to a problem might seem, it can never be regarded as the only possible solution (Note that this does not contradict the observation that, for instance, in mathematics, solutions are often fully determined by the operation one carries out to find them.) (Von Glasersfeld, 1995).

The viability of concepts and constructs has a hierarchy of levels that starts with simple repeatability in the sensory-motor domain. Next, on levels of higher abstraction, and then into operational coherence, and finally it ultimately concerns the non-contradictoriness of the entire repertoire of conceptual structures (Von Glasersfeld, 1996). In other words, knowledge is what we can do in our experiential world, the successful ways of dealing with the objects we call physical and the successful ways of thinking with abstract concepts (Von Glasersfeld, 1995).

In this sense, mathematics is the result of abstraction from operations on a level on which the sensory or motor material that provided the occasion for operation is disregarded (Von Glasersfeld, 1996).

The following principles of radical constructivism derived from Piaget's idea. And it could summarize the model of knowing in radical constructivism (Von Glasersfeld, 1990).

1. Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognizing subject.
2. a. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;
   b. Cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality.

From the principles of radical constructivism,, active engagement is crucial and necessary in learning, because of the first principle. According to the second principle, learners need to have opportunity to test a theory that they are learning to see if they are viable and make sense to

everyone in classroom.

D. Model of learning

In the radical constructivists' perspective, learning is not just a stimulus-response phenomenon as Behaviorism argued. Learning occurs through assimilation and accommodation to neutralize a perturbation and this is possible because human being is a self-organizing organism. The concept of accommodation is powerful enough to explain the modifications that might occur in children's mathematical knowledge as they interact in their environments (Steffe, 1996). In this respect, children do not simply receive adults' knowledge.

Von Glasesfeld considered leaning as a process of self-organization in which the subject reorganizes his or her activity in order to eliminate perturbations. And the learner also acknowledged that this constructive activity occurs as the cognizing individual interacts with other members of a community. Further, Von Glasersfeld argued that "learning is characterized by the subjective reconstruction of social means and models through negotiation of meaning in social interaction" (as cited in Cobb, 1996). And the process requires self-regulation and the building of conceptual structures through reflection and abstraction (Von Glasersfeld, 1995).

E. Misunderstandings of radical constructivism

The followings are some of the misunderstandings of constructivism. First is 'Radical constructivism regards memorization and rote learning as useless.' However, this is not what Radical Constructivism intended. As von Glasersfeld mentioned, there are indeed matters that can and perhaps must be learned in a purely mechanical way. However, since radical constructivism considers conceptual understanding only, they argue that memorization and learning by rote are not helpful for conceptual understanding (Von Glasersfeld, 1995).

Next misunderstanding is 'Radical Constructivism is nothing but solipsism; it denies reality and it does not take into account social interaction.' The statement that the construction of the experiential world is irrevocably subjective has been interpreted as a declaration of solipsism and as the denial of any "real" world. However, the concept of 'viability' shows it is not solipsism, rather, Radical Constructivism considers others' thoughts and try to make sense of our experiential world. If one begins with the assumption that all knowledge is derived from perceptual and conceptual experience, there is no way to deny that others and society have an influence on the individual's cognitive constructing. This also can be shown from Piaget that he reiterates that the most important occasions for accommodation arise in social interaction.

On the contrary, because others add up to a major part of the individual's experiential environment, they will have a considerable role in determining which behaviors, concepts, and theories are considered viable in the individual's physical and linguistic interactions with them. Nevertheless, as a constructivist, one will remain aware of the fact that these others and the society they constitute exist for the individual subject only to the extent to which they figure in that individual's experience. That is to say, they are for each subject what he or she perceives and conceives them to be (Von Glasersfeld, 1995).

Last misunderstanding might be 'Different solutions must be considered equally desirable.' However, if people achieve the desired goal, the preference for a particular way of doing cannot be justified by its rightness. Rather with reference to some other values such as speed, economy, convention, or elegance (Von Glasersfeld, 1995). One of the challenge that mathematics teachers have is that when they accept students' diverse solutions for a problem, then what would be the next step. For

this challenging situation, one could have discussion of which solution might be the most efficient in different situations or contexts.

2. Vygotsky's social-cultural theory of development

Vygotsky's social-cultural theory of development is regarded as a kind of constructivism. In this section, I summarized how social-cultural theory of development views conceptual development and identified key ideas of it. The views from radical constructivism on social-cultural theory of development was also introduced.

A. Conceptual development in young children

Vygotsky's social-cultural theory of development stresses the importance of both 'social interaction' and 'the use of cultural tools' (as cited in Cobb, 1996). Development is a process of acquiring cultural tools during joint activities with adults and peers who act as guide in an on-going cultural apprenticeship (Rogoff 1990, as cited in Renshaw, 1996). Vygotsky described conceptual development as an interaction between natural, spontaneous concepts and the organized systems of concepts referred to as 'scientific' concepts. The interaction between spontaneous and scientific concepts provides a window through which to view the relationship between education and the conceptual development of children. He also argued that spontaneous concepts provide the necessary but not sufficient conditions for progress toward more powerful forms of thinking. Scientific concepts cannot be grasped by children in a direct manner, as implied in a transmission model of teaching. A social process, such as effective teaching and learning in the zone of proximal development is required to develop a child's everyday concepts towards the more abstract scientific concepts (Renshaw, 1996).

Specifically, Vygotsky examined the relationship between speaking and thinking. In his analysis, thinking arises from engagement with others in activities. The changing functional relationship between speaking and thinking exemplifies the general developmental process where social tools, such as speech and gesture that initially serve social and communicative functions, are transformed into internal tools of thinking and problem solving. However, the movement from the social level of functioning to the internal plane of functioning requires active engagement by children in social interaction with peers and supportive adults. Hence, from the social-cultural point of view, the opportunity to use speech in collaborative activities with others is central to conceptual development (Renshaw, 1996).

Vygotsky argued that the spoken word and the meaning of a concept are hard to be considered to be the same. The underlying meaning of the spoken word varies for individuals depending on their developmental level, experience, etc. Adults re-interpret, elaborate, and provide broader contexts for children's ideas through on-going dialogues with children (Renshaw, 1996).

It is basically assumed by Vygotsky that an asymmetrical relationship between the child and the social environment is the normative case. He was in fact concerned about situations in which a peer group is left to cooperate without adult supervision. In Vygotsky's view, it is the adult's responsibility to help the child perform actions that are beyond his or her individual competence. As the child interacts with the adult, he or she gradually takes over and internalizes these inter-mental or socially distributed actions. Child's development is to a considerable extent equivalent to mastering these cultural tools. (Cobb, 1996)

B. Zone of Proximal Development

Imitating a sociocultural activity with the help of

others constitutes the essence of what Vygotsky called the 'Zone of Proximal Development'. According to Vygotsky, the zone of proximal development is the activity setting in which the child can learn with the greatest developmental effects. This zone is constructed in the cooperation between the child and the adult (parent, teacher) on the basis of what the child wants and the actions the child actually can carry out, as well as the help the child gets from the adult (Van Oers, 1996).

Vygotsky viewed the notion of 'pseudoconcept' as characterizing the thinking of pre-school and elementary school children and it was placed between complex concepts and the mature form of concepts. Even thouth peudoconcept appreas to be a mature form of thinking, the underlying meaning is less abstract and remains bound to specific contexts. The apparent similarity of the pseudoconcept and the mature concept enables adults to act toward children as if they were further advanced in understanding a concept than actually is the case. Such misunderstanding is productive because it reveals to children the underlying structure of their thinking and creates conditions for a more deliberate and reflective application of concepts. In this sense, pseudoconcept is a vehicle for instructional dialogue. Vygotsky mentioned that 'scientific concepts' grow downward through spontaneous concepts and 'spontaneous concepts' grow upward through scientific concepts (as cited in Steffe, 1996)

C. Interpretations of Vygotsky's social-cultural development theory and views from Radical constructivism

Van Oers interpreted Vygotsky's meaning of education was basically a process of enculturation. Similarly, Renshaw (1996) emphasized learning as a dynamic cultural apprenticeship, which he further elaborated as a dynamic process of internalization of shared social behavior. Internalization is regarded as more than the social transmission of preformed cultural knowledge. However, viewing learning as a dynamic cultural apprenticeship essentially leaves out Piaget's fourth factor contributing to development-autoregulation or equilibration-is equivalent to viewing the learner as an instructable rather than as a self-organizing system. Also Davydov's use of Piagetian theory to support his curriculum on quantity reveals a serious issue within socio-cultural approaches to education that are based on a belief in an objective ontological reality (Steffe, 1995).

3. Social Constructivism

Social Constructivism considers individual subjects and the realm of the social as inseparably interconnected. Social Constructivist research paradigm adopts a modified relativist ontology which implies that there is a world out there supporting the appearances we have shared access to, but we have no certain knowledge of it. They have emphasis on the essential and constitutive nature of language and social interaction. The underlying metaphor is 'persons in conversation', comparing persons in meaningful linguistic and extralinguistic interaction and dialogue. Mind is seen as part of a broader context, the "social construction of meaning." Language is regarded as the shaper of, as well as being the summative product of, individual minds (Ernest, 1996).

Social constructivism view mathematics as a social construction and it is a descriptive because it opposed to a prescriptive philosophy of mathematics, aiming to account for the nature of mathematics understood broadly, as in the adequacy criteria (Ernest, 1991).

The following three are main basis to understand mathematical knowledge as a social construction (Ernest, 1991).

1. The basis of mathematical knowledge is linguistic knowledge, conventions and rules, and language is a

social construction.
2. Interpersonal social processes are required to turn an individual's subjective mathematical knowledge, after publication, into accepted objective mathematical knowledge.
3. Objectivity itself will be understood to be social.

A central focus of social constructivism is the genesis of mathematical knowledge, rather than just its justification. Newly generated mathematical knowledge can be either subjective or objective knowledge and Social Constructivists argue that they consider both those forms of knowledge and link them in a cycle which each contributes to the renewal of the other. In this cycle, the path followed by new mathematical knowledge is from subjective knowledge to objective knowledge through publication. Objective knowledge is internalized and reconstructed by each individual, during the learning of mathematics, to become the individuals' subjective knowledge. Cobb viewed Social Constructivists' mathematical learning as both a process of active individual construction and a process of acculturation (as cited in Steffe, 1995). Hence subjective and objective knowledge of mathematics each contributes to the creation and re-creation of the other.

4. Social Constructionism

Sometimes Constructionism is regarded as constructivism by some educators, however, they are different in many aspects.

Social Constructionism quests explicating the processes by which people come to describe, explain, or otherwise account for the world (including themselves). Similar to Radical constructivism, Social Constructionism is also based on critique of problem of traditional view of knowledge. However, Social Constructionism do not assume the external world as its fundamental concern or with the individual mind. Social Constructionism place great emphasis on language. It regards discourse about the world not as a reflection or map of the world but as an artifact of communal interchange (Gergen, 1985).

Social Constructionism views knowledge as the culture's accumulation and within the process of social interchange (Gergen, 1985, 1995). Thus it primarily values a repository of linguistic artifacts such as texts, documents, and journals and in the classroom they focus on lectures, discussions, overhead projections, etc as important. Social Constructionism places propositional representations of everything from physics to psychology, geography to government and gain their legitimacy not by virtue of their capacities to map or picture the world, but through processes of social interchange. Thus Social Constructionism regards linguistic meaning is objective (Steffe, 1995). The followings are three main ideas in Social Constructionism (Gergen, 1995).

First, meaning in language is achieved through social interdependence. Meaning of language is achieved through the coordinate efforts of two or more persons which imply that there is no subjectivity in linguistic meaning from a constructionist point of view. What a person says remains nonsense until other person agree to its meaningfulness. Thus Social Constructionists quest to replace the individualistic ideology of the traditional conceptions of knowledge with a communal concern. For this reason, Social Constructionism places 'community' prior to the 'individual' and regard individual rationality mainly as a by-product of the social aspects. Specifically, they place the human relationship; the patterns of interdependent action at the micro-social lever, in the first place rather than attempting to explain via psychological process within persons (Gergen, 1995).

In this vein, Social Constructionsts hold cooperative or dialogic processes as central to the process of education and they account language in classrooms; lectures, discussions, overhead

projections, and etc., as transmission of knowledge in the classroom (Gergen, 1995).

Next, meaning in language is context dependent. As mentioned earlier, Social Constructionism tries to shift the orientation of knowledge from mind to language. They think knowledge is achieved when one transcends the particular and grasps the general, when one can abstract from the welter of detail and articulate a more general theory. In this vein, knowledgeable representations of the world, like rules of logic, are granted a form of unlimited power, they applicability transcending the limits of both history and culture.

Social Constructionists' agreements regarding the relationship of language to referents are always located within particular socio-historical circumstances. Agreements are typically generated for local purposes, and there is no principled means of ensuring their generality outside such circumstances. Also there is no means of guaranteeing consistency of meaning outside the conditions of agreement because there is nothing about the nature of propositions that defines the conditions under which they apply. Hence, Social Constructionists count "knowledgeable propositions" as dependent on socio-historical contingencies. However, in Radical Constructivism point of view, social constructionists tend to introduce the social context as an ontological given (Steffe, 1995).

Lastly, language primarily serves communal function. As discussed earlier, Social Constructivists argues that languages do not reflect or picture an independent world, nor do they represent expressions of individual minds. Moreover, relationships between propositions and particulars are developed within specific contexts, and there is little means of fixing such relationships for use in later contexts. The constructionist orientation shifts attention away from the truth-bearing view of language to its functions within ongoing relationships.

In this sense, Social Constructionism employs language not as a means of world reflection or self-expression, but as we might move within a game. A strong emphasis is placed on the pragmatic conditions and constraints of language use. There is no pure language because there are no intelligible statements outside some form of game. All language is applied in the sense that it carries out functions within some community.

III. Compare and Contrast among the Sources of Constructivism

1. View of knowledge

All of Radical Constructivism, Social Constructionism, Social Constructivism, and Vygotsky's cultural-historical theory of development accept the argument of the skeptics and reject the traditional assumption that knowledge should be a true representation of a reality that exists in and by itself. Further, each challenges the traditional view of the individual mind as a device for reflecting the character and conditions of an independent world. (Ernest, 1996; Gergen, 1995; Van Oers, 1996; Von Glasersfeld, 1995)

However, they emphasize differently in their view of knowledge. Radical Constructivism and Social Constructivism both of them emphasize that knowledge as a whole is problematized, not just the learner's subjective knowledge, including mathematical knowledge and logic. Their focus of concern is not just the learner's cognitions, but the learner's cognitions, beliefs, and conceptions of knowledge. Also although we can tentatively come to know the knowledge of others by interpreting their language and actions through our own conceptual constructs, the others have realities that are independent of ours. Indeed, it is the realities of others along without own realities that we humans strive to understand, but we can never take any of

these realities as fixed.

But Social Constructivism has more emphasis on mathematical knowledge is irrevocably bound up with texts and semiosis as a social construct. Also it has more emphasis on 'How the mind of the learner is formed by social interaction and how does this impact on the learner's conceptions and activities in the classroom?'.

On the other hand, Social Constructionism considers knowledge as the culture's accumulation starting with language and temporary locations in dialogic space- samples of discourse that are accorded status as 'knowledgeable tellings' on given occasions. Similarly, Social Constructivism adopts persons in conversation as their underlying metaphor and emphasize on human beings and their language in its account of knowing. Following the work of Wittgenstein, Vygotsky, symbolic interactionism, and activity theory, language is regarded as the shaper of, as well as being the summative product of, individual minds.

2. Social Interaction and accommodation

In Radical Constructivism, the coordinated efforts of two or more persons, or social interaction, is also regarded as a primary source of accommodation of linguistic meaning. However, accommodation apparently has no place in Social Constructionism. Accepting accommodation would have the advantage of opening up all of the work on constructive processes and learning to Social Constructionism because it implies the adaptation of one's use of a word to other people's use of it (Steffe, 1995).

3. Linguistic meaning

Meaning of language is subjective-the meaning of a word or combination of words is regarded as whatever aspects of one or more schemes that are associated with the word or words are within the awareness of the individual. The most essential difference between Radical Constructivism and Social Constructionism is that Social Constructivism considers starting with language as the fundamental given, while Radical Constructivism insistence that there is no exit from subjectivity. (Steffe, 1995; Von Glasersfeld, 1995)

4. Individual vs. Community

While Radical Constructivism place 'individual' first, Social Constructionism and Vygotsky's theory both place 'community' prior to the 'individual' which implies that both regard individual rationality as a by-product of the social aspects. However, an essential difference between them is that Social Constructivism places the 'human relationship' in the foreground; it centrally concerned with such matters as negotiation, cooperation, conflict, rhetoric, ritual, roles, social scenarios, and the like, but avoids, psychological explanations of micro-social process. But Vygotsky's theory centrally concerned with zone of proximal development which essentially a mental space between actual and potential cognitive functioning.

IV. Suggestions from Constructivism for Mathematics Education

Constructivism does not claim to have made earth-shaking inventions in the area of education; it merely claims to provide a solid conceptual basis for some of the things that, until now, inspired teachers had to do without theoretical foundation (Von Glasersfeld, 1995).

The teacher must be concerned with what goes on in the student' head and listen to the student, interpret what the student does and says, and try to build up a 'model' of the students' conceptual structures (Von Glasersfeld, 1995).

Having found a viable way to solve a problem does not necessarily eliminate all motivation to search further. The solution found may seem cumbersome, costly, or inelegant, and this may generate the motivation to find another, more satisfactory one. In this regard, teacher can be extremely effective in orienting the students' attention (Von Glasersfeld, 1995).

2+2=4 is not given by God or any other extra-human authority, but we come to construct units in a particular way and have agreed on how they are to be counted (Steffe, 1995). 'Misconceptions': whatever a student does or says in the context of solving a problem is what, at this moment, makes sense to the student. The misconceptions are viable in the students' point of view. Only when students can be led to see as their own a problem in which their approach is manifestly inadequate will there be any incentive for them to change it (Von Glasersfeld, 1995).

It is not helpful to assume that a student's ideas are simply misconceptions that have to be replaced by the conceptions that are considered correct by mathematicians, physicists, or other experts. A new structure should be built out of elements with which the students are familiar. In other words, students must be shown that there are elements in their experience that can be related differently from the way they habitually relate them; to make such changes desirable to students, they must be shown that the new way provides advantages in a sphere of living and thinking that reaches beyond passing exams and getting good grades (Steffe, 1995).

Arithmetic begins with the abstraction of the concept of number from acts of counting. Such abstraction cannot be given; they have to be made by the students themselves. The teacher, of course, can help by generating situations that allow or even suggest the abstraction. This is where manipulatives can have important roles, but it would be naïve to believe that the move from handling or perceiving objects to a mathematical abstraction is easy and automatic. The sensory objects, no matter how ingeniously they might be designed, merely offer an opportunity for actions from which the desired operative concepts may be abstracted; one should never forget that the desired abstractions, no matter how trivial and obvious they might seem to the teacher, are never obvious to the novice (Steffe, 1995).

There is no way of transferring meaning, that is, concepts and conceptual structures, from one head to another, teachers, who have the goal of changing something in students' head, must have some notion of what goes on in those other heads. Hence it would seem necessary for a teacher to build up a model of the student's conceptual world.

When a student has struggled to find an answer to a given problem, it is not only boorish but also counterproductive to dismiss it as "wrong", even if the teacher then shows the "right" way of proceeding. Such disregard for the student's effort inevitably demolishes the student's motivation. Instead, a wiser teacher will ask the student how he or she came to the particular answer. In the majority of cases, the student, in reviewing the path (i. e., reflecting on the operations carried out), will either discover a hitch or give the teacher a clue to a conceptual connection that does not fit into the procedure that is to be learned. The first is an invaluable element of learning: It provides students with an opportunity to realize that they themselves can see what works and what does not. The second provides the teacher with an insight into the student's present way of operating and thus with a clearer idea of where a change might be attempted (Steffe, 1995).

This paper investigated several sources of constructivism by revisiting articles about constructivism. Considering young students's thinking is the crucial aspect of constructivism rather than just making 'fun' games for teaching

mathematics in classroom. I hope this paper provides an appropriate understanding of constructivism for teachers and teacher educators.

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구성주의 의미의 탐색에 대한 소고: 구성주의의 재조명

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수학교육학에서 구성주의는 학습이론에 있어서 핵심적인 이론이라고 할 수 있으며 구성주의에 대한 이해와 적용은 학습자 중심 수학 교수에 있어서 중요하다. 그러나 일부 초등 교사와 예비 초등 교사들은 구성주의를 편협한 시각으로 이해하고 있다. 구성주의가 수학을 흥미롭게 만드는 것을 중요하게 여기는 이론으로 여기고, 수학적인 요소를 고려하지 않은 게임을 만드는 것이 그 예이다. 본 논문에서는 여러 가지 유형의 구성주의, 급진적 구성주의, 비고츠키의 사회적 문화적 발달이론, 사회적 구성이론**, 사회적 구성주의가 각기 주장하는 바를 고찰하고 비교하여 구성주의를 이해하는 데에 도움을 주고자 하였다. 논문의 말미에 구성주의가 수학 교육에 시사하는 점이 논의되었다.

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* 주제어 : 구성주의, 급진적 구성주의, 비고츠키의 사회적 문화적 발달이론, 사회적 구성이론, 사회적 구성주의

** Social Constructionism (Constructionism을 구성주의로 번역하는 경우도 있으나, 본 고에서는 구성주의와 함께 다루고 있으므로, 구별을 위해 구성이론으로 번역하였다.)