Nuclear Engineering and Technology 50 (2018) 599–605

Contents lists available at ScienceDirect

Nuclear Engineering and Technology

journal homepage: www.elsevier.com/locate/net

Original Article

Parameter identifiability of Boolean networks with application to fault diagnosis of nuclear plants

Zhe Dong*, Yifei Pan, Xiaojin Huang

Institute of Nuclear and New Energy Technology, Collaborative Innovation Centre of Advanced Nuclear Energy Technology, Key Laboratory of Advanced Reactor Engineering and Safety of Ministry of Education, Tsinghua University, Beijing 100084, China

ARTICLE INFO

Article history:
Received 25 January 2018
Received in revised form
8 March 2018
Accepted 8 March 2018
Available online 5 April 2018

Keywords:
Boolean Network
Fault Diagnosis
Nuclear Heating Reactor
Parameter Identifiability

ABSTRACT

Fault diagnosis depends critically on the selection of sensors monitoring crucial process variables. Boolean network (BN) is composed of nodes and directed edges, where the node state is quantized to the Boolean values of True or False and is determined by the logical functions of the network parameters and the states of other nodes with edges directed to this node. Since BN can describe the fault propagation in a sensor network, it can be applied to propose sensor selection strategy for fault diagnosis. In this article, a sufficient condition for parameter identifiability of BN is first proposed, based on which the sufficient condition for fault identifiability of a sensor network is given. Then, the fault identifiability condition induces a sensor selection strategy for sensor selection. Finally, the theoretical result is applied to the fault diagnosis–oriented sensor selection for a nuclear heating reactor plant, and both the numerical computation and simulation results verify the feasibility of the newly built BN-based sensor selection strategy.

1. Introduction

The function of process fault diagnosis is observing the fault symptoms and identifying which fault is the root cause. The efficiency of fault diagnosis depends critically on the selection of sensors monitoring crucial process variables. There are hundreds of process variables available for measurement in a nuclear plant, and sensor selection for efficient fault diagnosis is an important problem. Because fault propagation in a process system, i.e., the cause–effect behavior, can be qualitatively described by directed graph (DG), the sensor selection problem was usually converted to different DG-based optimization problems. In the studies by Bagajewicz [1–4], sensor selection strategies are given as the solutions of mixed-integer linear programming (MILP) problems focusing on optimizing cost or (and) reliability. In the study by Bhushan et al. [5], the criteria of robustness were added to the MILP problems. In studies by Sen et al. and Carballido et al. [6,7], genetic algorithms were proposed to solve the optimization problems for sensor selection. The MILP-based approach was applied to the sensor selection for fault diagnosis problems of the nuclear plant equipment such as the integral pressurized water reactor and helical-coil once-through steam generator [8,9]. Moreover, sensor selection is usually served as a basis of developing more advanced fault detection and diagnosis methods for complex process systems [10–12].

The Boolean network (BN) is a network with nodes and directed edges, where the state of a node is quantized to the values of True or False and is determined through logical rules by the network logical parameters and the states of other nodes with edges directed to this node. It is can be seen that the BN is the coupling of a DG, and a set of logical rules or functions defined on the logical states of BN nodes, which means that the BN can be applied to describe the fault propagation in a sensor network. In the late 1960s, the BN was first introduced by Kauffman for modeling and analyzing cellular regulation phenomenon in the field of theoretic biology [13]. It has proved that the BN can be a proper tool to describe cellular networks [14]. The study of BN from a viewpoint of systems and control begun around 2010. Cheng and Qi gave the state–space model of BN on its linear representation and then revealed some features such as fix points, cycles, and controllability [15–18]. Therefore, applying BN to solve the problem of sensor selection for fault diagnosis may give a more advanced and efficient method.

In this article, the nodes in a sensor network are regarded as the nodes of a BN, the faults are regarded as parameters of the BN, and

* *Corresponding author.*
E-mail address: dongzhe@mail.tsinghua.edu.cn (Z. Dong).

https://doi.org/10.1016/j.net.2018.03.015

the cause–effect behaviors are described by logical functions defined on the states of nodes and parameters. By thoroughly analyzing the parameter identifiability of an arbitrarily given BN, the sufficient condition of fault discriminability is proposed based on analyzing the steady state–space structure of the BN, which further leads to a new sensor selection strategy. Then, the BN-based sensor selection method is applied to realize the fault diagnosis–oriented sensor selection of a nuclear heating reactor (NHR), and numerical computation and simulation results show the feasibility of this new approach.

2. Boolean network and its linear representation

In this section, the BN is first defined, and then its linear representation is proposed based on the semi-tensor product (STP) of matrices.

2.1. Logical matrix and Boolean network

Definition 1 [16,17]. Matrix $\boldsymbol{A}\in M_{m\times n}$ is called a logical matrix if the columns of $\boldsymbol{A}$, denoted by Col($\boldsymbol{A}$), satisfy Col($\boldsymbol{A}$)$\subset\Delta_m$, where $\Delta_m=\{\delta_m^k|k=1,\ldots,m\}$, and δ_m^k is the kth column of $\boldsymbol{I}_{\mathrm{m}}$. The set of $m\times n$ logical matrices is denoted by $L_{m\times n}$, and Δ_2 is usually denoted by Δ.

Definition 2 (Boolean network) [14]. A BN with a set of nodes $\boldsymbol{s}_i\in\Delta$ ($i=1,2,\ldots,n$) and parameters $\boldsymbol{f}_j\in\Delta$ ($j=1,2,\ldots,m$) can be described as

$$\begin{cases}\boldsymbol{s}_1(t+1)=\boldsymbol{\sigma}_1[\boldsymbol{s}_1(t)\quad\cdots\quad\boldsymbol{s}_n(t)\quad\boldsymbol{f}_1\quad\cdots\quad\boldsymbol{f}_m],\\ \boldsymbol{s}_2(t+1)=\boldsymbol{\sigma}_2[\boldsymbol{s}_1(t)\quad\cdots\quad\boldsymbol{s}_n(t)\quad\boldsymbol{f}_1\quad\cdots\quad\boldsymbol{f}_m],\\ \qquad\vdots\\ \boldsymbol{s}_n(t+1)=\boldsymbol{\sigma}_n[\boldsymbol{s}_1(t)\quad\cdots\quad\boldsymbol{s}_n(t)\quad\boldsymbol{f}_1\quad\cdots\quad\boldsymbol{f}_m],\end{cases}\tag{1}$$

where $\boldsymbol{\sigma}_i$: $\Delta^{n+m}\rightarrow\Delta$ ($i=1,2,\ldots,n$) are logical functions.

Remark 1. For a sensor network, nodes $\boldsymbol{s}_i\in\Delta$ ($i=1,2,\ldots,n$) denote the sensor state is normal or abnormal, and parameters $\boldsymbol{f}_j\in\Delta$ ($j=1,2,\ldots,m$) denote whether the corresponding fault occurs.

2.2. Structure matrix of logical function

It can be seen from model (1) that logical functions are nonlinear, which leads to the difficulty in analyzing the properties of BNs. Therefore, it is necessary to give a linear form of the given nonlinear logical function, which is called the structure matrix.

Definition 3 (Semi-tensor product) [16,17]. Suppose $\boldsymbol{A}\in M_{m\times n}$ and $\boldsymbol{B}\in M_{p\times q}$, let t be the lowest common multiple of positive integers n and p. The STP of $\boldsymbol{A}$ and $\boldsymbol{B}$ is defined by

$$\boldsymbol{A}\boldsymbol{B}=\left(\boldsymbol{A}\otimes\boldsymbol{I}_{t/n}\right)\left(\boldsymbol{B}\otimes\boldsymbol{I}_{t/p}\right)\tag{2}$$

where $\otimes$ is Kronecker product, $\boldsymbol{I}$ is identity matrix, and the product in the right side is just the classical matrix product. In the following, the multiplication of matrices and vectors is under the meaning of STP.

Remark 2. Based on Definition 3, if $n=p$, then the STP degenerates to the classical product. Moreover, it can be seen that

$$\begin{cases}\boldsymbol{S}(t)=\boldsymbol{s}_1(t)\ldots\boldsymbol{s}_n(t)\in\Delta_{2^n},\\ \boldsymbol{F}=\boldsymbol{f}_1\ldots\boldsymbol{f}_m\in\Delta_{2^m}.\end{cases}\tag{3}$$

The following Lemma 1 reveals that every logical function has a linear representation.

Lemma 1. Every logical function $\boldsymbol{\sigma}$: $\Delta^n\rightarrow\Delta$ has a linear representation given by

$$\boldsymbol{\sigma}(\boldsymbol{a}_1,\cdots,\boldsymbol{a}_n)=\boldsymbol{L}_\sigma\prod_{i=1}^n\boldsymbol{a}_i,\tag{4}$$

where $\boldsymbol{a}_i=[q_i,1\text{-}q_i]^{\mathrm{T}}\in\Delta$, $q_i\in\{0,1\}$, $i=1,2,\ldots,n$, and $\boldsymbol{L}_\sigma\in L_{2\times 2n}$ is called the structure matrix of logical function $\boldsymbol{\sigma}$.

Proof: Based on the definition of STP, it can be seen that

$$\boldsymbol{A}_n=\prod_{i=1}^n\boldsymbol{a}_i=\boldsymbol{\delta}_{2^n}^k\in\Delta_{2^n},\tag{5}$$

where $\boldsymbol{A}_n$ is just the vector representation of binary number $Q_n=q_1q_2\ldots q_n$, and then it can be directly computed that

$$k=2^n-\sum_{i=1}^n q_i2^{n-i}.\tag{6}$$

From the equivalence of Δ^n and Δ_2^n, the definition domain of $\boldsymbol{\sigma}$ can be regarded as Δ_2^n. For the kth element $\delta_{2^n}^k$,

$$\boldsymbol{M}_\sigma=[\boldsymbol{\sigma}_1\quad\boldsymbol{\sigma}_2\quad\ldots\quad\boldsymbol{\sigma}_n]\in M_{2\times 2^n},\tag{7}$$

where

$$\boldsymbol{\sigma}_k=\boldsymbol{\sigma}\left(\boldsymbol{\delta}_{2^n}^k\right),$$

is just the matrix satisfying equation (4).

Remark 3. It can be verified by direct computation that the structure matrices of logical negation "¬," disjunction "∨," conjunction "∧," identity "I," and constant "F" are given respectively by

$$\boldsymbol{L}_\neg=\boldsymbol{\delta}_2[2\quad 1],\tag{8}$$

$$\boldsymbol{L}_\vee=\boldsymbol{\delta}_2[1\quad 1\quad 1\quad 2],\tag{9}$$

$$\boldsymbol{L}_\wedge=\boldsymbol{\delta}_2[1\quad 2\quad 2\quad 2],\tag{10}$$

$$\boldsymbol{L}_{\mathrm{I}}=\boldsymbol{\delta}_2[1\quad 2],\tag{11}$$

$$\boldsymbol{L}_{\mathrm{F}}=\boldsymbol{\delta}_2[2\quad 2].\tag{12}$$

Remark 4. From Lemma 1, the BN model can be rewritten as

$$\begin{cases}\boldsymbol{s}_1(t+1)=\boldsymbol{L}_1\boldsymbol{F}\boldsymbol{S}(t),\\ \boldsymbol{s}_2(t+1)=\boldsymbol{L}_2\boldsymbol{F}\boldsymbol{S}(t),\\ \qquad\vdots\\ \boldsymbol{s}_n(t+1)=\boldsymbol{L}_n\boldsymbol{F}\boldsymbol{S}(t),\end{cases}\tag{13}$$

where both $\boldsymbol{F}$ and $\boldsymbol{S}(t)$ are given in equation (3), and $\boldsymbol{L}_i$ is the structure matrix of logical function $\boldsymbol{\sigma}_i$ ($i=1,\ldots n$). It can be seen from models (13) and (1) that nonlinear model (1) has been transferred to linear model (13).

2.3. Linear representation of Boolean network

From model (13), it can be seen that

$$\boldsymbol{S}(t+1)=s_1(t+1)\ldots s_n(t+1)=\boldsymbol{L}_1\boldsymbol{F}\boldsymbol{S}(t)\ldots\boldsymbol{L}_2\boldsymbol{F}\boldsymbol{S}(t)=\prod_{i=1}^n\boldsymbol{L}_i\boldsymbol{F}\boldsymbol{S}(t).\tag{14}$$

Here, the problem is that whether there is a matrix $\boldsymbol{L}_{\mathrm{F}}$ so that (14) can be rewritten as state–space model

$$S(t+1) = L_F S(t),$$

which is also called the linear representation of BN (1). This problem is solved in the following parts of this section.

To give the linear representation of BN (1), the following Lemma 2 is introduced which shows an important property of $A_n \in \Delta_{2n}$ defined by (5).

Lemma 2. Suppose $a_i = [q_i, 1\text{-}q_i]^T \in \Delta$, $q_i \in \{0,1\}$, $i = 1, 2, \ldots, n$, and $A_n \in \Delta_{2n}$ defined by (5), then, A_n satisfies

$$A_n^2 = \Phi_n A_n, \tag{15}$$

where

$$\Phi_n = \mathrm{diag}\left(\left[\delta_{2^n}^1 \quad \ldots \quad \delta_{2^n}^{2^n}\right]\right) = \delta_{2^{2n}}\left[1 \quad \ldots \quad k+(k-1)2^n \quad \ldots \quad 2^{2n}\right] \in \mathrm{L}_{2^{2n}\times 2^n}. \tag{16}$$

Proof: Suppose

$$A_n = \delta_{2^n}^k, \tag{17}$$

where positive integer k is given by equation (6). Then, based on Definition 2 and by direct computation, we have

$$A_n^2 = \delta_{2^n}^k \delta_{2^n}^k = \left[O_{1\times(k-1)2^n} \quad \left(\delta_{2^n}^k\right)^T \quad O_{1\times(2^n-k)2^n}\right]^T = \delta_{2^{2n}}^{k+(k-1)2^n}, \tag{18}$$

and

$$\Phi_n A_n = \left[\delta_{2^{2n}}^1 \quad \ldots \quad \delta_{2^{2n}}^{k+(k-1)2^n} \quad \ldots \quad \delta_{2^{2n}}^{2^{2n}}\right]\delta_{2^n}^k = \delta_{2^{2n}}^{k+(k-1)2^n}. \tag{19}$$

From equations (18) and (19), it is apparent to see that equation (15) is well satisfied, which completes this proof.

Remark 5. By successively applying Lemma 2, it can be obtained that

$$A_n^q = \Phi_n^{q-1} A_n, \tag{20}$$

which means that the power of A_n is determined by itself. Then, for logical function σ with structure matrix L_σ,

$$\sigma(A_n^q) = L_\sigma A_n^q = L_\sigma \Phi_n^{q-1} A_n.$$

For giving the linear representation of BN (1), another useful lemma is introduced as follows.

Lemma 3. Let $x \in \mathrm{R}^m$ and $A \in \mathrm{M}_{p\times q}$. Then

$$xA = (I_m \otimes A)x. \tag{21}$$

Proof: Based on Definition 2, it can be computed directly that

$$xA = (x \otimes I_p)A = \mathrm{diag}([x_1 A \quad \ldots \quad x_m A]) = \mathrm{diag}([A \quad \ldots \quad A])x = (I_m \otimes A)x, \tag{22}$$

which means that equation (21) is well satisfied.

Based on the above discussions, the linear representation of BN (1) is summarized in the following Theorem 1.

Theorem 1. The linear representation of BN (1) can be represented as

$$S(t+1) = L_F S(t), \tag{23}$$

where

$$L_F = LF, \tag{24}$$

and

$$L = L_1 \prod_{k=2}^{n} [(I_{2^{m+n}} \otimes L_k)\Phi_{m+n}]. \tag{25}$$

Proof: From lemmas 2 and 3 as well as BN model (13), it can be derived that

$$\begin{aligned} S(t+1) &= \prod_{i=1}^{n} L_i F S(t) \\ &= L_1 F S(t) L_2 F S(t) \prod_{k=3}^{n} L_k F S(t) \\ &= L_1 (I_{2^{m+n}} \otimes L_2)[F S(t)]^2 \prod_{k=3}^{n} L_k F S(t) \\ &= L_1 [(I_{2^{m+n}} \otimes L_2)\Phi_{m+n}] F S(t) \prod_{k=3}^{n} L_k F S(t). \end{aligned} \tag{26}$$

Then, by iteratively applying lemmas 2 and 3, it can be finally obtained that

$$S(t+1) = L_1 \prod_{k=2}^{n} [(I_{2^{m+n}} \otimes L_k)\Phi_{m+n}] F S(t), \tag{27}$$

which is just the linear representation given by (25). The proof of Theorem 1 is completed.

Remark 6. Suppose the initial condition of state−vector $S(t)$ is S_0, then the response of linear system (23) is

$$S(t) = (L_F)^n S_0 \tag{28}$$

3. Parameter identifiability of Boolean network

After giving representing BN model (1) to linear state−space model (23), some important properties of the BN can be given based on linear model (23). In this section, a sufficient condition for the parameter identifiability of BN (1) is given, which is important for the applications such as sensor selection for fault diagnosis.

Definition 4 (Steady state and transition period). For a given parameter vector $F \in \Delta_2^n$, suppose there is a finite positive integer T so that

$$S(T+1) = S(T). \tag{29}$$

Then T and $S(T)$ are respectively called the transition period and steady-state under parameter F.

Definition 5 (Parameter identifiability). For parameter vectors F_i $(i = 1, \ldots, m)$, suppose the transition period of BN (1) under F_i with the same initial condition is T_i. Then, parameter vectors F_i are identifiable if

$$S(T_i) \neq S(T_j), \quad i \neq j. \tag{30}$$

Remark 7. Parameter identifiability means that different parameter vectors can be discriminative with each other based on the difference between their corresponding steady state.

The following Theorem 2 gives a sufficient condition for the parameters of BN (1) to be identifiable.

Theorem 2. Consider BN (1) and suppose that the transition period T_i with the same initial condition under parameter vector

$$\boldsymbol{F}_j = \boldsymbol{\delta}_{2^m}^{k_j}, \quad j = 1, \cdots, m, \tag{31}$$

is finite, where

$$k_j = 2^m - 2^{m-j}. \tag{32}$$

Then parameters $\boldsymbol{f}_j \in \Delta$ ($j = 1, 2, \ldots, m$) can be identifiable under initial condition $\boldsymbol{S}_0$ if matrix

$$\boldsymbol{Q} = \left[\left(\boldsymbol{L}\boldsymbol{\delta}_{2^m}^{k_1}\right)^{T_1} \quad \cdots \quad \left(\boldsymbol{L}\boldsymbol{\delta}_{2^m}^{k_j}\right)^{T_j} \quad \cdots \quad \left(\boldsymbol{L}\boldsymbol{\delta}_{2^m}^{k_m}\right)^{T_m} \right] \boldsymbol{S}_0 \tag{33}$$

has full column rank, i.e.,

$$\mathrm{rank}(\boldsymbol{Q}) = m. \tag{34}$$

Proof: From Definition 5, it can be seen that if

$$\mathrm{rank}([\boldsymbol{S}(T_1) \quad \ldots \quad \boldsymbol{S}(T_m)]) = m, \tag{35}$$

then the parameters $\boldsymbol{f}_j$ ($j = 1, 2, \ldots, m$) can be identifiable. Further, since

$$\boldsymbol{S}(T_j) = \left(\boldsymbol{L}\boldsymbol{\delta}_{2^m}^{k_j}\right)^{T_j} \boldsymbol{S}_0, \tag{36}$$

conditions (34) and (35) are equivalent with each other, which completes the proof of Theorem 2.

Remark 8. From equation (36), it can be seen that

$$\boldsymbol{S}(T_j) = \boldsymbol{\delta}_{2^n}^{l_j}. \tag{37}$$

Since $\boldsymbol{s}_i(t) = [q_i, 1\text{-}q_i]^{\mathrm{T}} \in \Delta$, $q_i \in \{0,1\}$, from (5) and (6), we have

$$\begin{cases} q_1 = \left\lfloor \dfrac{2^n - l_j}{2^{n-1}} \right\rfloor, \\ q_i = \left\lfloor \dfrac{2^n - l_j - \sum_{r=1}^{i-1} q_i 2^{n-i}}{2^{n-i}} \right\rfloor, \quad i = 2, \cdots, n, \end{cases} \tag{38}$$

where $\lfloor a \rfloor$ is the largest integer less or equal to a.

4. Application to sensor selection for fault diagnosis

Based on the discussion and analysis in Sections 2 and 3, the fault propagation in process networks such as a nuclear reactor can be modeled by the following BN

$$\boldsymbol{s}_l(t+1) = \boldsymbol{L}_{\vee}^{m+n-1} \left(\prod_{j=1}^{m} \boldsymbol{L}_{\mathrm{f},j} \boldsymbol{f}_j \right) \left(\prod_{i=1}^{n} \boldsymbol{L}_{\mathrm{s},i} \boldsymbol{s}_i \right), \quad l = 1, \cdots, n, \tag{39}$$

where $\boldsymbol{s}_i \in \Delta$ ($i = 1, 2, \ldots, n$) denote the sensor states and $\boldsymbol{f}_j \in \Delta$ ($j = 1, 2, \ldots, m$) denote the faults. Here, in equation (39), $\boldsymbol{L}_{\mathrm{f},j} = \boldsymbol{L}_{\mathrm{I}}$ if fault $\boldsymbol{f}_j$ can be detected by sensor $\boldsymbol{s}_l$, otherwise $\boldsymbol{L}_{\mathrm{f},j} = \boldsymbol{L}_{\mathrm{F}}$. Similarly, $\boldsymbol{L}_{\mathrm{s},i} = \boldsymbol{L}_{\mathrm{I}}$ if the fault can be propagated from the variable measured by sensor $\boldsymbol{s}_i$ to that measured by sensor $\boldsymbol{s}_l$, otherwise $\boldsymbol{L}_{\mathrm{s},i} = \boldsymbol{L}_{\mathrm{F}}$.

Based on lemmas 2 and 3,

$$\boldsymbol{s}_l(t+1) = \boldsymbol{L}_l \left(\prod_{j=1}^{m} \boldsymbol{f}_j \right) \left(\prod_{i=1}^{n} \boldsymbol{s}_i \right), \quad l = 1, \cdots, n, \tag{40}$$

where

$$\boldsymbol{L}_l = \boldsymbol{L}_{\vee}^{m+n-1} \left\{ \left[\prod_{i=1}^{m} \left(\boldsymbol{I}_{2^{i-1}} \otimes \boldsymbol{L}_{\mathrm{f},i} \right) \right] \left[\prod_{j=1}^{n} \left(\boldsymbol{I}_{2^{m+j-1}} \otimes \boldsymbol{L}_{\mathrm{s},j} \right) \right] \right\}. \tag{41}$$

Then, based on Theorem 1, we can obtain the linear representation of BN model (39).

To guarantee that all the faults can be detected by the sensor network, for any $k_j \in \{1, 2, \ldots, 2^{\mathrm{m}}\text{-}1\}$ ($j = 1, \ldots, m$), where $k_s \neq k_t$ for $s \neq t$, and any $\boldsymbol{S}_0 \in \Delta_{2n}$

$$\boldsymbol{S}(T_j) = \left(\boldsymbol{L}\boldsymbol{\delta}_{2^m}^{k_j}\right)^{T_j} \boldsymbol{S}_0 \neq \boldsymbol{\delta}_{2^m}^{2^m}, \tag{42}$$

which is just the fault detection condition. Then, based on Theorem 2, if both conditions (42) and (35) are well satisfied, then the faults can be detected and identified based on the responses of the sensor network. Here, the sensor response under a given fault is given by (39), from which we can obtain a proper sensor selections strategy.

5. Numerical verification

In this section, the BN-based sensor selection method proposed in Section 4 is applied to a NHR-based nuclear plant. Numerical computation for linear representation, steady-state under faults, and sensor selection strategy for fault identification are all given. Numerical simulation based on the dynamical model of a NHR plant show the feasibility of the theoretical result.

5.1. Background

The NHR is a typical integral pressurized water reactor developed by the Institute of Nuclear and New Energy Technology of Tsinghua University, which has the advanced safety features such as integral primary-circuit arrangement, full-power-range natural circulation, self-pressurization, and passive residual heat removing [19–21], which can be adopted for building electricity and process heat cogeneration plants [22]. The schematic diagram of an NHR-based nuclear steam supply system (NSSS) is shown in Fig. 1. The primary circuit (PC) transfer the heat produced by the fission reaction to the two intermediate circuits (ICs) via primary heat exchangers (PHEs) inside the reactor pressure vessel. The pressures of ICs are little higher than that of the PC, which effectively prevents the leakage of the radioactive fission production from the PC to IC. The heat stored in the ICs is transferred from the primary side to the secondary side of the U-tube steam generators so that the feedwater flow is heated up to be saturated live steam.

The sensors to be selected for fault diagnosis are those measuring the reactor neutron flux, reactor outlet temperature, IC hot leg temperature, and IC flowrate, which are all given in Table 1 with the measurement range and precision as well as the type of sensor output signal. The measurement range of each sensor corresponds with the output range of 4–20 mA. The faults to be detected or discriminated are given in Table 2, which include the abnormal reactivity injection, malfunction of the IC pump, and heat transfer degradation between two sides of the PHEs.

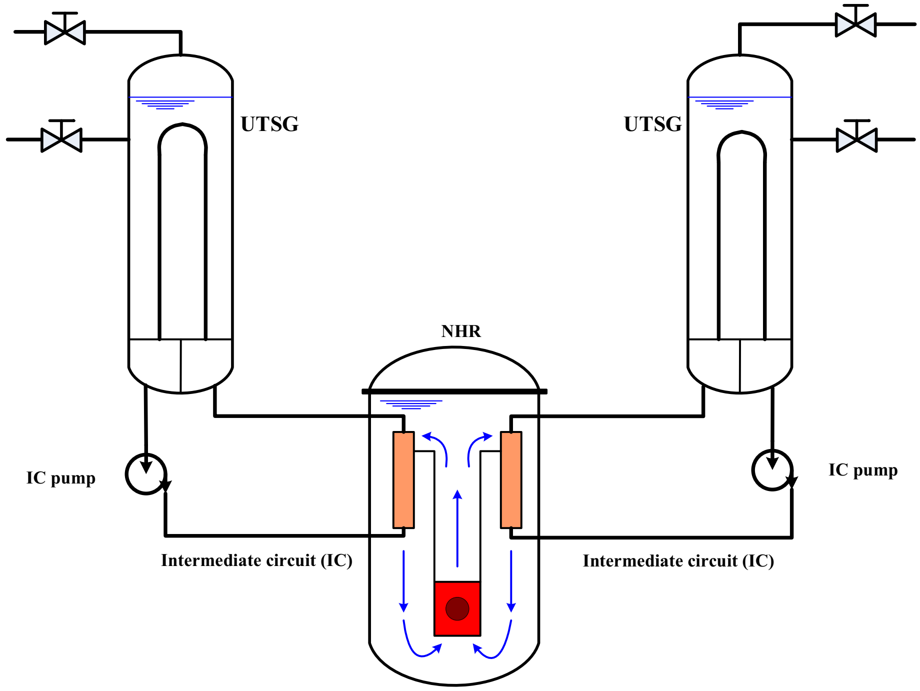

Fig. 1. Schematic diagram of the NHR-based NSSS.
NHR, nuclear heating reactor; NSSS, nuclear steam supply system; UTSG, U-tube steam generator.

Table 1
Available sensor nodes for selection.

Nodes	Description
s_1	Reactor neutron flux
s_2	Reactor core outlet temperature
s_3	IC hot leg temperature
s_4	IC coolant flowrate

IC, intermediate circuit.

Table 2
Faults to be detected or discriminated.

Nodes	Description
f_1	Abnormal reactivity injection to the reactor
f_2	Heat transfer degradation of PHE two sides
f_3	Malfunction of IC pump

IC, intermediate circuit; PHE, primary heat exchanger.

5.2. BN model for fault propagation

From the physical and thermal–hydraulic features of NHR-based NSSS, the following relationships between fault $\boldsymbol{f}_i$ ($i = 1, 2, 3$) and sensor $\boldsymbol{s}_j$ ($j = 1, 2, 3, 4$) can be observed:

(1) If fault $\boldsymbol{f}_1$ occurs, i.e., there is an abnormal positive or negative reactivity injection, then the response of neutron flux is abnormal which leads to the abnormality in the output signal of $\boldsymbol{s}_1$. Because the variation of neutron flux can directly result in the variations of primary coolant temperature, fault $\boldsymbol{f}_1$ can also lead to the abnormality in the acquired signal by $\boldsymbol{s}_2$. Because the variation of primary coolant temperature can result in the variation of secondary coolant temperature, the abnormality in the acquired signal by $\boldsymbol{s}_2$ can further lead to the abnormality of sensor $\boldsymbol{s}_3$. Moreover, since the IC coolant flowrate is controlled by the IC pump, there is no effect of fault $\boldsymbol{f}_1$ to sensor $\boldsymbol{s}_4$.

(2) If fault $\boldsymbol{f}_2$ occurs, i.e., the heat transfer between PC and IC is degraded, then the thermal resistance of PHEs is abnormal, which immediately leads to the abnormalities in the outputs of sensors $\boldsymbol{s}_2$ and $\boldsymbol{s}_3$. Here, fault $\boldsymbol{f}_2$ may be induced by the limescale inside the tubes of PHE.

(3) If fault $\boldsymbol{f}_3$ occurs, i.e., the IC pump is malfunctioned, then there must be the abnormality of IC flowrate which induces the abnormality in the output of $\boldsymbol{s}_4$. Since the hot leg temperature is very sensitive to the IC flowrate, the abnormality in the output of $\boldsymbol{s}_4$ can further lead to the abnormality of $\boldsymbol{s}_3$. Because of the heat transfer between PC and IC, this fault can further lead to the abnormality in the output of $\boldsymbol{s}_2$.

Based on the above analysis, the BN model for the fault propagation can be written as

$$\begin{cases} \boldsymbol{s}_1(t+1) = \boldsymbol{f}_1, \\ \boldsymbol{s}_2(t+1) = \boldsymbol{f}_2 \vee \boldsymbol{s}_1(t) \vee \boldsymbol{s}_3(t), \\ \boldsymbol{s}_3(t+1) = \boldsymbol{f}_2 \vee \boldsymbol{s}_2(t) \vee \boldsymbol{s}_4(t), \\ \boldsymbol{s}_4(t+1) = \boldsymbol{f}_3. \end{cases} \tag{43}$$

Then, based on equations (25), (16), and (41), the linear representation of BN model (43) can be given by canonical form (23) with structural matrix given by

$$\boldsymbol{L} = \delta_{16}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 3 & 1 & 3 & 1 & 1 & 5 & 5 & 1 & 3 & 5 & 7 \\ 2 & 2 & 2 & 2 & 2 & 4 & 2 & 4 & 2 & 2 & 6 & 6 & 2 & 4 & 6 & 8 \\ 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 \\ 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \\ 9 & 9 & 9 & 9 & 9 & 11 & 9 & 11 & 9 & 9 & 13 & 13 & 9 & 11 & 13 & 15 \\ 10 & 10 & 10 & 10 & 10 & 12 & 10 & 12 & 10 & 10 & 14 & 14 & 10 & 12 & 14 & 16 \end{bmatrix}, \tag{44}$$

where this representation of matrix $\boldsymbol{L}$ is a simplification, for example,

$$\boldsymbol{\delta}_{16}[1 \quad 2] = \left[\boldsymbol{\delta}_{16}^{1} \quad \boldsymbol{\delta}_{16}^{2}\right] \in \mathrm{L}_{16\times 2}, \tag{45}$$

and δ_m^k is the kth column of $\boldsymbol{I}_{\mathrm{m}}$ as defined in Definition 1.

5.3. Fault identifiability and sensor selection strategy

From equation (6), choose $k_1 = 4$, $k_2 = 6$ and $k_3 = 7$.

Furthermore, from equations (24) and (31), it can be computed that

$$\begin{aligned}\boldsymbol{L}_{\mathrm{F1}} &= \boldsymbol{L}\boldsymbol{F}_1 = \boldsymbol{L}\boldsymbol{\delta}_8^4 \\ &= \boldsymbol{\delta}_{16}[\ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 4\ \ 2\ \ 4\ \ 2\ \ 2\ \ 6\ \ 6\ \ 2\ \ 4\ \ 6\ \ 8\ ],\end{aligned} \tag{46}$$

$$\begin{aligned}\boldsymbol{L}_{\mathrm{F2}} &= \boldsymbol{L}\boldsymbol{F}_2 = \boldsymbol{L}\boldsymbol{\delta}_8^6 \\ &= \boldsymbol{\delta}_{16}[\ 10\ 10\ 10\ 10\ 10\ 10\ 10\ 10\ 10\ 10\ 10\ 10\ 10\ 10\ 10\ 10\ ],\end{aligned} \tag{47}$$

$$\begin{aligned}\boldsymbol{L}_{\mathrm{F3}} &= \boldsymbol{L}\boldsymbol{F}_3 = \boldsymbol{L}\boldsymbol{\delta}_8^7 \\ &= \boldsymbol{\delta}_{16}[\ 9\ \ 9\ \ 9\ \ 9\ \ 9\ \ 11\ \ 9\ \ 11\ \ 9\ \ 9\ \ 13\ \ 13\ \ 9\ \ 11\ \ 13\ \ 15\ ],\end{aligned} \tag{48}$$

from which it can be seen that

$$\boldsymbol{L}_{\mathrm{F1}}^4 = \boldsymbol{L}_{\mathrm{F1}}^3, \quad \boldsymbol{L}_{\mathrm{F2}}^2 = \boldsymbol{L}_{\mathrm{F2}}, \quad \boldsymbol{L}_{\mathrm{F3}}^4 = \boldsymbol{L}_{\mathrm{F3}}^3\,.$$

Moreover, from Definition 4, it can be seen that the transition period T_i under faultf_i $(i = 1, 2, 3)$ satisfies $T_1 = T_3 = 3$ and $T_2 = 1$

Consider initial state $S_0 = \delta_{16}^{16}$, which means that all the sensor states areδ_2^2. Then, it can be computed from (33) that

$$\boldsymbol{P} = \boldsymbol{\delta}_{16}[2 \quad 10 \quad 9]. \tag{49}$$

From equation (49), it can be seen that matrix $\boldsymbol{P}$ satisfies the identifiability condition. Thus, sensor selection given by Table 1 is reasonable for fault diagnosis of NHR. Further, based on (49) it can be seen that the sensor nodes sensitive to $\boldsymbol{f}_1$ are $\{\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{s}_3\}$, the sensor nodes sensitive to $\boldsymbol{f}_2$ are $\{\boldsymbol{s}_2, \boldsymbol{s}_3\}$, and the sensor nodes sensitive to $\boldsymbol{f}_3$ are $\{\boldsymbol{s}_2\ \boldsymbol{s}_3\ \boldsymbol{s}_4\}$. Then, it is reasonable to use $\boldsymbol{s}_1$ to identify $\boldsymbol{f}_1$, to use $\boldsymbol{s}_3$ to identify $\boldsymbol{f}_2$, and to use $\boldsymbol{s}_4$ to identify $\boldsymbol{f}_3$.

5.4. Numerical simulation

The above sensor selection strategy for fault diagnosis of NHR plant is verified by numerical simulation on the full-scale simulation programs [21], and the power controller is online in the closed-loop.

In this simulation, fault $\boldsymbol{f}_1$ is simulated by injecting a negative reactivity of 0.2 \$. Fault $\boldsymbol{f}_2$ is simulated by stepping the heat transfer coefficient of PHE down to 80% of its current value. Fault $\boldsymbol{f}_3$ is simulated by a negative step decrease of IC flowrate of 100 kg/s. Initially, the NSSS operates at full power-level, and a fault occurs at 3000s. The responses of the normalized nuclear power n_{r}, reactor core outlet coolant temperature T_{cout}, IC hot leg temperature T_{hlg}, and IC flowrate G_{p}, under faults $\boldsymbol{f}_1, \boldsymbol{f}_2$, and $\boldsymbol{f}_3$ are all shown in Fig. 2.

Here, the responses of n_{r}, T_{cout}, T_{hlg}, and G_{p} correspond to sensor $\boldsymbol{s}_1$, $\boldsymbol{s}_2$, $\boldsymbol{s}_3$, and $\boldsymbol{s}_4$, respectively. Based on the comparison among the responses of n_{r}, T_{cout}, T_{hlg}, and G_{p} as well as the output signal of sensor $\boldsymbol{s}_i$ $(i = 1, 2, 3, 4)$ under faults $\boldsymbol{f}_1, \boldsymbol{f}_2$, and $\boldsymbol{f}_3$, it can be seen that sensor $\boldsymbol{s}_1$ is sensitive to $\boldsymbol{f}_1$, $\boldsymbol{s}_3$ is sensitive to $\boldsymbol{f}_1, \boldsymbol{f}_2$, and $\boldsymbol{f}_3$, and $\boldsymbol{s}_4$ is sensitive to $\boldsymbol{f}_3$. Then, it is reasonable to choose $\boldsymbol{s}_1$ to identify $\boldsymbol{f}_1$, choose $\boldsymbol{s}_4$ to identify $\boldsymbol{f}_3$, and choose $\boldsymbol{s}_3$ to detect whether a fault occurs. Here, f_2 can also be identified based on the outputs of s_1, s_2, and s_3. Actually, if

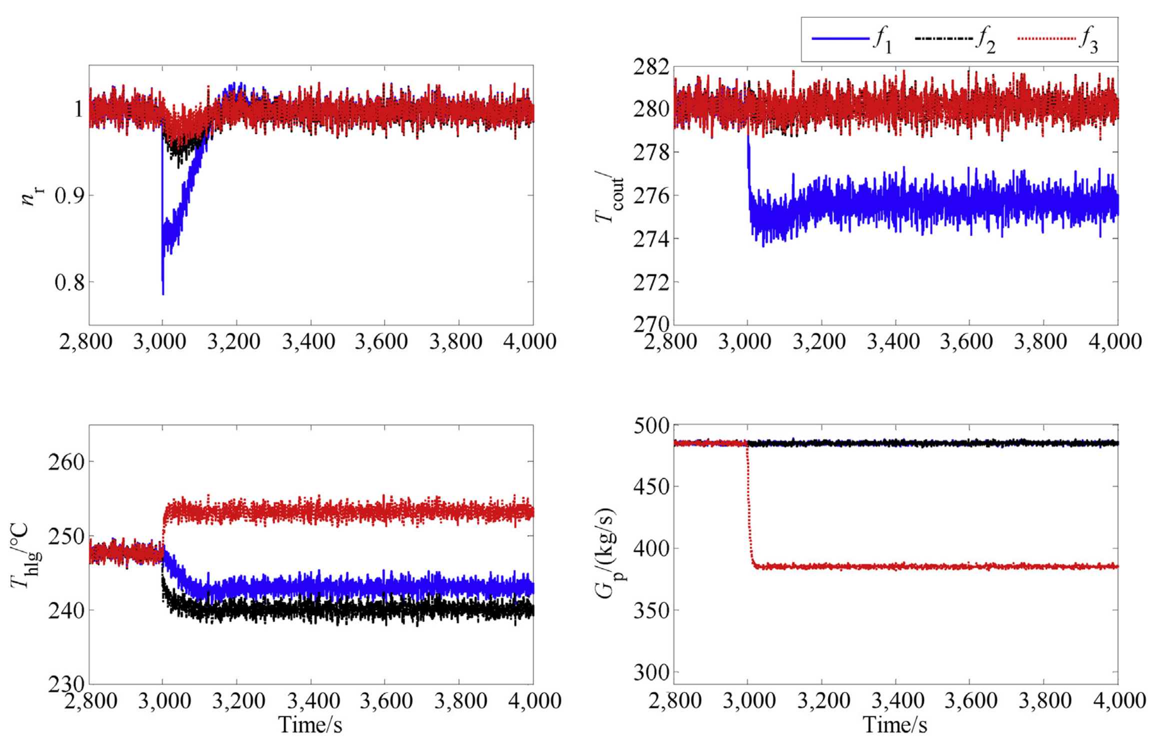

Fig. 2. Dynamic responses of NHR-based NSSS under faults $\boldsymbol{f}_i$ $(i = 1, 2, 3)$, n_{r}, normalized nuclear power; T_{cout}, reactor core outlet temperature; T_{hlg}, IC hot leg temperature; G_{p}, IC flowrate.

IC, intermediate circuit; NHR, nuclear heating reactor; NSSS, nuclear steam supply system

$$\begin{cases} \boldsymbol{s}_1 = \boldsymbol{s}_3 = \boldsymbol{\delta}_2^2 \\ \boldsymbol{s}_2 = \boldsymbol{\delta}_2^1 \end{cases}$$

then it can be seen that $\boldsymbol{f}_2$ occurs. Therefore, the numerical simulation result is well in accordance with the sensor selection strategy and verifies the theoretical result.

6. Conclusion

Owing to the strength of BN in describing dynamics of logical systems, the BN is applied to solve the fault diagnosis–oriented sensor selection problem in this article. The nodes and faults in a sensor network are regarded as the nodes and parameters of a BN, respectively, and the cause–effect behaviors are described by logical functions defined on the states of nodes and parameters. The fault discriminability is transferred to parameter identifiability of the BN. A sufficient condition for BN parameter identifiability is given, which further leads to not only the sufficient condition for the faults propagated in a sensor network to be identifiable but also the strategy of sensor selection. Finally, the BN-based sensor selection method is applied to realize the fault diagnosis–oriented sensor selection of a NHR. Numerical computation for linear representation, steady-states under faults, and sensor selection strategy are all given, and dynamical simulation based on the model of a NHR plant shows the feasibility of the theoretical result.

Conflict of interest

There is no conflict of interest.

Acknowledgment

The work in this article is jointly supported by the National Science and Technology Major Project of China (Grant No. ZX06901) and the Natural Science Foundation of China (NSFC) (Grant No. 61773228).

References

[1] M. Bagajewicz, Design and retrofit of sensor networks in process plants, AIChE J. 43 (1997) 2300–2306.
[2] M. Bagajewicz, A review of techniques for instrumentation and upgrade in process plants, Can. J. Chem. Eng. 80 (2002) 3–16.
[3] M. Bagajewicz, E. Cabrera, New MILP formulation for instrumentation network design and upgrade, AIChE J. 48 (2002) 2271–2282.
[4] M. Bagajewicz, A. Fuxman, A. Uribe, Instrumentation network design and upgrade for process monitoring and fault detection, AIChE J. 50 (2004) 1870–1880.
[5] M. Bhushan, S. Narasimhan, R. Rengaswamy, Robust sensor network design for fault diagnosis, Comp. Chem. Eng. 32 (2008) 1067–1084.
[6] S. Sen, S. Narasimhan, K. Deb, Sensor network design of linear processes using genetic algorithms, Comp. Chem. Eng. 22 (1998) 385–390.
[7] J.A. Carballido, I. Ponzoni, N.B. Brignole, CGD-GA: a graph-based genetic algorithm for sensor network design, Inform. Sci. 177 (2007) 5091–5102.
[8] F. Li, B.R. Upadhyaya, Design of sensor placement for an integral pressurized water reactor using fault diagnostic observability and reliability criteria, Nucl. Technol. 173 (2011) 17–25.
[9] F. Li, B.R. Upadhyaya, S.R.P. Perillo, Fault diagnosis of helical coil steam generator systems of an integral pressurized water reactor using optimal sensor selection, IEEE Trans. Nucl. Sci. 59 (2012) 403–410.
[10] S.M. Namburu, M.S. Azam, J. Luo, K. Choi, K.R. Pattipati, Data-driven modeling, fault diagnosis and optimal sensor selection for HVAC chillers, IEEE Trans. Autom. Sci. Eng. 4 (2007) 469–473.
[11] N. Najjar, S. Gupta, J. Hare, S. Kandil, R. Walthall, Optimal sensor selection and fusion for heat exchanger fouling diagnosis in aerospace systems, IEEE Sens. J. 16 (2016) 4866–4881.
[12] D. Li, Y. Zhou, G. Hu, C.J. Spanos, Optimal sensor configuration and feature selection for AHU fault detection and diagnosis, IEEE Trans. Ind. Inform. 13 (2017) 1369–1380.
[13] S.A. Kauffman, Metabolic stability and epigenesist in randomly constructed genetic nets, J. Theor. Biol. 22 (1969) 437–467.
[14] C. Farrow, J. Heidel, H. Maloney, J. Rogers, Scalar equations for synchronous Boolean networks with biological applications, IEEE Trans. Neural Netw. 15 (2004) 348–354.
[15] H. Qi, D. Cheng, Logic and logic-based control, J. Contr. Theory Appl. 6 (2008) 123–133.
[16] D. Cheng, H. Qi, Controllability and observability of Boolean control networks, Automatica 45 (2009) 1659–1667.
[17] D. Cheng, H. Qi, A linear representation of dynamics of Boolean networks, IEEE Trans. Automatic Contr. 55 (2010) 2251–2258.
[18] D. Cheng, H. Qi, State-space analysis of Boolean networks, IEEE Trans. Neural Netw. 55 (2010) 2251–2258.
[19] D. Wang, C. Ma, D. Dong, J. Lin, Chinese nuclear heating test reactor and demonstration plant, Nucl. Eng. Design 136 (1992) 91–98.
[20] D. Wang, The design characteristics and construction experiences of the 5 MW nuclear heating reactor, Nucl. Eng. Design 143 (1993) 19–24.
[21] W. Zheng, D. Wang, NHR-200 nuclear energy system and its possible applications, Progr. Nucl. Energy 29 (1995) 193–200.
[22] Z. Dong, Y. Pan, A lumped-parameter dynamical modal of a nuclear heating reactor cogeneration plant, Energy 145 (2018) 638–656.