



## Original Article

## Enhanced reasoning with multilevel flow modeling based on time-to-detect and time-to-effect concepts

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## ARTICLE INFO

*Article history:*

Received 31 January 2018

Received in revised form

6 March 2018

Accepted 13 March 2018

Available online 23 March 2018

*Keywords:*

Multilevel Flow Modeling

Time-series Data

Time-to-Detect

Time-to-Effect

## ABSTRACT

To easily understand and systematically express the behaviors of the industrial systems, various system modeling techniques have been developed. Particularly, the importance of system modeling has been greatly emphasized in recent years since modern industrial systems have become larger and more complex.

Multilevel flow modeling (MFM) is one of the qualitative modeling techniques, applied for the representation and reasoning of target system characteristics and phenomena. MFM can be applied to industrial systems without additional domain-specific assumptions or detailed knowledge, and qualitative reasoning regarding event causes and consequences can be conducted with high speed and fidelity.

However, current MFM techniques have a limitation, i.e., the dynamic features of a target system are not considered because time-related concepts are not involved. The applicability of MFM has been restricted since time-related information is essential for the modeling of dynamic systems. Specifically, the results from the reasoning processes include relatively less information because they did not utilize time-related data.

In this article, the concepts of time-to-detect and time-to-effect were adopted from the system failure model to incorporate time-related issues into MFM, and a methodology for enhancing MFM-based reasoning with time-series data was suggested.

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## 1. Introduction

To easily understand and systematically express the behaviors of industrial systems, various system modeling techniques have been developed. Particularly, the importance of system modeling has been greatly emphasized in recent years since modern industrial systems have become larger and more complex.

Although various modeling techniques are classified according to various criteria such as domain characteristics or modeling purposes, all modeling techniques are either classified as a quantitative modeling technique or a qualitative modeling technique according to the type of underlying models. If the system is well-understood to acquire analytic solutions, quantitative modeling techniques based on concrete mathematical and physical backgrounds can be applied, and the phenomena within such a system

can be analyzed with computational approaches. However, existing systems that are not always well-understood cannot apply quantitative modeling techniques, even though these techniques may involve many assumptions and simplifications. In many cases, a system's internal causalities and correlations are known qualitatively rather than quantitatively, and sometimes qualitative analyses are more feasible than quantitative analyses due to practical reasons such as computation time problems.

Multilevel flow modeling (MFM) is one of the qualitative modeling techniques, applied for the representation and reasoning of target system (usually for systems that cannot be modeled quantitatively) characteristics and phenomena. This model represents a system with several interconnected levels of means and part-whole abstractions and goals and functions with flows (mass, energy, and information) and their interactions [1]. Based on these characteristics, MFM can be applied to industrial systems without additional domain-specific assumptions or detailed knowledge, and qualitative reasoning regarding event causes and consequences can be conducted with high speed and fidelity.

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Owing to these advantages, MFM has been applied for not only modeling of the system itself [2] but also for failure mode analysis [3,4], fault diagnosis [5], finding of counter-actions [6,7], and procedure validation [8]. It is already proven that MFM is undoubtedly a useful tool for intuitive yet powerful system representation. However, current MFM techniques have a limitation, namely they are not able to consider the dynamic features of the target system because time-related concepts are not involved. The applicability of MFM has been restricted since time-related information is essential for the modeling of dynamic systems. Specifically, the results from the reasoning processes include relatively less information because they did not utilize time-related data. Therefore, in this article, the concepts of time-to-detect (TTD) and time-to-effect (TTE) adopted from the system failure model were incorporated into the MFM method, and a methodology for enhancing MFM-based reasoning with time-series data was suggested.

The organization of the rest of this article is as follows. Section 2 briefly explains the characteristics of the MFM and system failure models. Section 3 addresses the methodology for enhanced MFM-based reasoning with newly adopted concepts and time-series data. Section 4 includes discussions, and Section 5 presents concluding remarks and future work outlooks.

## 2. Preliminaries

### 2.1. Characteristics of MFM

MFM is a methodology for the qualitative modeling of industrial processes. It represents a system's hierarchical structure with means-end and part-whole abstractions and represents the goals and functions of the system with mass, energy, and information flows and their interactions [1]. MFM models are simple yet can include many fundamental features of the target system. Fig. 1 presents the basic MFM symbols including the function symbols and relation symbols.

Because MFM is based on fundamental laws of energy and mass conservation, the entire system can be accurately modeled, and the models are easy to understand. Moreover, this characteristic enables users to conduct qualitative reasoning, which is a process that infers the causes and consequences of observed phenomena [9].

The major characteristics of MFM can be summarized as follows:

- System representation with flows and interactions: MFM represents the target system functions with elementary flows and

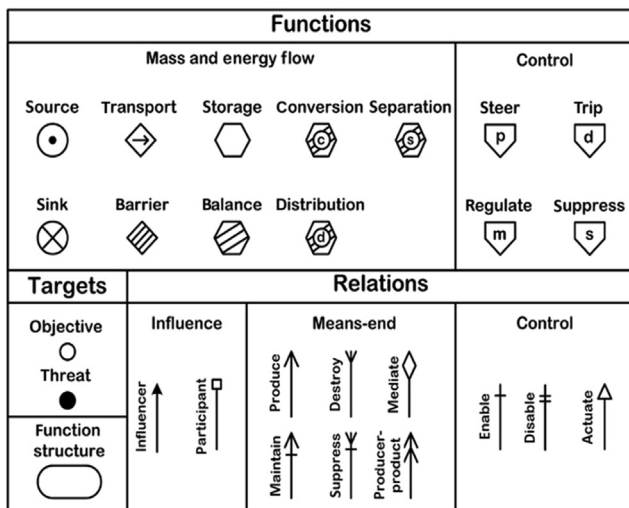


Fig. 1. Basic MFM symbols [1].  
MFM, multilevel flow modeling.

the corresponding control functions that form function structures. Accordingly, most of existing systems can be modeled easily and accurately without additional domain-specific assumptions or detailed knowledge.

- Qualitativeness: Since MFM is a qualitative modeling technique, a system can be modeled without detailed quantitative relations, and therefore the technique can be easily applied to most systems. However, the application of MFM would not be suitable if quantitative modeling is available or required.
- Model-based reasoning: Reasoning with MFM is based on pre-defined models. Once a model is established, additional empirical data are not considered during a reasoning process unless the model is revised.
- Snap-shot evidence and results: Since current MFM methods do not involve time-related concepts, they cannot consider time-related issues and accordingly cannot consider the dynamic features of the systems. During cause reasoning, it is not possible to aggregate serial observations, and therefore it is necessary to repeatedly conduct cause reasoning for every updated observation. During consequence reasoning, the occurrence of a specific event that will eventually occur can be inferred, but when the event will happen cannot be inferred.

Among these features, the fourth characteristic (snap-shot evidence and results) is regarded as one of the main drawbacks of MFM because, in many cases, time-related data such as the order of the event occurrence or the time gap between event occurrences are utilized as valuable evidence for cause and consequence reasoning.

To treat the dynamic features of a system by MFM, two kinds of approaches can be considered. The first approach is to combine a quantitative reasoning technique with MFM. As shown in several studies [10,11], it is proven that this approach can be applied well to consider dynamic features of a system. However, in order to apply this approach, it is essential to have detailed knowledge of a system (i.e., quantitative physical relations), and accordingly it is only possible for limited cases.

In contrast, the second approach is to extend the modeling capability of MFM. Because it is necessary to expand the modeling capability in a line that does not greatly impair MFM's own characteristics such as simplicity and qualitativeness, this approach has the disadvantage that it is forced to consider the dynamic feature in a much more restricted way than the first approach. Nonetheless, this approach will allow dynamic features to be considered for a wider range of cases, and thus MFM is enhanced according to this approach in this article.

If MFM can consider dynamic features, it is expected that more delicate cause reasoning is possible since time-related data can be utilized as additional evidence. Additionally, more detailed consequence reasoning would be possible, which includes information regarding when will the "event-of-interest" happens.

### 2.2. System failure model

A system failure model (tentative name) was suggested as a core concept of functional fault analysis (FFA). FFA is a systematic design methodology, in which the integration of system health management concepts into the early design stage of complex systems (such as spaceships) is based on a high-level functional model of the system that captures the physical architecture. Among the various concepts of an FFA, the system failure model was established to consider the propagation of effects of various failure modes and the timing by which the fault effects propagate along the modeled physical paths [12].

Accordingly, the system failure model involves various timing definitions. These timing definitions are represented in Fig. 2.

However, the system failure model was established specifically for case of spaceship (recommendation to abort, time to escape, etc.), and therefore minor modifications of the model should be applied to permit generality. The modified system failure model, which can be applied to general industrial systems, is represented in Fig. 3, and the corresponding timing definitions are as follows.

- TTE: The time from the “onset of failure” to the point when its effects are potentially detectable.
- TTD: The time from the “onset of failure” to the confirmation of fault existence. For cases in which failure is not detected, TTD is not definable.
- Time-to-diagnosis: The time from the “onset of failure” to the identification of the fault (e.g., fault location, fault type, etc.). For cases in which failure is not diagnosed, time-to-diagnosis is not definable.
- Time-to-mitigation: The time from the “onset of failure” to the complete prevention of critical system failure. For cases in which failure is not mitigated, time-to-mitigation is not definable.
- Time-to-criticality: The time from the “onset of failure” to critical system failure.

### 3. Application of TTD and TTE concepts to MFM

The concepts introduced from the modified system failure model were applied to the MFM method to ensure that MFM was capable of addressing time-related issues. In this section, the processes for the application of the TTD and TTE concepts are addressed, and enhanced reasoning based on these concepts is introduced.

#### 3.1. Modified definitions of TTD and TTE concepts from an MFM perspective

Multilevel flow model-based qualitative reasoning is conducted for cases in which one or more functions are not in normal states, which include failed states (i.e., the set of failed states is a proper subset of the set of not in normal states). Therefore, it is necessary to refine and redefine the timing definitions from the modified system failure model from an MFM perspective to adopt such concepts into the MFM.

Among the various timing definitions, only TTD and TTE are relevant to the MFM. The other timing definitions were filtered out because they are related to the processes of diagnosis and mitigation of failure, which are out of the MFM scope.

If there is a simple system with only two interconnected functions (function A and function B; function A affects function B) and the corresponding instrumentation systems (instrumentation system A for function A and instrumentation system B for function B), then this system can be represented as shown in Fig. 4. The MFM technique usually does not represent the instrumentation systems separately, but they are shown separately to provide better understanding.

If a state alteration in function A occurs and is detected by instrumentation system A, then the TTD for function A can be redefined as the time from the “actual state alteration of function A” to the “detection of the state alteration of function A”. Similarly, if a state alteration in function A occurs and it induces a state alteration in function B, then the TTE between function A and function B can be redefined as the time from the “actual state alteration of function A” to the point when it induces the “actual state alteration of function B”. Here, the word “actual” is used to distinguish the detection of the state alterations from the real state alterations.

These concepts can be applied to any multilevel flow models since every function node in a multilevel flow model with its corresponding instrumentation has its own TTD, and every influencing relation between function nodes in the model has its own TTE.

While introducing these concepts to MFM, it is implicitly assumed that the effect propagates to the subsequent component only after the state of the antecedent component is “sufficiently” changed, even in cases in which the multiple components are serially connected. This is because the conventional MFM represents the component states discretely rather than continuously, and it neglects small changes such as slight turbulence within certain levels of magnitude.

#### 3.2. Enhanced reasoning based on TTD and TTE concepts

For better understanding, it is convenient to start from cases in which the TTD and TTE values are fixed. In such cases, it is possible to deterministically infer which cause (among multiple cause suspects) actually induced the observed events during cause reasoning

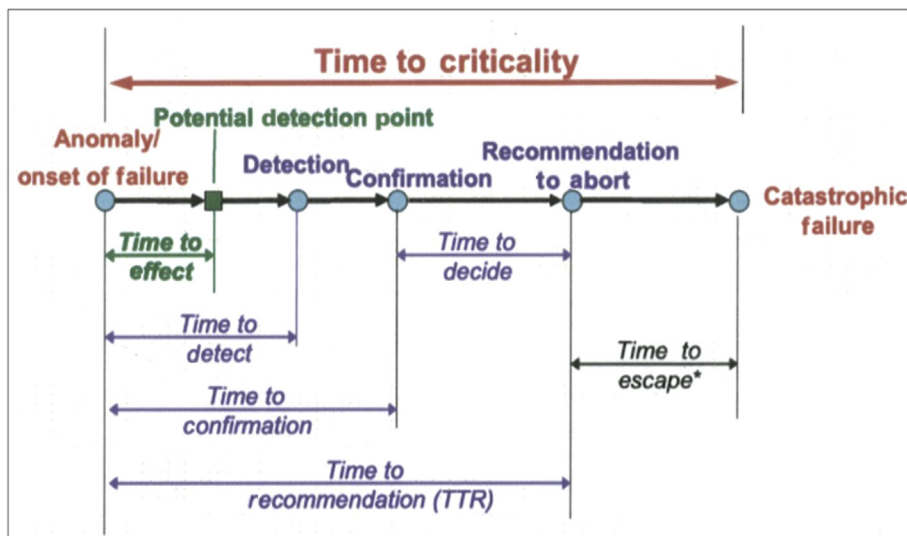


Fig. 2. Schematic of system failure model and its timing definitions [12].

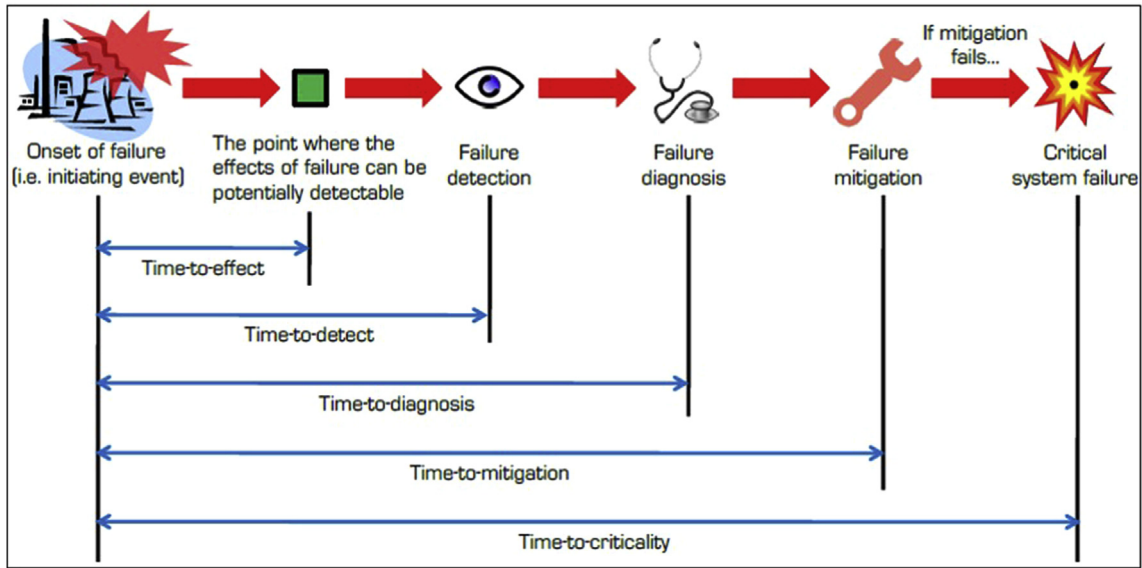


Fig. 3. Schematic of modified system failure model and its timing definitions.

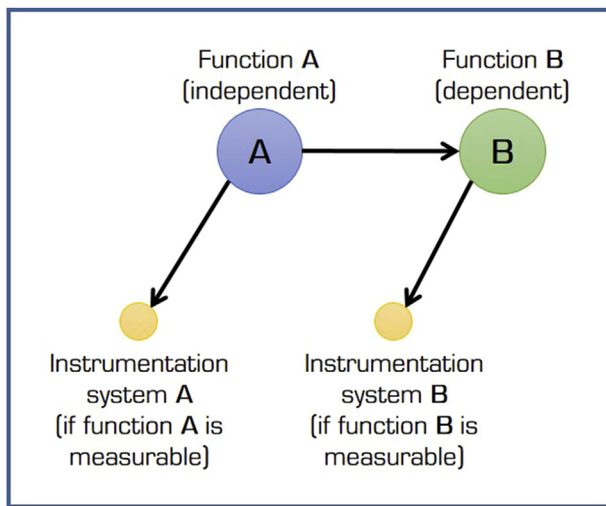


Fig. 4. Diagram of simple two-function system.

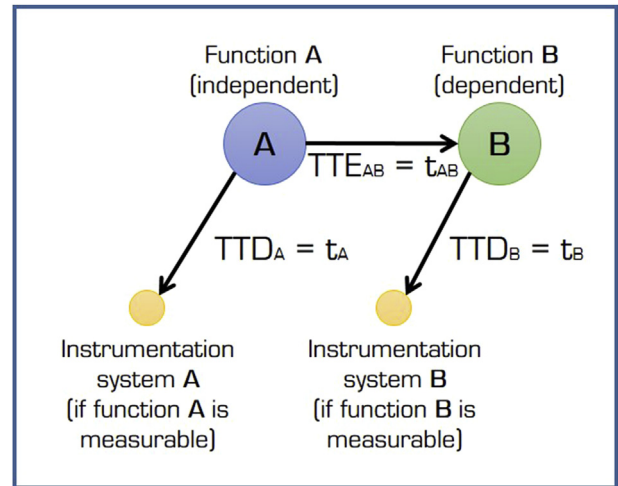


Fig. 5. Diagram of simple two-function system with corresponding TTDs and TTEs. TTD, time-to-detect; TTE, time-to-effect.

and the specific latent event's expected occurrence time during consequence reasoning.

As a brief illustration of an application of these concepts, assume that the TTD and TTE values for each function and corresponding instrumentation system (represented in Fig. 4.) are given as shown in Table 1 and Fig. 5.

If an alteration in the state of function A is detected at time  $t = 0$ , for given TTD and TTE values, then it can be inferred that the state of function A was actually altered at  $t = -t_A$ . The state of function B

will be altered at  $t = -t_A + t_{AB}$ , and the alteration in the state of function B will be detected at  $t = -t_A + t_{AB} + t_B$ .

These inferring processes can be easily applied to larger and more complex systems if the TTD and TTE values are properly determined. Additionally, the results from these inferring processes can be exploited for enhanced cause and consequence reasoning, which provides more information than that of conventional multilevel flow model-based reasoning.

In Sections 3.2.1 and 3.2.2, examples of enhanced cause and consequence reasoning are introduced.

3.2.1. Enhanced cause reasoning example

To demonstrate enhanced cause and consequence reasoning based on the TTD and TTE concepts, a case study for a simple water supply system was conducted.

The example water supply system consists of two domains: one energy (electricity) domain and one mass (water) domain. The objective of this system is to provide a sufficient water supply, and both domains should work properly to achieve this objective.

Table 1 Notation and value of each function.

Notation	Meaning	Value
$TTD_A$	TTD for function A	$t_A$
$TTD_B$	TTD for function B	$t_B$
$TTE_{AB}$	TTE from function A to function B	$t_{AB}$

TTD, time-to-detect; TTE time-to-effect.

In the energy domain, first, the inlet electricity is received from the external power grid and is transmitted through the power line. Then, the power distributor distributes the electricity to Pump 1 and Pump 2 in the mass domain through each power line. In the mass domain, first, the inlet water is received from the external water source and flows through Pump 1 to Water tank 1 and then through Pump 2 to Water tank 2. Then, part of the water in Water tank 2 is supplied, and the remainder of the water is discarded. A schematic of the example water supply system is provided in Fig. 6, and the corresponding multilevel flow model is represented in Fig. 7.

As an example of enhanced cause reasoning, assume that there is information regarding the TTD and TTE values as shown in Fig. 8 (the numbers near the arrows represent corresponding TTE values, and the numbers near the function symbols represent the corresponding TTD values). Notice that the instrumentation systems are sparse, which implies that only the water levels in Water tank 1 and Water tank 2 are detectable. Therefore, only two TTDs that correspond to Water tanks 1 and 2 exist, while TTEs still exist among every influence relation.

Suppose that water level reduction in Water tank 1 is detected. In this situation, there can be many types of cause suspects. To simplify the problem, only two cause suspects, including the inlet water flow rate reduction and power distributor malfunction are considered in this case study.

If the inlet water flow rate reduction is a root cause, the water level of Water tank 2 will be affected by the reduction in the level of Water tank 1 only. Therefore, a water level reduction in Water tank 2 will be detected relatively slowly. Specifically, if the inlet water flow rate reduction occurred at  $t = 0$ , then the

- water level reduction in Water tank 1 is detected at  $t = (2 + 10) + 1 = 13$
- water level reduction in Water tank 2 is observed at  $t = (2 + 10 + 6 + 15) + 1 = 34$

In this case, the time gap between the detection of the water level reduction in Water tanks 1 and 2 is 21 unit time, which means that the water level reduction in Water tank 2 will be detected after the 21st time unit of the detection of the water level reduction in Water tank 1.

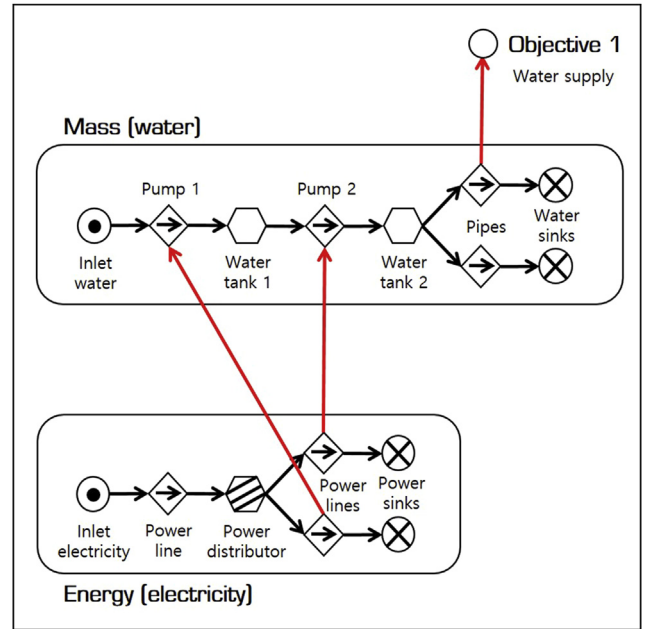


Fig. 7. Multilevel flow model of example water supply system.

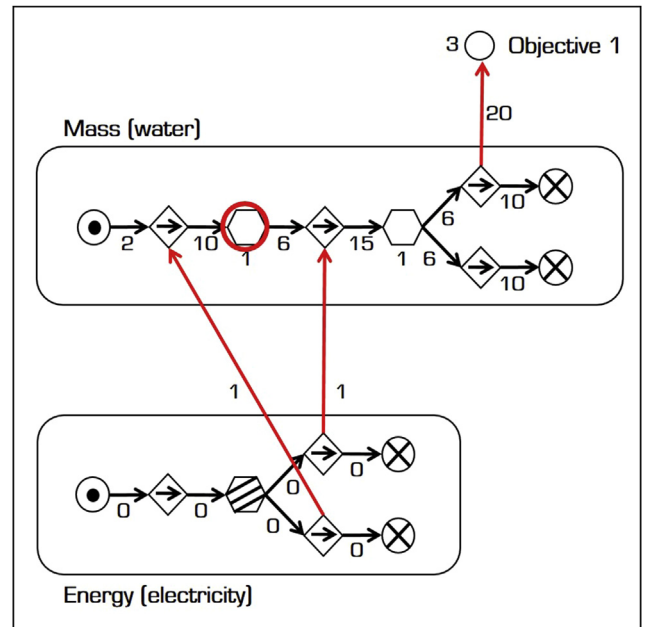


Fig. 8. Multilevel flow model of example water supply system with TTD and TTE values—enhanced cause reasoning example (red circle: observed point). TTD, time-to-detect; TTE time-to-effect. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

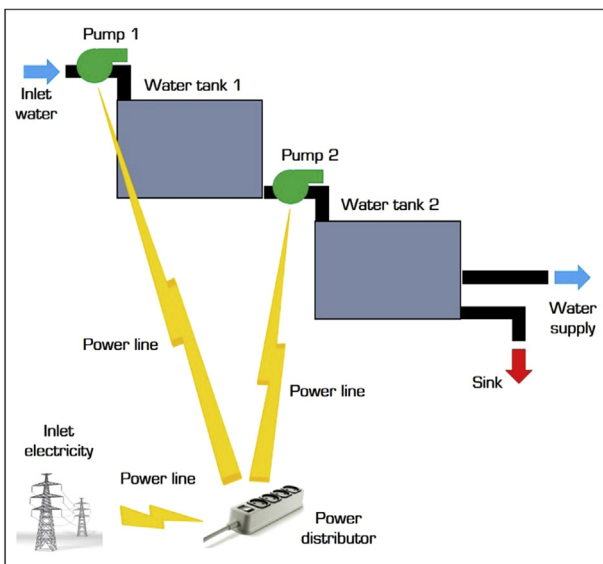


Fig. 6. Schematic of example water supply system.

If power distributor malfunction is a root cause, then the water level of Water tank 2 will be affected by both the Water tank 1 level reduction and the Pump 2 performance reduction. Therefore, the water level reduction in Water tank 2 will be detected relatively faster. Particularly, if a power distributor malfunction occurred at  $t = 0$ , then the

- water level reduction in Water tank 1 is detected at  $t = (1 + 10) + 1 = 12$

- water level reduction in Water tank 2 is detected at  $t = (1 + 15) + 1 = 17$

In this case, the time gap between detections of the water level reduction in Water tanks 1 and 2 is 5 unit time, which means that the water level reduction in Water tank 2 will be detected after the 5th time unit of the detection of the water level reduction in Water tank 1.

Therefore, if there is an additional water level reduction in Water tank 2, it can be inferred that it is the true root cause. Without the TTD and TTE concepts, conventional MFM cannot infer the true root cause between these two types of cause suspects, although the same evidence exists regarding the water level reduction in Water tanks 1 and 2.

### 3.2.2. Enhanced consequence reasoning example

As an example of enhanced consequence reasoning, suppose that a water level reduction in Water tank 2 is detected (see Fig. 9), it is obvious that the objective will eventually fail. However, when the TTD and TTE concepts are adopted, the model will infer not only how the objective's status will change but also when it will change.

If it is assumed that the water level reduction in Water tank 2 is detected at  $t = 0$ , then it can be inferred that the

- water level reduction in Water tank 2 actually occurred at  $t = 0 - 1 = -1$
- change in the objective's status actually occurred at  $t = -1 + 6 + 20 = 25$
- change in the objective's status will be detected at  $t = (-1 + 6 + 20) + 3 = 28$

In this case, the time gap between the detection of water level reduction in Water tank 2 and the failure of the objective is 38, which means that the water supply will become insufficient after the 38th time unit of the detection of the water level reduction in Water tank 2. Without the TTD and TTE concepts, conventional

MFM can provide information regarding “how” an objective's status will change but not “when” an objective's status will change.

### 3.3. Probabilistic reasoning

Practically, the TTD and TTE values should be represented as distributions rather than fixed values because they may include uncertainties, and the values can change because of many types of factors such as the degree of anomaly and the taken control actions. If reasoning processes are conducted based on these distributions, the corresponding cause and consequence reasoning results will become probabilistic. In detail, the probability for each cause suspect can be obtained during cause reasoning, and the probability of an occurrence of a specific event within a specific time can be obtained during consequence reasoning.

To conduct probabilistic reasoning based on the TTD and TTE distributions, it is necessary to consider the summation of two or more distributions. If it is assumed that all the TTD and TTE distributions are independent of each other, then this problem can be regarded as the summation of the distributions of independent random variables, which is solvable through a convolution operation.

The probability distribution of the sum of two or more independent random variables can be calculated by applying a convolution operator to the individual distributions. For continuously distributed random variables with probability density functions  $f$  and  $g$ , the general formula for the distribution of the sum  $Z = X + Y$  is as follows.

$$h(z) = (f * g)(z) = \int_{-\infty}^{\infty} f(z - t)g(t)dt \quad (1)$$

In Section 3.3.1, methods for the estimation of the TTD and TTE distributions are discussed. Then, in Sections 3.3.2 and 3.3.3, probabilistic cause and consequence reasoning based on the estimated TTD and TTE distributions are described.

#### 3.3.1. Estimation of the TTD and TTE distributions

Theoretically, the introduced concepts can be applied to general multilevel flow models without any difficulties. However, to apply these concepts for the solving of practical problems, it is essential to estimate the TTD and TTE distributions with proper accuracy and precision.

In the case of TTDs, most existing instrumentation systems are both theoretically and empirically well defined, and such instrumentation systems are applied to real-world systems. In this regard, issues related to the estimation of the TTD distribution were not considered in this article.

However, an estimation of TTEs is expected to be much harder than that of TTDs because most of the functions are serially connected and vary due to many types of factors, including state thresholds, input conditions, and causes of a single function's state alteration. Moreover, since MFM is not likely to be applied to systems that are well-understood for solving differential equations, analytical methods are not suitable for an estimation of a TTE distribution.

Alternatively, empirical approaches for the estimation of a TTE distribution can be considered. If the time gap can be measured between the original event and the latent event and the number of observations is sufficient, then the TTE distributions can be obtained through data aggregation methods. In the following sections, estimations of the TTE distributions based on a Bayesian update and a non-Bayesian probability distribution approximation algorithm are briefly introduced.

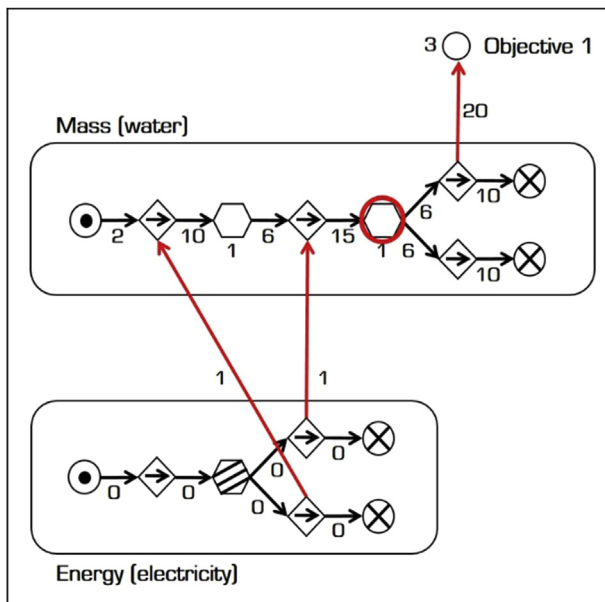


Fig. 9. Multilevel flow model of example water supply system with TTD and TTE values—enhanced consequence reasoning example (red circle: observed point). TTD, time-to-detect; TTE time-to-effect. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

### 3.3.1.1. Estimation of TTE distributions based on a Bayesian update.

A Bayesian update, also widely known as Bayesian inference, is a method of statistical inference that can be used to update the probability for a hypothesis based on newly obtained evidence. With its concrete mathematical background, namely the Bayes' theorem, the Bayesian update has served as a useful and reliable method for approximating the true distribution of a population from a sample.

If the hypothesis is represented as a probability distribution, it is necessary to define the form of the prior distribution and likelihood. If the event propagation from function A to function B is observed  $\mathbf{k}$  times, then the TTE distribution can be obtained through  $\mathbf{k}$  times of updates from the prior distribution.

Bayesian updating processes are highly affected by the forms of the prior distribution and likelihood. The beta distribution is widely used as the prior distribution and likelihood because it can approximate many other types of distributions.

However, not only the beta distribution but also many other types of commonly used distributions are unsuitable for representing multimodal distributions (distributions with multiple peaks). Accordingly, it is difficult to consider multimodal distributions with a Bayesian update although multimodal distributions frequently emerge in real-world data. Many studies have been conducted to solve multimodal problems in a Bayesian framework, but the investigations are still ongoing and not perfectly solved [13,14].

### 3.3.1.2. Estimation of TTE distributions based on non-Bayesian probability distribution approximation algorithm.

As mentioned, commonly used distributions for the Bayesian update are not suitable for representing multimodal distributions since it is difficult to represent various multimodal distributions in a general formula. Instead, multimodal distributions are expressed as linear combinations of multiple unimodal distributions (distributions with a single peak). Still, this does not change the fact that the Bayesian update is not good for approximating multimodal distributions.

Alternatively, studies regarding non-Bayesian approaches for the approximation of multimodal distributions have been actively conducted; these approaches do not require definitions regarding the forms of prior and posterior distributions [15–18]. Although these studies are still ongoing and it cannot be guaranteed that any of them will be applicable to any type of multimodal distribution, many non-Bayesian probability distribution approximation methods are more capable of multimodal distributions than is the Bayesian framework.

If an event propagation from function A to function B is observed  $\mathbf{k}$  times, then the TTE distribution can be obtained by simply merging all the evidence and applying an approximation algorithm. In general, to apply non-Bayesian probability distribution approximation algorithms, a relatively larger number of observations are needed because there is no prior information regarding the form of the distribution.

However, it is not necessary to choose only one approach for the TTE distribution estimation. A mixed approach can be considered that applies the Bayesian update when the amount of data is small and applies another method when the data are sufficiently collected. If it is expected that the target TTE distribution is unimodal, a continuous application of the Bayesian update can be considered.

Because empirical approaches are inevitably highly dependent on the observed or measured data and their uncertainty, the quality of the collected data should be sufficiently high, and data uncertainty should be precisely defined.

### 3.3.2. Probabilistic cause reasoning

To simplify the problem, assume that there are two event paths (cause suspects) that can affect both function A and function B. Because the two event paths involve different functions, their TTD and TTE profiles will also be different. In this case, the distribution of the time gap between the “detection of the state alteration of function A” and the “detection of the state alteration of function B” can be calculated for each event path through a series of convolution operations.

As an example, consider the problem introduced in Section 3.2.1 and define the event path that starts from the inlet water flow rate reduction to the water level reduction in Water tank 2 as event path 1, and the event path that starts from the power distributor malfunction to water level reduction in Water tank 2 as event path 2. A schematic of the MFM model of example water supply system and the two event paths is provided in Fig. 10.

If the TTD and TTE distributions for all involved relations are available, it is possible to represent the time gap between detections of the water level reduction in Water tanks 1 and 2 as a probabilistic distribution according to each event path, with a method similar to that introduced in section 3.2.1.

After the time-gap distribution for each event path is calculated and the actual time gap is observed, the probabilities of event occurrences due to *event path 1* and *event path 2* can be calculated. If the actual time gap is denoted as  $t_m$  and the time-gap distributions for event path 1 and event path 2 are denoted as  $pd_1$  and  $pd_2$ , respectively, then the probabilities of the event occurrences due to event path 1 ( $\mathbf{P}_1$ ) and event path 2 ( $\mathbf{P}_2$ ) can be represented as follows (see Fig. 11).

$$P_1(t = t_m) = \frac{pd_1(t_m)}{pd_1(t_m) + pd_2(t_m)} \quad (2)$$

$$P_2(t = t_m) = \frac{pd_2(t_m)}{pd_1(t_m) + pd_2(t_m)} \quad (3)$$

If the actual time gap is measured and corresponding probability density values of  $pd_1$  and  $pd_2$  are 0.1 and 0.05, respectively, it is possible to deduce that

- the probability of the event occurrence due to event path 1 is  $\mathbf{P}_1 = 0.1/(0.1 + 0.05) = 66.7\%$
- the probability of the event occurrence due to event path 2 is  $\mathbf{P}_2 = 0.05/(0.1 + 0.05) = 33.3\%$

This approach can be generalized for multiple (larger than two) event path cases. If there are  $\mathbf{n}$  possible event paths, then the probability of event occurrence due to event path  $\mathbf{x}$  ( $\mathbf{P}_x$ ) can be represented as follows

$$P_x(t = t_m) = \frac{pd_x(t_m)}{\sum_{k=1}^n pd_k(t_m)} \quad (4)$$

### 3.3.3. Probabilistic consequence reasoning

To simplify the problem, assume that the state alteration of function A is observed and can induce the state alteration of function B. If the TTD distributions for functions A and B are well defined and the TTE distributions between function A and function B are well defined, then the time-gap distribution between the “detection of the state alteration of function A” and “detection of the state alteration of function B” can be easily calculated through a series of convolution operations. The calculated time-gap distribution itself implies the probability of when the state alteration of

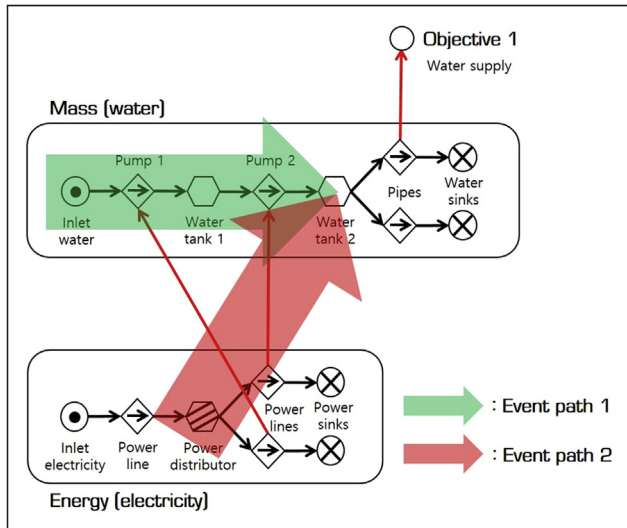


Fig. 10. Schematic of MFM model of example water supply system and two event paths. Multilevel flow modeling.

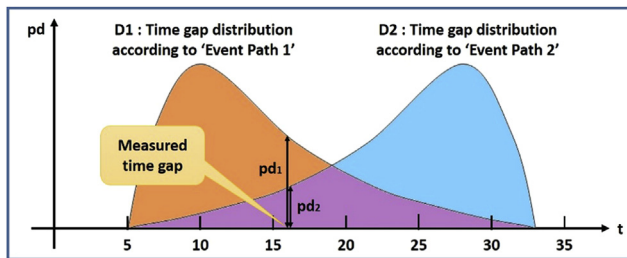


Fig. 11. Schematic of probabilistic cause reasoning.

function B will be detected, which is an example of advanced probabilistic consequence reasoning.

If new observations regarding the specific function's state alteration between function A and function B become available, the time of function B's state alteration detection can be predicted with a reduced uncertainty (i.e., narrower time-gap distribution).

#### 4. Discussion

The MFM's main characteristics such as simplicity and qualitiveness can be regarded as both advantageous and disadvantageous. From an applicability perspective, these characteristics are definitely advantageous. On the other hand, from the perspective of precision, these characteristics are disadvantageous. This is why the simplicity and qualitiveness of MFM are regarded as characteristics rather than advantages or disadvantages.

Thus, an improvement in the MFM should be conducted without harm to the model's characteristics. However, this is a dilemma because both advantages and disadvantages are based on the same characteristics, meaning that eliminating the disadvantages could induce the elimination of the advantages at the same time.

MFM can abstract various systems into general flows of mass and energy but cannot include their detailed physical properties. Accordingly, to impart quantitiveness to MFM, it is necessary to implement domain-specific information, which may harm the baseline characteristics of the model. Alternatively, to quantify the multilevel flow model and avoid serious harm to its characteristics, the only variable that can be applied equally to every system was quantified in this study, namely time.

Concepts for the quantification of time proposed in this study are similar to those in critical path method (CPM), which is an algorithm for scheduling of project activities suggested by Kelley and Walker [19]. However, there are several differences between CPM and the proposed concepts. First, although CPM was originally suggested as an algorithm for project scheduling and accordingly includes the concept of time, it has not been widely applied to the field of functional modeling of industrial systems due to its lack of simplicity in application to big projects or large systems and its lack of flexibility when considering various factors that affect time values. In contrast, MFM has been actively applied for functional modeling of various industrial systems; however, MFM itself does not include any quantitative elements. For this reason, this study tried to overcome the limitations of each method by introducing the TTD and TTE concepts into the framework of MFM, similar to the form of CPM.

Additionally, although the conventional CPM set the bounds for expected time to each element, it does not consider the detailed distributional factors. This is due to the only slight need to consider the "excessively detailed" time distribution since the CPM is usually applied for the scheduling of long-term projects. However, because the main purpose of this study is to conduct enhanced reasoning based on time-related evidence with relatively smaller time units, distributional factors of TTD and TTE were considered throughout the study. Moreover, Bayesian- and non-Bayesian-based methods for the estimation of corresponding distributions were suggested.

#### 5. Conclusion

In this article, concepts of TTD and TTE were adopted from the system failure model to provide dynamic feature capabilities to MFM. The system failure model was used along with the modified system failure model for applications related to general industrial systems; the definitions and the definitions of the TTD and TTE concepts were redefined from an MFM perspective. Additionally, enhanced reasoning based on these concepts was introduced, including both deterministic (when TTDs and TTEs are given as fixed values) and probabilistic (when TTDs and TTEs are given as distributions) cases with a simple case study on a water supply system.

It is expected that as a result of this study, because more evidences have become available while conducting the reasoning processes, the multilevel flow model's applicability to various systems will be enhanced compared to that of the conventional MFM. Especially, this type of enhancement can be emphasized more for sparse instrumentation systems, which implies that less data are available.

For future studies, it is necessary to conduct additional case studies for the examination of the practical applicability of the suggested concepts and methods. Furthermore, the continuous monitoring of TTE distribution estimation methods including both the Bayesian update and non-Bayesian methods should be conducted.

#### Conflict of interest

All authors have no conflicts of interest to declare.

#### Acknowledgments

This research was supported by the National R&D Program through the National Research Foundation of Korea (NRF) funded by the Korean Government. (MSIP: Ministry of Science, ICT and Future Planning) (No. NRF-2016R1A5A1013919).



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