Current Optics and Photonics
Vol. 2, No. 3, June 2018, pp. 280-285

ISSN: 2508-7266(Print) / ISSN: 2508-7274(Online)
DOI: https://doi.org/10.3807/COPP.2018.2.3.280

Design of a See-through Off-Axis Head-Mounted-Display Optical System with an Ellipsoidal Surface

Junhua Wang, Qing Zhou, Jie Chen, Lexin Hou, and Min Xu*

Shanghai Engineering Center of Ultra-precision Optical Manufacturing, Department of Optical Science and Engineering, Fudan University, Shanghai 200433, China

(Received April 27, 2018 : revised May 21, 2018 : accepted May 21, 2018)

A new method to design a see-through off-axis head-mounted-display (OA-HMD) optical system with an ellipsoidal surface is proposed, in which a tilted ellipsoidal surface is used as the combiner, which yields the benefits of easier fabrication and testing compared to a freeform surface. Moreover, we realize a coaxial structure in the relay lens group, which is simple and has looser tolerance requirements, thus making assembly easier. The OA-HMD optical system we realize has a simple structure and consists of a combiner and 7 pieces of coaxial relay lenses. It has a 48° × 36° field of view (FOV) and 12-mm exit pupil diameter.

Keywords : Head-Mounted-Display, Lens system design, Geometric optical design, Image analysis
OCIS codes : (220.3620) Lens system design; (080.2740) Geometric optical design; (110.2960) Image analysis

I. INTRODUCTION

In recent decades, many different types of HMD optical systems with high resolution and wide FOV design requirements have been reported [1-8]. Compared to other types of HMDs, OA-HMD optical systems can minimize the size of the system, inhibit stray light, and avoid ghost images, while having a wider FOV, large exit pupil diameter, and long pupil distance [9-11]. Figure 1 is the layout of a see-through OA-HMD optical system. We can see that an OA-HMD optical system usually consists of a combiner, an image source, and a relay lens group. The rays of light emitted by the image source form an intermediate image through the relay lens group, which is positioned in the eye's line of sight by reflection from the combiner, forming a virtual image in the far field. Through the combiner, the outside scene and the information generated by the computer can be simultaneously observed.

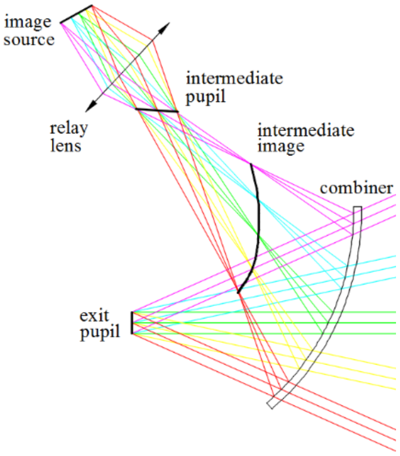


FIG. 1. Layout of the see-through OA-HMD optical system.

There are many designs using a freeform surface and noncoaxial relay lens groups to correct off-axis aberrations, and thus attain good imaging quality. Zhenrong Zheng *et al.* presented a design for a see-through OA-HMD optical system with 40° × 30° FOV and 70-mm eye relief by using a sphere combiner and a noncoaxial relay lens group with a freeform surface, in 2010 [9]. Junhua Wang *et al.* reported a see-through OA-HMD optical system with 40° × 30° FOV and 15-mm exit pupil diameter by using a

---

*Corresponding author: minx@fudan.edu.cn, ORCID 0000-0002-4901-0088

Color versions of one or more of the figures in this paper are available online.

TABLE 1. Specifications of the see-through OA-HMD optical system

Parameter	Specification
FOV	48° × 36°
Eye relief	> 60 mm
Exit pupil diameter	12 mm
Image quality	MTF > 0.5 at 30 lp/mm

7-piece coaxial relay lens group and a freeform surface combiner, in 2015 [10]. Lidong Wei *et al.* reported a see-through OA-HMD optical system with 24° × 18° FOV and 7-mm exit pupil diameter by using an ellipsoidal-surface combiner and noncoaxial relay lens group with a freeform surface, in 2018 [11]. However, it is difficult to fabricate and test a freeform surface, and the complex optomechanical structure of a noncoaxial relay lens group can make assembly more difficult.

A new method to design a see-through OA-HMD optical system with simple structure and wider FOV is proposed. We use a tilted ellipsoidal surface as the combiner, which is easier to fabricate and test compared to a freeform surface. Moreover, a coaxial structure is realized in the relay lens group. The coaxial structure is simple and has looser tolerance requirements, and thus makes the OA-HMD optical system easy to realize. Table 1 lists the main performance parameters of our proposed OA-HMD.

II. ABERRATION ANALYSIS AND CORRECTION METHOD OF THE OA-HMD OPTICAL SYSTEM

2.1. Aberration Analysis of the OA-HMD Optical System

OA-HMD optical systems have complex optical structure, severe aberrations, and low resolution, which limit their development and application [2]. Through the analysis of the optical structures of previous OA-HMD optical systems, the difficulty in correcting the off-axis aberrations is found to be due to mainly following problems: (1) the combiner has tilt and decentering, which gives the aberration field of the optical system an inconsistent distribution, producing many off-axis aberrations and introducing off-axis aberration that is difficult to corrected by the relay lens group; (2) the tilted and curved intermediate image is difficult to corrected by the relay lens group; and (3) the lateral chromatic aberration is difficult to corrected by the relay lens group. The reason for these problems is not using the initial structure of the combiner and the relay lens to balance the aberration.

2.2. Correction Method of the OA-HMD Optical System

To solve the above problems, the initial layout of an OA-HMD optical system must offer the following correction features: (1) reach the “pupil-stop conjugation”, to solve the inconsistent distribution of the aberration field, reduce the off-axis aberration, and improve the off-axis aberration correction by the relay lens group [10, 12]; (2) make the relay lens group meet the Scheimpflug image condition to correct the tilted intermediate image, and introduce a field curve to correct the curved intermediate image; and (3) have an intermediate pupil located near the front surface of the relay lens group, which can correct lateral chromatic aberration.

III. DESIGN PROCESS

3.1. Combiner Design

To achieve wide FOV and good imaging ability, many OA-HMD optical systems used a freeform surface as combiner. However, it is difficult to fabricate and test a freeform surface; thus we want to search for a quadric surface to substitute for the freeform surface. The ellipsoid is a type of quadric surface that has following two advantages: (1) the ellipsoidal surface is easier to fabricate and test than the freeform surface, and (2) the optical paths are equal between the foci of the ellipsoid, which means rays emerging from one focus must converge at the other after reflection by the ellipsoidal surface. Thus, the foci of the ellipsoid are aberration-free. When the exit pupil is located at a focal point, the chief rays will converge at the other focal point after reflection by the tilted ellipsoidal surface, without introducing any off-axis aberrations, so using a tilted ellipsoidal surface can achieve the “pupil-stop conjugation” requirement. However, a tilted ellipsoidal combiner introduces several critical off-axis aberrations, which must be balanced by the relay lens group to meet the resolution requirement.

To render the aberration field produced by the combiner uniformly distributed, the relay lens group is considered as an ideal lens when solving for the ellipsoidal surface’s parameters. The spot-diagram distribution of an optical system composed of a combiner and an ideal lens is used to evaluate whether the ellipsoidal surface’s parameters meet the criteria that the spot diagram is uniformly distributed on the aberration field, and that the spot-diagram RMS radius is as small as possible. The surface sag of the ellipsoid is given by Eq. (1). The initial parameters for the final ellipsoidal combiner are $K = -0.16$ and $R = -100$, and the rotation angle is 75°.

$$z = \frac{cr^2}{1+\sqrt{1+(1+\mathrm{k})\mathrm{c}^2 r^2}}, \text{where} \qquad (1)$$
$$r = \sqrt{x^2+y^2}.$$

Here c is the curvature (reciprocal of the radius) and k is the conic constant, $-1 < k < 0$.

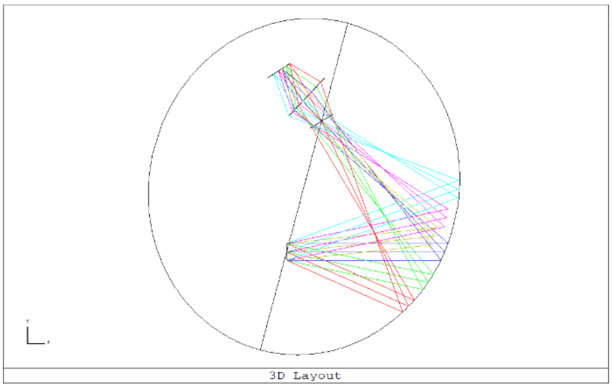
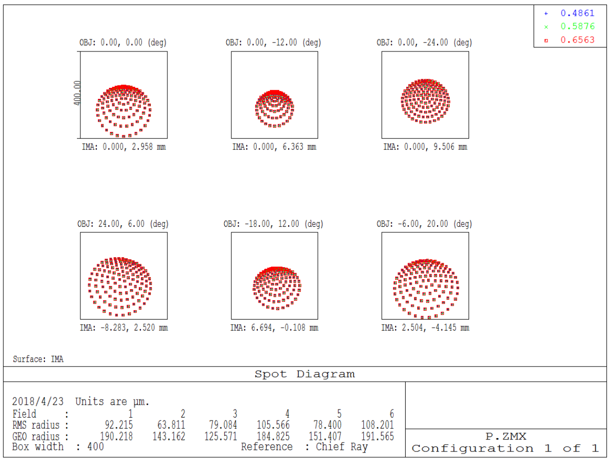


FIG. 2. Layout of the optical system composed of combiner and ideal lens.



```
2018/4/23  Units are µm.
Field      :        1         2         3         4         5         6
RMS radius :   92.215    63.811    79.084   105.566    78.400   108.201
GEO radius :  190.218   143.162   125.571   184.825   151.407   191.565
Box width  : 400                       Reference  : Chief Ray
```

FIG. 3. Spot diagram of the optical system composed of combiner and ideal lens.

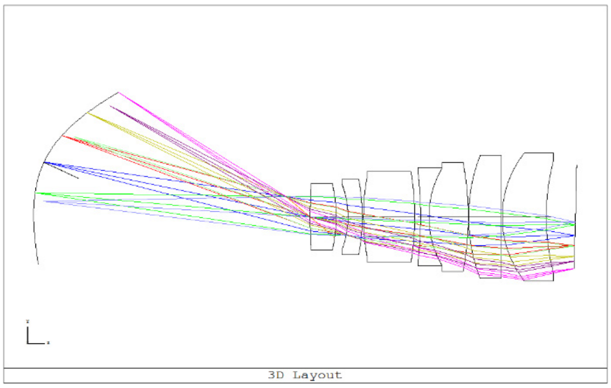
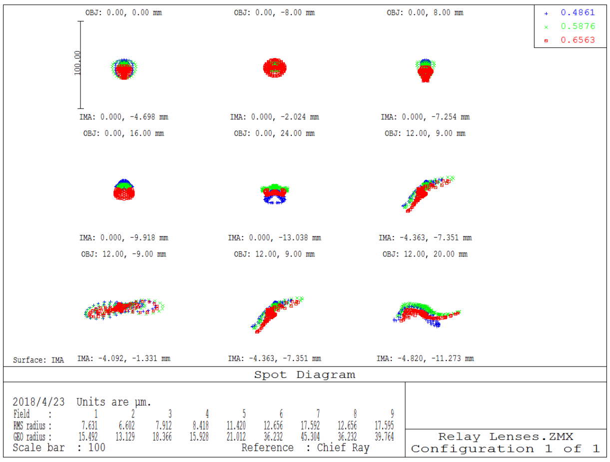


FIG. 4. Initial structure of the relay lens group.



```
2018/4/23  Units are µm.
Field      :      1       2       3       4       5       6       7       8       9
RMS radius :  7.631   6.602   7.912   8.418  11.420  12.656  17.592  12.656  17.595
GEO radius : 15.492  13.129  18.366  15.928  21.012  36.232  45.304  36.232  39.764
Scale bar  : 100                         Reference  : Chief Ray
```

FIG. 5. Spot diagram of the relay lens group.

As shown in Fig. 2, the rays of light are reflected by the combiner to form a curved and titled intermediate image, together with a tilted intermediate pupil. As shown in Fig. 3, spot diagrams are uniformly distributed on the aberration field, which is favorable for the correction of off-axis aberrations.

3.2. Relay Lens Group Design

In an OA-HMD optical system, the intermediate image formed by rays of light reflected from the ellipsoidal combiner is tilted and curved, and must be corrected by the relay lens group to meet the imaging requirements. An OA-HMD optical system has stringent requirements for the initial structure of relay lens group; otherwise, rays of light will spill out when matched with the combiner, and the optimization cannot be performed. Therefore, we need the initial structure of the relay lens group to be sufficiently good. The initial structure of the relay lens group should satisfy the Scheimpflug condition to correct the tilted intermediate image, and the field curvature introduced can correct the curved intermediate image. At the same time, when solving the initial structure of the relay lens group, the pupil matching, object-image matching, and FOV matching of the relay lens group should be considered together with the ellipsoidal combiner, which avoids rays of light spilling out, and is beneficial to the optimization of the optical system.

The ellipsoidal combiner satisfies a “pupil-stop conjugate” requirement between the stop (exit pupil) and intermediate pupil, so that the aberration field is uniformly distributed, which reduces the generation of aberration and enables a simplified structure of the relay lens group. By analyzing the intermediate image and the intermediate pupil parameters, a relay lens group consisting of 7 pieces of coaxial relay lenses can correct tilted and curved intermediate images. When solving for the initial structure of the relay lens group, the evaluation criteria are that the spot diagram must be uniformly distributed on the aberration field, and the spot diagram’s RMS radius must be as small as possible.

Figure 4 illustrates the initial structure of the relay lens group, which consists of 7 pieces of coaxial spherical lenses, with tilt and decentering about the optical axis. The image plane also has a small tilt about the optical axis. As shown in Fig. 5, the RMS radius of the spot diagram is about 12 µm, and the aberration of the initial structure of the relay lens group is corrected.

3.3. The Initial Structure of the OA-HMD

Figure 6 illustrates the initial layout of the OA-HMD optical system, which consists of an ellipsoidal combiner and a relay lens group, with an exit pupil diameter of 8 mm and 48° × 36° FOV. As shown in Fig. 7, the RMS radius of the spot diagram is about 40 μm and the imaging quality is low, so the optical system needs to be optimized to meet the imaging requirements.

IV. OPTIMIZATION

The 8-mm exit pupil diameter does not meet the design requirements in the initial structure of the OA-HMD optical system. Therefore, the purpose of the pre-optimization is to optimize the optical system to meet the design requirements of the exit pupil diameter. In the optimization process, the tilt and decentering of the relay lens group are set as variables, together with the curvature and interval of the lens, so that the exit pupil diameter meets the design requirements. The purpose of post-optimization is to improve the imaging quality of the optical system. In practical application, the diameter of the human eye's pupil is approximately 3 mm [13]. As shown in Fig. 8, nine 3-mm subpupils were selected within the 12-mm exit pupil diameter, for optimal design and image-quality analysis. In the optimization process, all of the parameters of the OA-HMD optical system are set as variables, so that the OA-HMD optical system can meet the image-quality requirements.

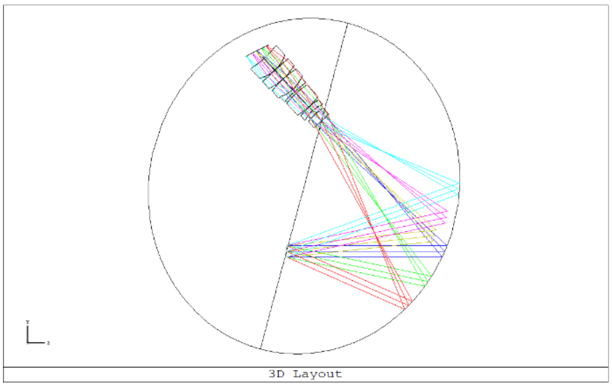
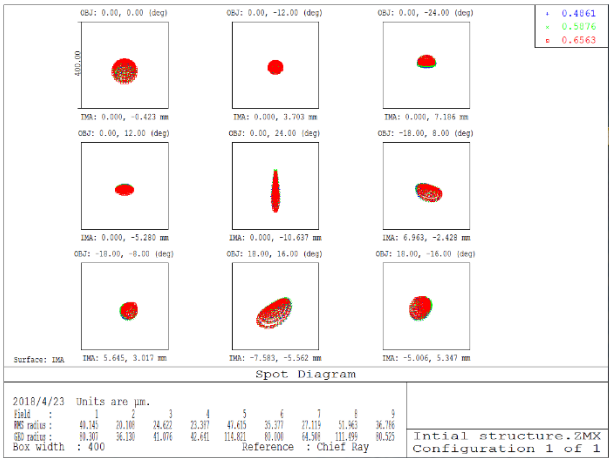


FIG. 6. Initial structure of the OA-HMD optical system.



```
2018/4/23  Units are µm.
Field      :      1       2       3       4       5       6       7       8       9
RMS radius :  40.145  20.108  24.622  23.387  47.615  35.377  27.119  51.963  36.786
GEO radius :  80.307  36.130  41.076  42.641 114.821  80.000  64.508 111.499  80.525
Box width  : 400                      Reference  : Chief Ray
```

```
Intial structure.ZMX
Configuration 1 of 1
```

FIG. 7. Spot diagram of the initial OA-HMD optical system.

V. DESIGN RESULTS

5.1. The Ultimate Structure

Figure 9 shows the ultimate structure of the see-through OA-HMD optical system, which consists of an ellipsoidal combiner and 7 pieces of coaxial relay lenses. Obviously, this simple structure has many advantages. The coaxial structure of the relay lens group makes the mechanical design and assembly easier, compared to a noncoaxial

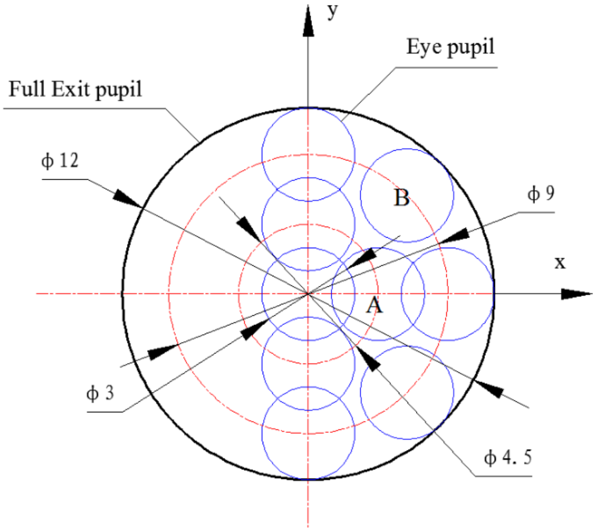


FIG. 8. Evaluation points in the 12-mm exit pupil.

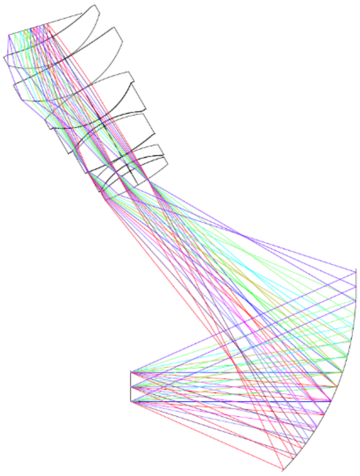


FIG. 9. Ultimate structure of the OA-HMD optical system.

relay lens group. Moreover, the ellipsoidal surface can be fabricated and tested easily.

5.2. Imaging-quality Analysis

Figure 10 shows the spot diagram for the OA-HMD optical system at different sub-pupils. It can be seen from the figure that the RMS radius of the spot diagram is about 5 μm, which is much smaller than an image source pixel. Figure 11 shows the best MTF curves among the different 3-mm-diameter evaluation points for which the MTF curves are greater than 0.7 at 30 lp/mm, corresponding to area A in Fig. 8. Shown in Fig. 12 are the worst MTF curves among the different 3-mmdiameter evaluation points, for which the MTF curves are greater than 0.7 at 30 lp/mm, corresponding to area B in Fig. 7. The average MTF is greater than 0.5 at 30 lp/mm. As shown in Fig. 13, the distortion is about -19.56%, while the FOV is 48° × 36°. The distortion is mainly caused by tilt of the combiner and is difficult to correct by optical methods. Correction of the distortion by digital-domain methods [8] is strongly recommended.

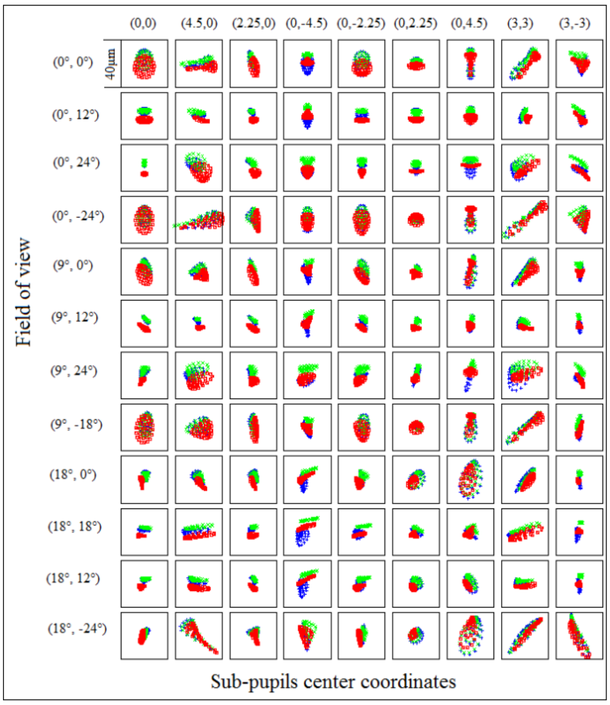


FIG. 10. Spot diagram of the OA-HMD optical system at different sub-pupils.

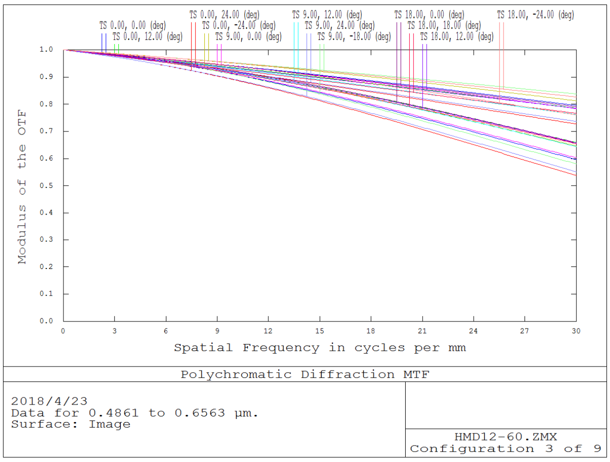


FIG. 11. MTF curves of the OA-HMD optical system at evaluation point A.

5.3. Tolerance Analysis

Tolerance analysis is a tradeoff among the performance requirements of an optical system, processing costs of the optical elements, and assembly costs of an optical system, thereby determining its processing parameters and assembly requirements [10]. The tolerance analysis of the OA-HMD optical system uses 1000 Monte Carlo simulations to analyze the MTF value of the 3-mm subpupils, and uses the normal distribution to analyze the MTF. Table 2 shows

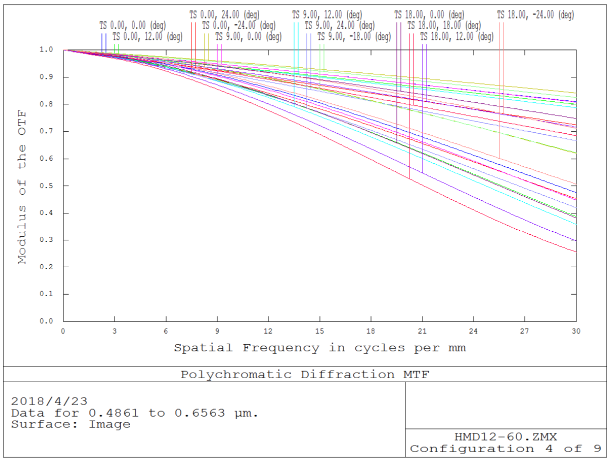


FIG. 12. MTF curves of the OA-HMD optical system at evaluation point B.

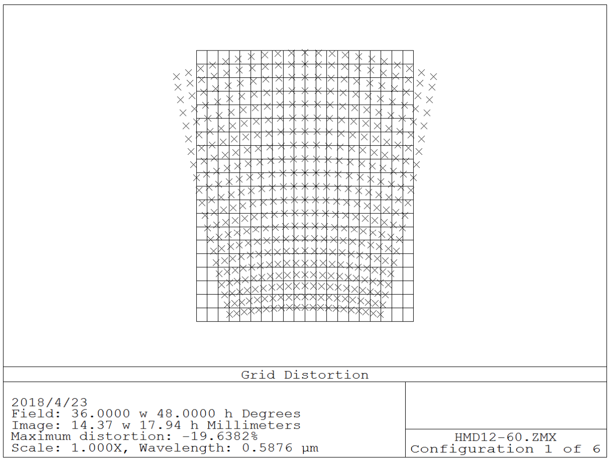


FIG. 13. The grid distortion of the OA-HMD optical system.

TABLE 2. Tolerance Values of the OA-HMD optical system

Parameter	Value
Radius (fringes)	±3
S+A irregularity (fringes)	±0.5
Thickness (mm)	±0.05
Surface decentering (mm)	±0.03
Surface tilt (′)	±1
Element decentering (mm)	±0.03
Element tilt (′)	±1
Glass material (n_d, v_d)	±0.0005, ±0.5%

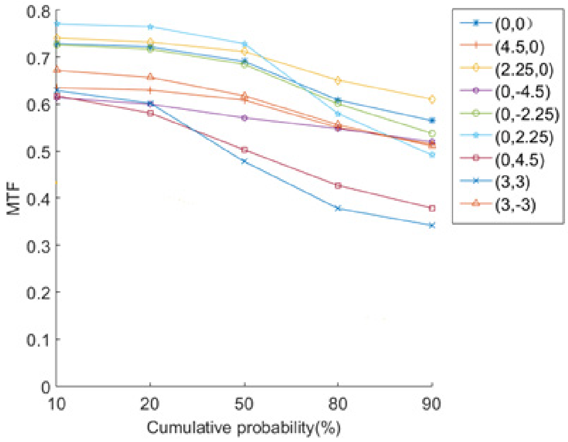


FIG. 14. The possible values of MTF curves at different evaluation points.

the tolerance value of the OA-HMD optical system calculated by data from the Zemax software. It is obvious that the optical system has a loose tolerance and can be assembled easily.

Figure 14 shows the cumulative probabilities for different MTF values under the conditions of Table 2. There is an 80% probability that the MTF is higher than 0.5 at 30 lp/mm. We can see that the OA-HMD optical system has a loose tolerance and can be easily fabricated.

VI. CONCLUSIONS

A new method to design an OA-HMD with wider FOV is proposed. We use an ellipsoidal combiner to realize the “pupil-stop conjugation” requirement, which can make the aberration field of the optical system uniformly distributed, reducing transverse aberration and simplifying the optical structure of the relay lens group. The OA-HMD optical system we have realized includes an ellipsoidal combiner and 7 pieces of coaxial relay lenses, with a 48° × 36° wide FOV and 12-mm exit pupil diameter. Moreover, the ellipsoidal combiner can be easily fabricated and tested, the coaxial structure of relay lenses can be easily assembled, and the OA-HMD optical system has a loose tolerance and can be easily fabricated.

ACKNOWLEDGMENT

This study was supported by the National Key Research and Development Program of China (2017YFB1104700) and the Science Challenging Program (JCKY2016212A506-0106).

REFERENCES

1. D. W. Cheng, Y. T. Wang, H. Hua, and M. M. Talha, “Design of an optical see-through head-mounted display with a low f-number and large field of view using a freeform prism,” Appl. Opt. **48**, 2655-2668 (2009).
2. J. P. Rolland, “Wide-angle, off-axis, see-through head-mounted display,” Opt. Eng. **39**, 1760-1767 (2000).
3. J. M. Yang, W. Q. Liu, W. Z. Lv, D. L. Zhang, F. He, Z. L. Wei, and Y. Kang, “Method of achieving a wide field-of-view head-mounted display with small distortion,” Opt. Lett. **38**, 2035-2037 (2013).
4. J. M. Yang, P. Twardowski, P. Gérard, and J. Fontaine, “Design of a large field-of-view see-through near to eye display with two geometrical waveguides,” Opt. Lett. **41**, 5426-5429 (2016).
5. J. H. Wang, Y. C. Liang, and M. Xu, “Design of a low distortion head-mounted display with freeform reflective mirror based on two ellipsoids structure,” J. Opt. Soc. Korea **20**, 234-238 (2016).
6. H. J. Choi, J. M. Ryu, and J. H. Kim, “Tolerance analysis of focus-adjustable head-mounted displays,” Curr. Opt. Photon. **1**, 474-490 (2017).
7. B. Chen and A. M. Herkommer, “Alternate optical designs for head-mounted displays with a wide field of view,” Appl. Opt. **56**, 901-906 (2017).
8. M. K. Tehrani and S. S. M. Fard, “Design of diffraction limited head mounted display optical system based on high efficiency diffractive elements,” Curr. Opt. Photon. **1**, 150-156 (2017).
9. Z. R. Zheng, X. Liu, H. F. Li, and L. Xu, “Design and fabrication of an off-axis see-through head-mounted display with an x-y polynomial surface,” Appl. Opt. **49**, 3661-3668 (2010).
10. J. H. Wang, Y. C. Liang, and M. Xu, “Design of a see-through head-mounted display with a freeform surface,” J. Opt. Soc. Korea **19**, 614-618 (2015).
11. L. D. Wei, Y. C. Li, J. J. Jing, L. Feng, and J. S. Zhou, “Design and fabrication of a compact off-axis see-through head-mounted display using a freeform surface,” Opt. Express **26**, 8550-8565 (2018).
12. J. H. Wang, Y. C. Liang, and M. Xu, “Design of panoramic lens based on ogive and aspheric surface,” Opt. Express **23**, 19489-19499 (2015).
13. H. C. Peng, D. W. Cheng, J. Han, C. Xu, W. T. Song, L. Z. Ha, J. Yang, Q. X. Hu, and Y. T. Wang, “Design and fabrication of a holographic head-up display with asymmetric field of view,” Appl. Opt. **53**, H177-H185 (2014).