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MATHEMATICAL MODELING FOR THE OBESITY DYNAMICS WITH PSYCHOLOGICAL AND SOCIAL FACTORS

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Abstract. We develop a mathematical model for the obesity dynamics to investigate the long term obesity trend with the consideration of psychological and social factors due to the increasing prevalence of obesity around the world. Many mathematical models for obesity dynamics adopted the modeling idea of infectious disease and treated overweight and obese people infectious and spreading obesity to normal weight. However, this modeling idea is not proper in obesity modeling because obesity is not an infectious disease. In fact, weight gain and loss are related to social interactions among different weight groups not only in the direction from overweight/obese to normal weight but also the other way around.

Thus, we consider these aspects in our model and implement personal weight gain feature, a psychological factor such as body image dissatisfaction, and social interactions such as positive support on weight loss and negative criticism on weight status from various weight groups. We show that the equilibrium point with no normal weight population will be unstable and that an equilibrium point with positive normal weight population should have all other components positive. We conduct computer simulations on Korean demography data with our model and demonstrate the long term obesity trend of Korean male as an example of the use of our model.

1. Introduction

The prevalence of overweight, obesity and the body image dissatisfaction is increasing at an alarming rate in many developed and even developing countries in the world [1]. Body image dissatisfaction is defined as subjective negative evaluation of one's physical appearance [2]. In fact, pursuing a thin body or masculine body is getting more popular among young people in many countries [1, 3]. Hence, whether the subject populations are children/adolescents

aged 8 to 18 or young adults aged over 20, these studies show some association between obesity and body image dissatisfaction, low self-esteem, depression or abnormal eating behavior. These results imply that the body image and obesity induced psychological problems such as abnormal eating behavior or depression, which could grow to larger health problems in the future.

There are many studies modeling obesity dynamcis [4, 5, 6, 7]. However, most of these studies treated the overweight and obese population classes with epidemiological characteristics such that the overweight and obese populations are socially contagious and hence infect normal weight people to gain weight. However, people can gain weight with any personal reasons and not just through a relationship with overweight and obese people. Moreover, from the previous studies related to body image dissatisfaction and obesity [4, 8, 9] people can gain or lose weight via social interactions such as positive support on weight loss and negative criticism about weight status delivered from normal weight and overweight people, respectively. These social interactions can lead people to lose weight successfully or sometime to development of abnormal eating behavior. Thus, the social influence on weight loss or gain can occur not only in the direction from overweight/obese classes to normal weight class but also the other way around from normal weight class to overweight/obese classes. Thus, we develop a mathematical model that can describe the social interactions among different weight classes to analyze the relationship between obesity and body image dissatisfaction and their influence to each other. We conduct computer simulations to show the long term trend of obesity of Korean male.

The organization of our paper is as follows. We formulate our model with assumptions in Sec. 2. Then, we show the analysis of our model in Sec. 3, and we conduct computer simulations with Korean male demography data in Sec. 4. Finally, we discuss our result and future work in Sec. 5.

2. Model Formulation and Assumptions

We consider three weight groups such as normal weight, overweight and obese groups according to body mass index (BMI) [10] and abnormal eating behavior with body image dissatisfaction and social interactions such as positive support on weight loss and negative criticism on weight status in our model. We use a compartment model that divides a whole population into subgroups called *compartments*. For our model, we consider eight subpopulation groups (compartments) such that

- N_w is individuals classified as normal weight ($18.5 \leq \text{BMI} < 25$) without both abnormal eating behavior and body image dissatisfaction;
- N_w^{ds} is individuals classified as normal weight with body image dissatisfaction but without abnormal eating behavior;
- O_w is Individuals classified as overweight ($25 \leq \text{BMI} < 30$) without both abnormal eating behavior and body image dissatisfaction;

- O_w^{ds} is individuals classified as overweight with body image dissatisfaction but without abnormal eating behavior;
- O_b is individuals classified as obese ($30 \leq$ BMI) without abnormal eating behavior;
- AE_n, AE_w, and AE_b, are normal weight people, overweight people and obese people with abnormal eating behavior, respectively.

For the model we have the following assumptions.

(A1) Teasing, peer pressure, criticism on weight from family members and friends are considered negative social interactions which play a role in weight gain or loss;

(A2) Individuals can gain weight with any reasons including malnutrition, physical inactivity and social influences as mentioned;

(A3) Abnormal eating behavior of a normal weight person means extreme diet (almost no eating). For an overweight or obese people, abnormal eating behavior means either extreme diet or eating too much unconsciously (called binge eating);

(A4) Whenever people (overweight or obese classes) are on a diet, the weight loss maintenance period may be relatively short. Hence, these people can gain more weight or develop abnormal eating behavior later;

(A5) Normal weight people with the driving for thinness or masculinity can be considered as having body image dissatisfaction;

(A6) Overweight individuals can face a pressure from family and peers for weight loss which will make them unsatisfied with their body image, and will move to the overweight class with body image dissatisfaction class (O_w^{ds}). Also, if they try unhealthy ways to lose weight (if they fail weight loss), they may develop abnormal eating behavior. As a result, they can either gain more weight and hence move to the obese class (O_b). On the other hand, overweight people with body image dissatisfaction can follow a healthy diet with positive support from their family members and peers. Once, they succeed in losing enough weight, they can move to the normal weight class (N_w).

(A7) Positive support from family and peers will affect weight loss process for both overweight and obese classes and help those to have healthy diet.

(A8) Abnormal eating behavior can be developed by psychological factors, emotions, peer pressure and life style, etc.

(A9) People with abnormal eating behavior can recover naturally without external help.

Then, the dynamics of the eight compartments is governed by

$$\frac{dN_w}{dt} = \frac{br}{c}H(c-H) - \delta_w N_w - e_N N_w + r_1 AE_n$$
$$+ l_w O_w(\frac{sN_w}{H}) + l_w O_w^{ds}(\frac{sN_w}{H}) - \mu N_w$$

$$\frac{dN_w^{ds}}{dt} = e_N N_w - q N_w^{ds} - \delta_w N_w^{ds} + \tilde{r}_1 AE_n - \mu N_w^{ds}$$

$$\frac{dO_w}{dt} = \delta_w N_w - \delta_b O_w - e_O O_w - O_w(\frac{pN_w + \tilde{p}N_w^{ds}}{H}) + r_2 AE_w$$
$$+ l_b O_b(\frac{sN_w + \beta O_w}{H}) - l_w O_w(\frac{sN_w}{H}) - \mu O_w$$

$$\frac{dO_w^{ds}}{dt} = e_O O_w + \delta_w N_w^{ds} + O_w(\frac{pN_w + \tilde{p}N_w^{ds}}{H})$$
$$- l_w O_w^{ds}(\frac{sN_w}{H}) - b O_w^{ds} + \tilde{r}_2 AE_w - k O_w^{ds} - \mu O_w^{ds}$$

$$\frac{dO_b}{dt} = \delta_b O_w + b O_w^{ds} - l_b O_b(\frac{sN_w + \beta O_w}{H})$$
$$- \tilde{l}_b O_b(\frac{pN_w + \alpha O_w + \tilde{p}N_w^{ds} + \tilde{\alpha}O_w^{ds}}{H}) + r_3 AE_b - \mu O_b$$

$$\frac{dAE_n}{dt} = q N_w^{ds} - (r_1 + \tilde{r}_1)AE_n - \mu AE_n$$

$$\frac{dAE_w}{dt} = k O_w^{ds} - (r_2 + \tilde{r}_2)AE_w - \mu AE_w$$

$$\frac{dAE_b}{dt} = \tilde{l}_b O_b(\frac{pN_w + \alpha O_w + \tilde{p}N_w^{ds} + \tilde{\alpha}O_w^{ds}}{H}) - r_3 AE_b - \mu AE_b, \qquad (1)$$

where $H = N_w + N_w^{ds} + O_w + O_w^{ds} + O_b + AE_n + AE_w + AE_b$ is the total population and

- br is the birth rate, μ is the death rate, and c is a carrying capacity;
- q is the proportion of normal weight people with body image dissatisfaction (N_w^{ds}) who develop abnormal eating behavior while trying a diet;
- k is the proportion of overweight people with body image dissatisfaction (O_w^{ds}) who develop abnormal abnormal eating behavior while trying a diet;
- b is the proportion of overweight people with body image dissatisfaction (O_w^{ds}) who fail diet and gain more weight;

- δ_w and δ_b are the progression rates from normal weight (N_w) to overweight (O_w) and from overweight (O_w) to obese (O_b) classes by gaining weight, respectively;
- e_N and e_O are the rates of driving for thinness or masculinity and media impact for a thin or masculine body desire of the normal weight and overweight classes, respectively;
- l_w and l_b are rates of losing weight of the overweight and obese classes, respectively;
- $\tilde{l}_b$ is the proportion of obese people (O_b) who are pressured to do diet by family and peers but develop abnormal eating behavior as a result of failing diet;
- s and β are the rate of positive support of normal weight and overweight people on weight loss, respectively;
- p, α, $\tilde{p}$, and $\tilde{\alpha}$ are rates of weight loss pressure (teasing / comparing / peer pressure) of the normal weight, the overweight, the normal weight with body image dissatisfaction, the overweight with body image dissatisfaction, respectively;
- r_1, $\tilde{r}_1$, r_2, $\tilde{r}_2$, and r_3 are natural recovery rates from abnormal eating behavior of the normal weight, the normal weight with body image dissatisfaction, the overweight, the overweight with body image dissatisfaction and the obese, respectively.

Figure 1 is the conceptual diagram for our model.

3. Analysis of Model

Note that all variables remain nonnegative for $t \geq 0$, i.e. $\dfrac{dK}{dt} > 0$ if $K = 0$, where $K \in \{N_w, N_w^{ds}, O_w, O_w^{ds}, O_b, AE_n, AE_w, AE_b\}$. Also, by adding all equations in Eq (1) we have

$$\frac{dH}{dt} = (br - \mu)H - \frac{br}{c}H^2, \tag{2}$$

where $H = N_w + N_w^{ds} + O_w + O_w^{ds} + O_b + AE_n + AE_w + AE_b$ is the total population, and br and μ are birth and death rates, respectively. Then, we have the following:

Theorem 3.1. *The total population $H(t)$ stays nonnegative and satisfies*

$$H(t) \leq \max\{H(0), \frac{c(br - \mu)}{br}\} \tag{3}$$

for $t \geq 0$, where c is the carrying capacity and br and μ are the birth and death rates.

Proof. Since each subpopulation class stays nonnegative for $t \geq 0$, the total population stays nonnegative for $t \geq 0$. The existence of the upper bound
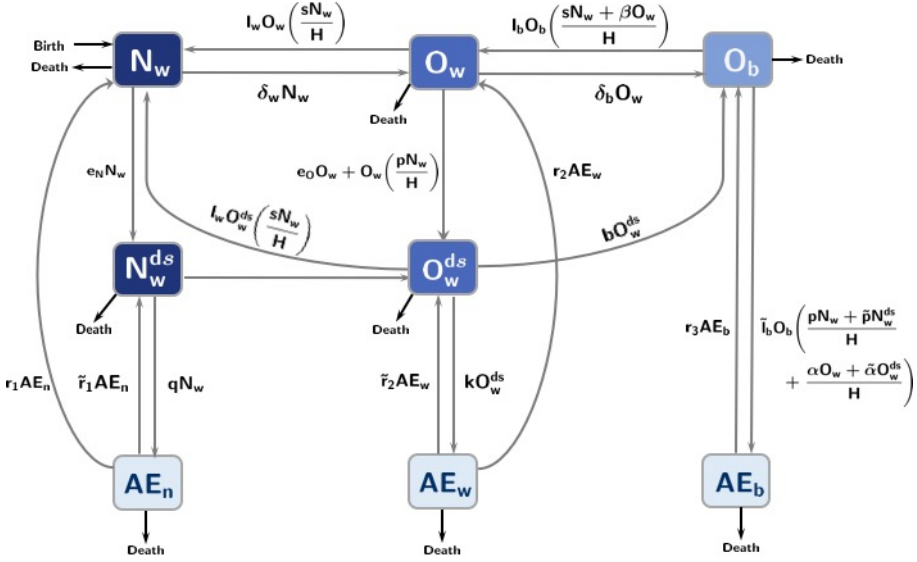
FIGURE 1. Conceptual diagram of the compartment model. The population is divided into eight subgroups N_w, N_w^{ds}, O_w, O_w^{ds}, O_b, AE_n, AE_w, AE_b. The interactions among subpopulations are indicated through arrows.

of $H(t)$ can be proven by the standard technique for Bernoulli equation for $n = 2$.   □

An equilibrium point can be obtained by setting $\dfrac{dK}{dt} = 0$ for each K, $K \in \{N_w, N_w^{ds}, O_w, O_w^{ds}, O_b, AE_n, AE_w, AE_b\}$ in Eq (1) and solving for each variable K. Consider an equilibrium point E^* of the system in Eq (1) as

$$E^* = (N_w^*, N_w^{ds^*}, O_w^*, O_w^{ds^*}, O_b^*, AE_n^*, AE_w^*, AE_b^*). \tag{4}$$

For the obesity profile in the long term, we need the following proposition before stating a main theorem for this case.

Proposition 3.2. [11] *[Chapter 6 Corollary 6.1.3] The eigenvalues of $A = [m_{ij}]_{n \times n}$, an n by n matrix, are in the union of n discs*

$$\bigcup_{j=1}^{n} \{z \in \mathbb{C} : |z - a_{jj}| \le C_j'(A)\}, \tag{5}$$

where $\mathbb{C}$ is the set of complex numbers, a_{jj} are the diagonal entries in A, $\{z \in \mathbb{C} : |z - a_{jj}| \le C_j'(A)\}$, is a disc in $\mathbb{C}$ centered at a_{jj} with the radius $C_j'(A)$, and

$$C_j'(A) = \sum_{i \ne j} |a_{ij}|, \ j = 1, \cdots, n, \tag{6}$$

which is the absolute column sum without the the diagonal entry in the jth column.

The above proposition is known as *Geršgorin disc theorem*. By using Proposition 3.2, we have the following result:

Theorem 3.3. *The equilibrium point E_1^* given by*

$$E_1^* = (0, 0, O_w^*, O_w^{ds^*}, O_b^*, 0, AE_w^*, AE_b^*) \tag{7}$$

exists but is unstable.

Proof. By setting $N_w = N_w^{ds} = AE_n^* = 0$ and $\dfrac{dK}{dt} = 0$ for each $K \in \{N_w, N_w^{ds},$ $O_w, O_w^{ds}, O_b, AE_n, AE_w, AE_b\}$ in Eq (1) and solving for each variable K, E_1^* in (7) can be obtained with

$$O_w^* = u_2 u_5, \; O_w^{ds^*} = u_5, \; O_b^* = u_3, \; AE_w^* = u_1 u_5, \text{ and } AE_b^* = u_4 u_5, \tag{8}$$

where

$$u_1 = \frac{k}{r_2 + \tilde{r}_2 + k}, \; u_2 = (b + k + \mu) + \frac{\tilde{r}_2 k}{r_2 + \tilde{r}_2 + \mu},$$

$$u_3 = \frac{c}{\beta l_b} \left((\delta_b + e_O + \mu) - \frac{r_2 k}{r_2 + \tilde{r}_2 + \mu} \right),$$

$$u_4 = \frac{u_3 \tilde{l}_b (\alpha u_2 + \tilde{\alpha})}{c(r_3 + \mu)}, \text{ and } u_5 = \frac{c - u_3}{u_1 + u_2 + u_4 + 1}.$$

By linearizing Eq (1) about E_1^* we have a Jacobian matrix J given by

$$\begin{bmatrix}
-a_1 & -br & -br & -br & -br & -br + r_1 & -br & -br \\
e_N & -b_2 & 0 & 0 & 0 & \tilde{r}_1 & 0 & 0 \\
\delta_w - \frac{pO_w^*}{c} + c_1 & -\frac{\tilde{p}}{c}O_w^* - A & -c_3 & A & \frac{l_b\beta}{c^2}O_w^* - A & A & r_2 - A & -A \\
\frac{p}{c}O_w^* - \frac{sl_w}{c}O_w^{ds^*} & \delta_w + \frac{\tilde{p}}{c}O_w^* & e_O & -d_4 & 0 & 0 & \tilde{r}_2 & 0 \\
-(c_1 + h_1) & A - h_2 & \delta_b - \tilde{c}_3 - h_3 & b + A - h_4 & -h_5 & A + B & A + B & A + B + r_3 \\
0 & q & 0 & 0 & 0 & -i_6 & 0 & 0 \\
0 & 0 & 0 & k & 0 & 0 & -j_7 & 0 \\
h_1 & h_2 & h_3 & h_4 & \tilde{h}_5 & -B & -B & -k_8
\end{bmatrix},$$

$$\tag{9}$$

where

- $a_1 = br + \delta_w + e_N + \mu - \dfrac{sl_w}{c}(O_w^* + O_w^{ds^*})$ and $b_2 = q + \delta_w + \mu$;

- $c_1 = \dfrac{l_b}{c^2}O_b^*(sc - \beta O_w^*)$;

- $c_3 = \delta_b - e_O - \mu - \tilde{c}_3$, $\tilde{c}_3 = \dfrac{l_\beta}{c^2}(c - O_w^*)$, and $d_4 = -b - k - \mu$;

- $A = \dfrac{l_b\beta O_b^* O_w^*}{c}$ and $B = \dfrac{\tilde{l}_b O_b^*}{c^2}(\alpha O_w^* + \tilde{\alpha}O_w^{ds*})$;

- $h_1 = \dfrac{\tilde{l}_b O_b^*}{c^2}(pc - (\alpha O_w^* + \tilde{\alpha}O_w^{ds*}))$ and $h_2 = \dfrac{\tilde{l}_b O_b^*}{c^2}(\tilde{p}c - (\alpha O_w^* + \tilde{\alpha}O_w^{ds*}))$;

- $h_3 = \dfrac{\tilde{l}_b O_b^*}{c^2}(\alpha c - (\alpha O_w^* + \tilde{\alpha}O_w^{ds*}))$ and $h_4 = \dfrac{\tilde{l}_b O_b^*}{c^2}(\tilde{\alpha}c - (\alpha O_w^* + \tilde{\alpha}O_w^{ds*}))$;

- $h_5 = \mu - A + \dfrac{l_b\beta O_w^*}{c^2} + \tilde{h}_5$ and $\tilde{h}_5 = \dfrac{\tilde{l}_b(1 - O_b^*)}{c^2}(\alpha O_w^* + \tilde{\alpha}O_w^{ds*})$;

- $i_6 = r_1 + \tilde{r}_1 + \mu$, $j_7 = r_2 + \tilde{r}_2 + \mu$, and $k_8 = B + r_3 + \mu$.

From the Jacobian matrix J we can calculate $C_j'(J) = \sum_{i \neq j}|a_{ij}|$, $j = 1, \cdots, 8,$. Then, we have

$$\bigcup_{j=1}^{8}\{z \in \mathbb{C} : |z - a_{jj}| \leq C_j'(J)\}, \tag{10}$$

where the discs for $j = 1, \cdots, 8$ may or may not cross the origin and the right half plane of $\mathbb{C}$. Note that the 8th disc is given by

$$\{z \in \mathbb{C} : |z - (-B - r_3 - \mu)| \leq B + r_3 + br + 2A\} \tag{11}$$

which is the disc centered at $-B - r_3 - \mu$ with the radius $C_8'(J) = B + r_3 + br + 2A$. Note the birth rate $\geq$ death rate, i.e. $br \geq \mu$. $br < \mu$ implies the extinction of the population, which is not realistic. Thus, we conclude

$$B + r_3 + \mu < B + r_3 + br + 2A. \tag{12}$$

Thus, the 8th disc in (11) surely crosses the origin and the right half plane of $\mathbb{C}$. Thus, E_1^* in (7) is unstable. $\square$

Theorem 3.4. *If $N_w^* \neq 0$ in E^* in (4), then the other components must be positive in E^*.*

Proof. We start with the following claim.
Claim 1. O_w^*, $O_w^{ds^*}$, and O_b^* cannot be zero at the same time.
Suppose not, i.e. Then, $O_w^* = O_w^{ds^*} = O_b^* = 0$. Then, $\dfrac{dAE_w}{dt} = 0$ and $\dfrac{dAE_b}{dt} = 0$ in Eq (1) imply $AE_w^* = AE_b^* = 0$ in E^* in Eq (4). Then, $\dfrac{dO_w}{dt} = 0$ gives $N_w^* = 0$, which is a contraction to $N_w^* \neq 0$. This proves Claim 1.

Suppose that $O_b^* = 0$ in E^* in Eq (4). Then, $AE_b^* = 0$. Consider $\dfrac{dO_b}{dt} = 0$ in Eq (1). Then, we have

$$\frac{dO_b}{dt} = 0 \quad \Rightarrow \quad \delta_b O_w + b O_w^{ds} = 0 \quad \Rightarrow \quad O_w = O_w^{ds} = 0,$$

which is a contraction to Claim 1. Thus, $O_b^* \neq 0$.

Assume that $O_w^{ds^*} = 0$ in E^* in Eq (4). Then, $AE_w^* = 0$. Consider $\dfrac{dO_w^{ds}}{dt} = 0$ in Eq (1). Then, we have

$$\frac{dO_w^{ds}}{dt} = 0 \Rightarrow e_O O_w + \delta_w N_w^{ds} + O_w(\frac{pN_w + \tilde{p}N_w^{ds}}{H}) = 0.$$

This is not possible unless $O_w = N_w^{ds} = N_w = 0$. But $N_w \neq 0$. Thus, $O_w^{ds^*} \neq 0$ and consequently $AE_w^* \neq 0$ in E^* in Eq (4). Suppose that $O_w = 0$ and consider $\dfrac{dO_w}{dt} = 0$ in Eq (1). Then, we have

$$\frac{dO_w}{dt} = 0 \Rightarrow \delta_w N_w + r_2 AE_w + l_b O_b\left(\frac{sN_w}{H}\right) = 0 \Rightarrow AE_w = -\frac{1}{r_2}\delta_w N_w + l_b O_b\left(\frac{sN_w}{H}\right) < 0.$$

This is a contradiction to the fact that $N_w > 0$, $O_b > 0$ and AE_w is nonnegative. Hence, $O_w \neq 0$.

Finally, suppose $N_w^{ds^*} = 0$ in E^* in Eq (4). Then, $AE_n^* = 0$. From $\dfrac{dN_w^{ds}}{dt} = 0$ in Eq (1) we have

$$\frac{dN_w^{ds}}{dt} = 0 \quad \Rightarrow \quad e_N N_w = 0 \quad \Rightarrow \quad N_w = 0$$

which is a contradiction to the fact that $N_w^* \neq 0$. Hence, $N_w^{ds^*} \neq 0$. Therefore if $N_w^* \neq 0$, all other components in E^* in Eq (4) are not zero. Since all components stay nonnegative for all $t \geq 0$, all components in E^* in Eq (4) are positive for $N^* \neq 0$. This completes the proof. □

Remark 1. In Theorem 3.4, $N_w^* \neq 0$ can be guaranteed if the birth rate is greater than or equal to the death rate in a population. The birth rate greater than or equal to the death rate is natural to assume otherwise the population will be extinct due to the death rate being higher than the birth rate. Thus, E^* in Eq (4) with all positive components will be achieved. This implies that obesity and abnormal eating free equilibrium point will not be obtained by itself unless an intervention on obesity is mandated in real life. Without an intervention, there will be always a certain level of obesity existing in the population. On the other hand, Theorem 3.3 is the worst situation that no normal weight people will exist in the long term. However, as proven, although this type of equilibrium point can exist but will be unstable. The reason is that there will be a continuous birth with the birth rate greater than or equal to the death rate and hence these new population can always remain normal weight, progress to

higher weight groups, or develop abnormal eating behavior.

4. Long Term Obesity Trend of Korean Male

In this section, we will show the obesity trend of Korean male with the Korean demography data and obesity rate in 2001. The birth and death rates of Korean male are $br = 0.208$ and $\mu = 0.00597$, respectively as in 2016 shown in Table 1. Then, we set the initial values as $N_w^0 = 1,356,520$, $N_w^{ds^0} = 100,000$, $O_w^0 = 427,500$, $O_w^{ds^0} = 209,440$, $O_b^0 = 50,000$, and $AE_n^0 = AE_w^0 = AE_b^0 = 1000$ that reflect the 32% of overweight/obese Korean male whose age is above 19 years old estimated in 2001 [12]. Then, we conduct computer simulations over the time span from 2000 to 2070 with the initial condition above and the parameter values given in Table 1.

As shown in Figure 2, overweight and obese Korean male will steadily increase after 2010 and surpass the normal weight population around 2045. Around 2070 the proportion of overweight and obese population will reach close to 68%. Although our model projection is not the exact number of overweight/obese population among Korean male, it can be a good reference for a potential obesity problem in Korea.
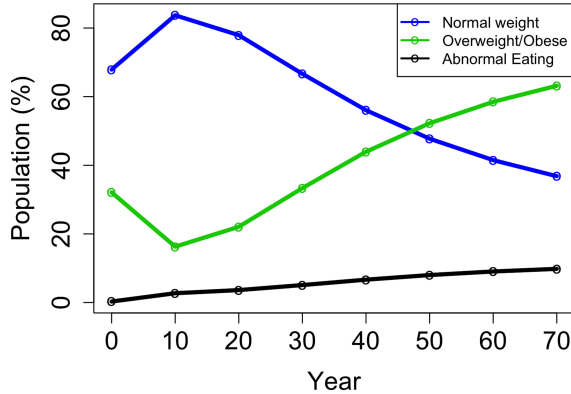

FIGURE 2. Long term obesity trend of Korean Male from 2001 till 2070. Proportion of the eight population subgroups with the parameter values in Table 1 and the initial data $N_w^0 = 1,356,520$, $N_w^{ds^0} = 100,000$, $O_w^0 = 427,500$, $O_w^{ds^0} = 209,440$, $O_b^0 = 50,000$, and $AE_n^0 = AE_w^0 = AE_b^0 = 1000$

Table 1. Literature based and estimated parameter values for
the numerical simulations.

Parameter	Characteristics	Reference
$br = 0.208$	Average Korean male birth rate	[13]
$\delta_w = 0.025$	Progression rate from normal to overweight	
$\delta_b = 0.01$	Progression rate from overweight to obese	
$e_N = 0.03$	Rate of driving for thinness of the normal weight	
$e_O = 0.065$	Rate of driving for thinness of the overweight	
$l_w = 0.033$	Rate of weight loss of the overweight	[7]
$l_b = 0.01$	Rate of weight loss of the obese	[7]
$\tilde{l}_b = 0.01$	Proportion of the obese developing abnormal eating behavior (AEB) due to peer pressure	
$\mu = 0.00597$	Average Korean male death rate	[13]
$s = 0.3$	Rate of supportive action of the normal weight	
$\beta = 0.03$	Rate of supportive action of the overweight for obese	
$p = 0.5$	Rate of negative criticism (*) of the normal weight	
$\tilde{p} = 0.7$	(*) of the normal weight with body image dissatisfaction (BID)	
$\alpha = 0.1$	Rate of weight loss pressure (**) of the overweight	
$\tilde{\alpha} = 0.3$	(**) of the overweight with BID	
$k = 0.0066$	Proportion of the overweight developing AEB	
$b = 0.0198$	Proportion of the overweight failing diet and gaining weight	
$r_1 = 0.03$	Natural recovery rate from AEB in the normal weight	
$\tilde{r}_1 = 0.018$	Natural recovery rate from AEB in the normal weight with BID	
$r_2 = 0.02$	Natural recovery rate from AEB in the overweight	
$\tilde{r}_2 = 0.012$	Natural recovery rate from AEB in the overweight with BID	
$r_3 = 0.01$	Natural recovery rate from AEB in the obese	
$q = 0.01$	Rate of developing AEB of the normal weight	
$c = 25,000,000$	Carrying capacity	

   Figure 3 illustrates the proportion of people having abnormal eating behavior
among overweight/obese and normal weight Korean males from 2001 to 2070
as well as the proportion of normal and overweight/obese Korean males in bar
charts. In Figure 3(b) the proportion of population having abnormal eating
behavior seems to increase both in normal and overweight/obese Korean males
although the magnitude is no more than 10% along the time. This shows
how the social interactions among people regarding weight gain and weight loss

and body image dissatisfaction can affect the development of abnormal eating behavior among different weight groups.

In fact, the annual health at a glance report indicated 26.4% of Korean boys aged 5 to 17 were either overweight or obese as of 2013 [14]. Korea web-based study revealed 45.1 % of teenage girls and 23.1% of boys attempting diets and also mentioned that 18.8 % of girls bought over-the-counter weight-loss medication and laxatives, and even forced themselves to vomit after meals to lose weight [15]. Moreover, the study by the Korea Food and Drug Administration of 7,000 teens in 2010 revealed 10 % of boys experienced abnormal eating behaviors [16]. Thus, the problem of abnormal eating behavior exists among Korean male and the increasing trend of obesity and abnormal eating behavior among Korean male from our simulation results will be viable information to policy makers.
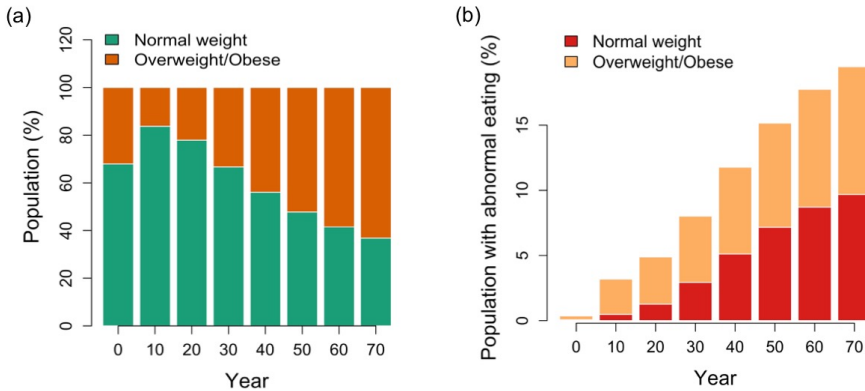

FIGURE 3. Long term change in the proportion of normal, overweight, obese classes. (a) Proportions of the normal weight and overweight/obese population, respectively (b) Proportion of the population with abnormal eating behavior among the normal weight people, overweight/obese people, respectively with the parameter values in Table 1 and the initial data $N_w^0 = 1,356,520$, $N_w^{ds^0} = 100,000$, $O_w^0 = 427,500$, $O_w^{ds^0} = 209,440$, $O_b^0 = 50,000$, and $AE_n^0 = AE_w^0 = AE_b^0 = 1000$

5. Discussion

We developed a mathematical model for the obesity dynamics with the consideration of psychological and social factors such as body image dissatisfaction, positive support on weight loss and negative criticism about weight status. In fact, many studies used the notion of infection called *social contagion of obesity* in obesity modeling adopted from epidemic modeling. That is, overweight or

obese people are thought to be contagious to normal weight people. In other words, the weight gain of normal weight people is due to the contact of overweight and obese people. However, this idea may not be able to explain why people gain weight in many cases. Rather, this type of modeling shows a limitation of adopting epidemic modeling to obesity modeling.

Hence, we considered a weight gain as a solely personal issue and focused more on social interactions among different weight groups for weight gain and loss. As a result, we constructed a compartment model with eight subpopuations with three weight groups, three groups with abnormal eating behaviors and two groups with body image dissatisfaction in order to investigate how the psychological and social factors influence the obesity trend in a population. We show that the equilibrium point with no normal weight populations will be unstable and that an equilibrium point with positive normal weight population should have all other components positive. Then, we demonstrated our model via computer simulations with the Korean demography data for the long-term obesity trend of Korean male from 2001 to 2070.

For the future work, we would like to develop more stability results of our model and estimate necessary parameter values for a numerical study when a specific country is considered.

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