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A Study on Moored Floating Body using Non-linear FEM Analysis

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Abstract

In this study, the behavior of the coupled mooring system and floating body is analyzed. The related works are introduced for the mooring analysis of the floating body. Equations motion are introduced for calculating mooring force connected with the floating body. For formulating the equations of motion, the concept of the constrained force is applied for compact expression of it. The input and output data of the module for calculating mooring force is defined. The static analysis and quasi-static analysis are performed. For the analysis, equilibrium equation for elastic catenary mooring line is used by employing finite element method, and the C# solver is developed in this research. The analysis results are validated by comparing with other research results.

Keywords: Mooring; Finite element method(FEM), Multibody system

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1. Introduction

Needs for accurate analysis of dynamic behavior have been increasing in many engineering fields, including the shipbuilding industry. There are several simulation tool which is focused on the traditional mechanical systems, such as car and machinery. In general, however, shipbuilding and offshore industry is different from conventional mechanical system in aspect of their purpose, size, and shape. Therefore, there are some limits to apply analysis tools, which are developed for conventional mechanical system, to the shipbuilding and offshore industry. Fig. 1 shows several kinds of the offshore operations in shipbuilding industry.

Due to these reasons, simulation tools are developed in several researches for the dynamic analysis of the shipbuilding and offshore operation process (Ku et al., 2014; Roh et al., 2014). The simulation programs developed from these researches can deal with multibody system, so that more accurate dynamic analysis can be completed.

These programs define the mooring line as a linear spring, and the spring constants are determined with some assumption and linearization. However, in some offshore operation, such as installation and decommissioning, mooring analysis can be one of the most important analysis point. Therefore, this paper presents about dynamic analysis for the moored floating multibody system. The analysis of mooring line is performed using non-linear finite element method(FEM).

The remainder of this paper is as follows. Section 2 reviews previous works related to this study. In Section 3, the research scope and input/output of the analysis system is defined. The equations for the analysis follows in Section 4. In Section 5, the simulation results are presented. Finally, the last section summarizes

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this study and briefly discusses the next study.

2. Related Works

There are several researches about mooring line analysis. In some researches, the mooring line is assumed as linear spring line for the motion analysis. In this kind of assumption, determining spring constant are very important. Moreover using linear spring model, the complicate motion of floating body and mooring line will be approximated, so the results of the analysis cannot be precise enough. However, this assumption has advantage in aspect of simulation time. (Yamanoto et al., 1980; Tang et al., 2011).

In the fully coupled method, the motion of the vessel and the mooring system is analyzed. The equations of motion are formulated for both of floating body and mooring line. The numerical models and procedures that provide accurate and efficient global modeling presented (Garrett, 2005; Kim et al., 2013a; Kim et al., 2013b). These methods normally needs long time to get simulation results, however, the results are more accurate than the simulation results from the linear spring methods.

In the introduced research, the floating structures are assumed as a single body. Since, however, needs for accurate analysis of dynamic behavior have been increasing in shipbuilding and offshore industry, simulation tools are developed (Ku et al., 2014; Roh et al., 2014). The programs developed from these researches are based on multibody dynamics, so that more sophisticated motion of the vessels and cranes can be simulated. Moreover, the researches proposed and implemented the functions of creating and editing scenario such as shipbuilding production, installation, or decommissioning processes. Fig. 2 shows an example of the program GUI, and its application.

These program also consider the mooring effects. However, the mooring line is model as linear spring. Therefore, it has limitation to use these program for analysis of offshore operation when mooring system is the key equipment, and significantly effects to the motion.

Fig. 1. Several kinds of the offshore operations in shipbuilding industry.

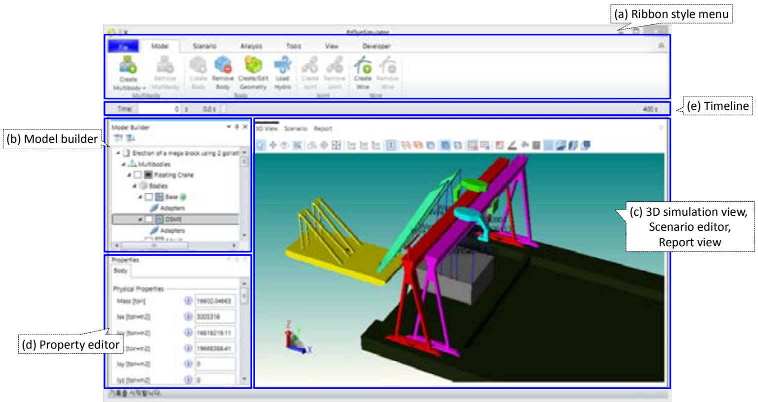


Fig. 2. GUI of the simulation program for motion analysis of multi-body system, and its application (Ku et al., 2014)

Table 1. Input and output data for the mooring analysis

	Input data	Output data
Static analysis	Mooring line specification - Length of mooring line - Weight of mooring line per meter - Young's modulus - Section area For FEM analysis - Stiffness of the seabed - Coefficient of coulomb friction - Number of element For static analysis - Start point of mooring line: anchoring position - End point of mooring line: fairlead position on the floating body	- Line tension - Line position
Dynamic analysis	- All data for static analysis - Inertia of a node - Initial position of a nodes - Added mass of a node - Mass density of water - Inertia coefficient - Drag coefficient - Projected area of element - Volume of element - Fluid velocity - Fluid acceleration - Node velocity - Position of the vessel - Velocity of the vessel	- Line tension - Line position - Acceleration of the lumped mass - Acceleration of the vessel

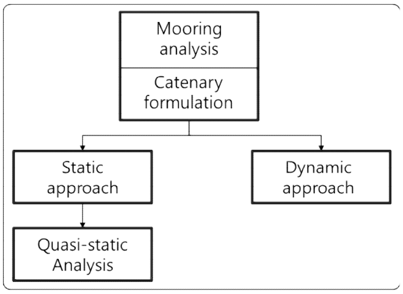


Fig. 3. Category of the mooring analysis - static, quasi-static, and dynamic approach

In this research, therefore, equations for analysis of mooring line is presented. The mooring lines are defined as finite element, so that the structural non-linearity can considered for the analysis, and the floating body is defined as multi-body, so that the non-linearity of the floating multi-body can be considered. Using these equations, it can be expected to simulate static and dynamic analysis. This paper presents the equations for static and dynamic analysis. The static analysis and quasi-static analysis are implemented and verified. The equations for dynamic analysis is only presented, but not implemented and verified, and it will be done in future research.

3. Definition of the Research Scope

As mentioned in previous chapter, the mooring line is assumed as catenary, and modeled as finite element. Fig. 3 shows the category of the mooring analysis. Basically, the mooring analysis is categorized into the static and dynamic approach, and using static approach, the quasi-static analysis can be performed. In this research, the equations for static, quasi-static, and dynamic analysis are presented.

The input and output data for the static analysis and dynamic analysis are shown in Table 1.

4. Equations for Static Analysis

There are analytic solution of the un-stretchable catenary mooring line (Arbabi, 1991). In this research, however, the mooring line can be assumed stretchable, and there are several numerical solution for it. Among the previous works, the equation presented by Kim(2010) is adopted to this research. The equations for static analysis using finite element method is as follows.

$$[K]\{u\} = \{f_n\} + \{f\} \tag{1}$$

In this equation, u is the position of the node for each finite elements. By solving this static equation, the static equilibrium position, tension, and mooring forces can be calculated. Eqs. (1.1) ~ (1.13) describe each term of Eq. (1).

$$[K] = \sum_e \begin{bmatrix} \left[K_{ge}\right] & \{D_g\} \\ \{D_g\}^T & \{z\} \end{bmatrix} \tag{1.1}$$

$$\{z\} = -\frac{s_0}{EA} \tag{1.2}$$

, where s_0 is the initial length of the each finite element of mooring line, E is the Young's modulus of the mooring line, A is the section area of the mooring line.

$$\{D_g\} = [R]^T \{D\} \tag{1.3}$$

$$\{D\} = \{B\} + [C]\{u_e\} \tag{1.4}$$

$$\{B\} = \frac{2}{J+1}[-100100]^T \tag{1.5}$$

$$\{u_e\} = [R]\{u_{ge}\} \tag{1.6}$$

, where u_{ge} is global position of the node for each finite elements, R is transformation matrix which transform the global vector to the local vector, u_e is local position of the node for each finite elements.

$$\left[K_{ge}\right] = [R]^T [K_e][R] \tag{1.7}$$

$$[K_e] = \lambda\left[C\right] \tag{1.8}$$

$$[C]=\frac{1}{(J+1)l}\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ & 1 & 0 & 0 & -1 & 0 \\ & & 1 & 0 & 0 & -1 \\ & & & 1 & 0 & 0 \\ S & y & m & m & 1 & 0 \\ & & & & & 1 \end{bmatrix} \tag{1.9}$$

$$J=\sqrt{(1+u'_x)^2+u'^2_y+u'^2_z} \tag{1.10}$$

$$\{u\}=\begin{Bmatrix} \{u_{ge}\} \\ \lambda \end{Bmatrix} \tag{1.11}$$

$$\{f_n\}=\sum_e \begin{Bmatrix} -\{f_{we}\} \\ s_0-l \end{Bmatrix} \tag{1.12}$$

$$\{f_{we}\}=\frac{Jwl}{2}[001001]^T \tag{1.13}$$

, where λ is Lagrange multiplier, l is length between the local coordinates, w is weight of the unit length of mooring line.

For solving the Eq. (1), Newton-Rapson method is adopted, and the solver is developed using C# language. For the quasi-static analysis, moreover, the calculation results of the static analysis, such as tension and mooring force, are used as external forces in the dynamic simulation of the floating body.

5. Equations for Dynamic Analysis

For more accurate dynamic analysis, coupled effects, which is occurred between the floating body and mooring line, should be considered (Paulling and Webster, 1986). There are several research about this issues (Colby et al., 2000; Garrett, 2005; Kim 2010). Among the previous works, the equation presented by Kim(2013) is adopted to this research. In previous research, however, the floating body is assumed as single body, while this research assumes the floating body as multi-body. Therefore, for the formulation of the equations of motion, concept constraint force is adopted.

$$[M]\{\ddot{u}\}+[K]\{u\}=\{f_n\}+\{f\} \tag{2}$$

$$[M_B+M_{add}(\infty)]\{\ddot{u}_B\}+[\int_0^t R(t-\tau)d\tau]\{\dot{u}_B\}+[K_B]\{u_B\}=\{f_B\}+\{f_m(u)\} \tag{3}$$

Eq. (2) is the equations of motion for mooring line. Eq. (3) is the equations of motion for single floating body. M represents mass/inertia matrix of the mooring line, M_B and M_{add} represent mass/inertia matrix, and added mass respectively. R is memory function, and K_B is hydrostatic restoring force.

Kim(2013) calculate the solution of the Eq. (2) and Eq. (3) respectively, and the convergence point is found during iterative calculation. However, in this research, concept constraint force is adopted as shown in Eq. (4).

$$C(u_{fairlead}, u_{B,fairlead}) = u_{fairlead} - u_{B,fairlead} = 0 \qquad (4)$$

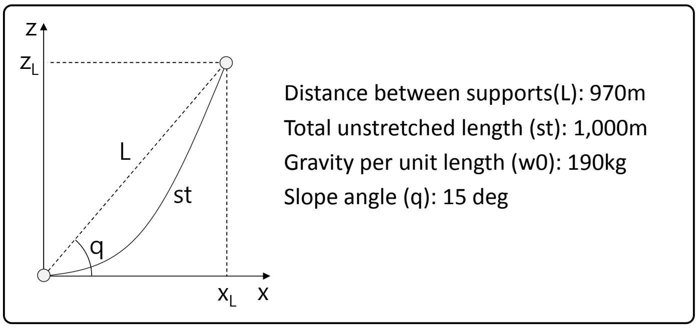
By merging Eq. (2) ~ Eq. (4), Eq. (5) can be derived.

$$\begin{bmatrix} M_{total} & C_{\mathbf{u}}^{T} \\ C_{\mathbf{u}} & 0 \end{bmatrix} \{\ddot{u}_{total}\} + [K_{total}]\{u_{total}\} = \{f_{total}\} \qquad (5)$$

Eq. (5) is more compact than Eq. (2), (3), and seems to be more useful in the computational aspect.

6. Simulation Results

In previous chapter, equations for static and dynamic analysis are presented. In this research, first of all, static analysis solver is developed. For the verification, the simulation results and the analytic solution (Arbabi, 1991) is compared. The example of the mooring line is shown in Fig. 4. Fig. 5 shows the comparison results. In the graph, line represents simulation results using developed solver, and dots represent analytic solution, and they are exactly same.



Distance between supports(L): 970m
Total unstretched length (st): 1,000m
Gravity per unit length (w0): 190kg
Slope angle (q): 15 deg

Fig. 4. Numerical example of the mooring line

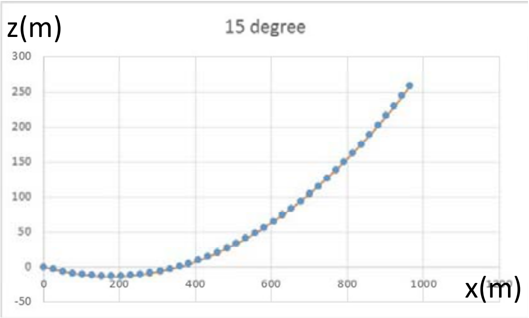


Fig. 5. Comparison with the analytic solution and the simulation results using developed program.

Fig. 6 ~ Fig. 13 show the static equilibrium position and tension of the numerical example. The slop angle q is varied from 0 deg. to 60 deg. Every analysis results shows the parabolic catenary curve shape. As axial stiffness, which is proportional to EA, decreases, the sag are quadratically increases, and the tension of the mooring line decreases, which means that the mooring force also decreases.

Fig. 14 ~ Fig. 17 shows the restoring force caused by mooring line of the numerical example. The position of the floating body is varied from 0 m to 30 m. The angle q is also varied from 0 deg. to 60 deg. As shown in this results, as the angle of the slop increases, the restoring force decreases. Moreover, more stiff mooring line gives greater restoring force.

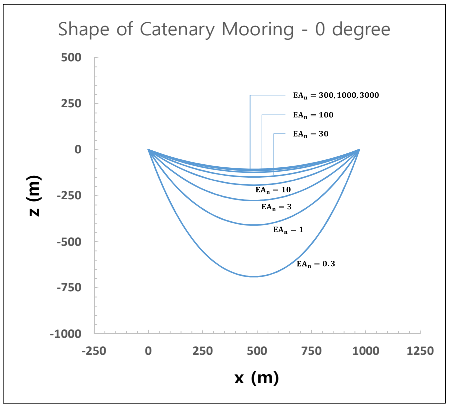


Fig. 6. Shape of catenary mooring line, when q is 0 deg.

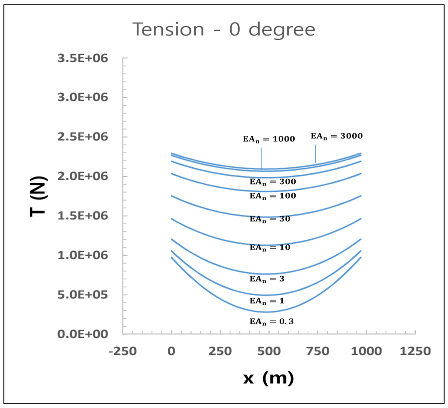


Fig. 7. Tension of catenary mooring line, when q is 0 deg.

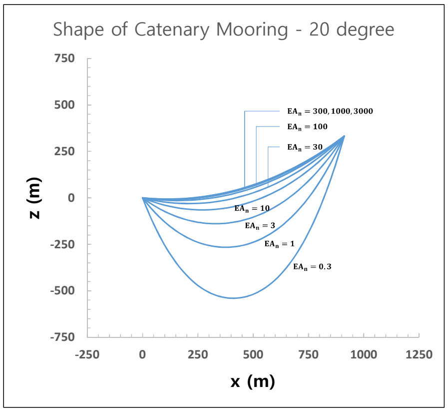


Fig. 8. Shape of catenary mooring line, when q is 20 deg.

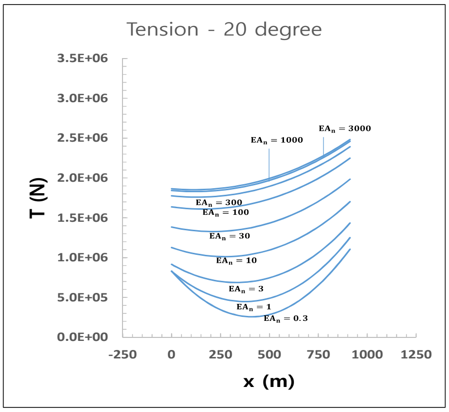


Fig. 9. Tension of catenary mooring line, when q is 20 deg.

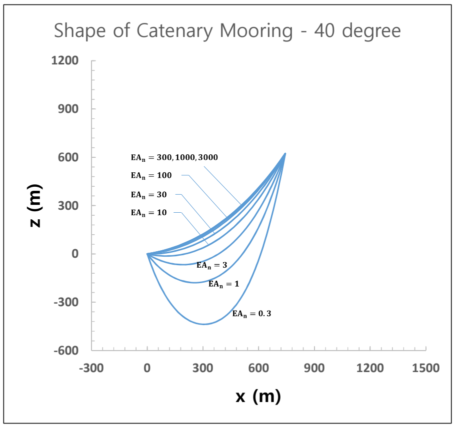


Fig. 10. Shape of catenary mooring line, when q is 40 deg.

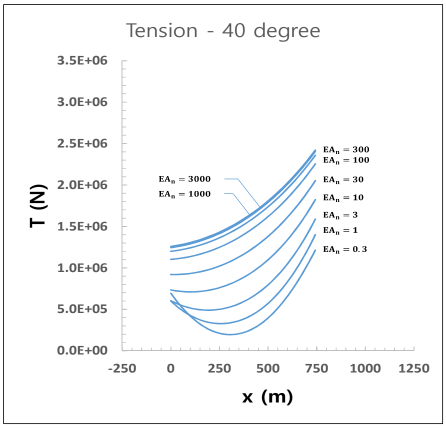


Fig. 11. Tension of catenary mooring line, when q is 40 deg.

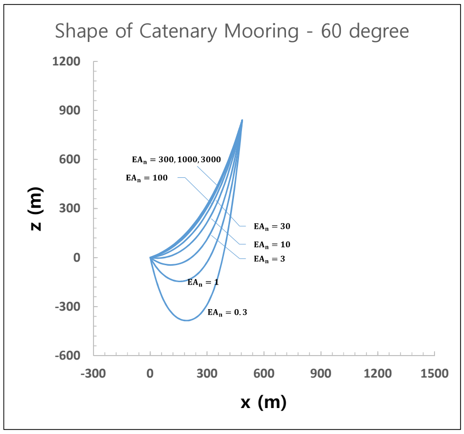


Fig. 12. Shape of catenary mooring line, when q is 60 deg.

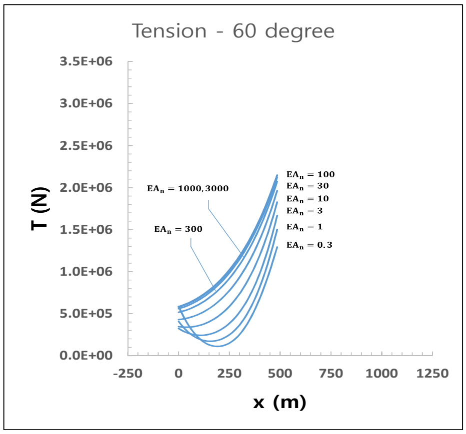


Fig. 13. Tension of catenary mooring line, when q is 60 deg.

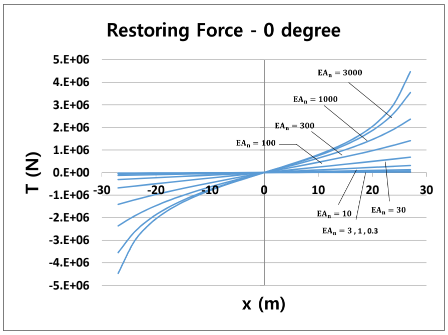


Fig. 14. Restoring force of catenary mooring line, when q is 0 deg.

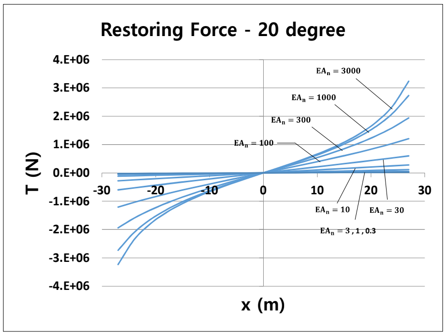


Fig. 15. Restoring force of catenary mooring line, when q is 20 deg.

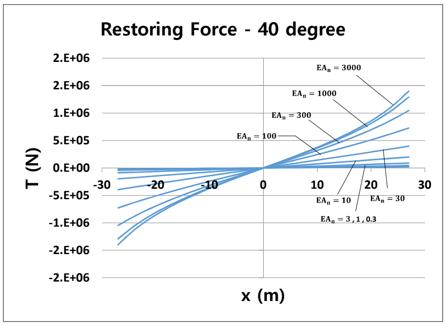


Fig. 16. Restoring force of catenary mooring line, when q is 40 deg.

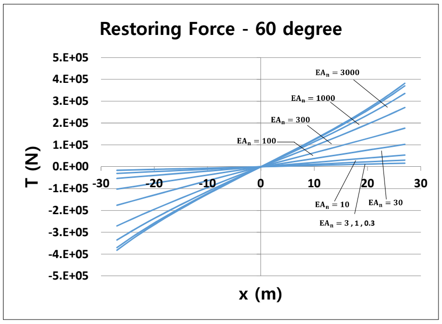


Fig. 17. Restoring force of catenary mooring line, when q is 60 deg

Lastly, quasi-static analysis is conducted. For the simple test initial status of the analysis is assumed as follows.

- Waterline area of floating body: 110x46 m^2
- Weight of the floating body: 9,500ton
- The floating body is modeled as a particle moving on x-z plane.
- Two catenary mooring line is used.

Fig. 18 shows the initial status for the quasi-static analysis, and Fig. 19 shows the results of the analysis.

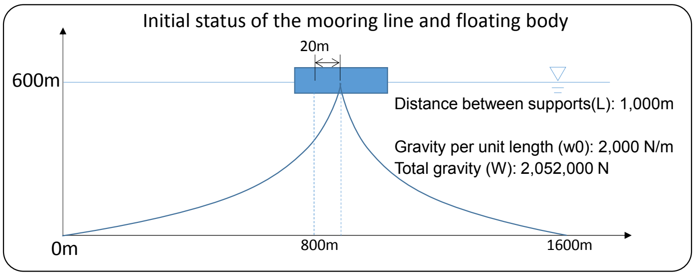


Figure 18. Initial status for the quasi-static analysis

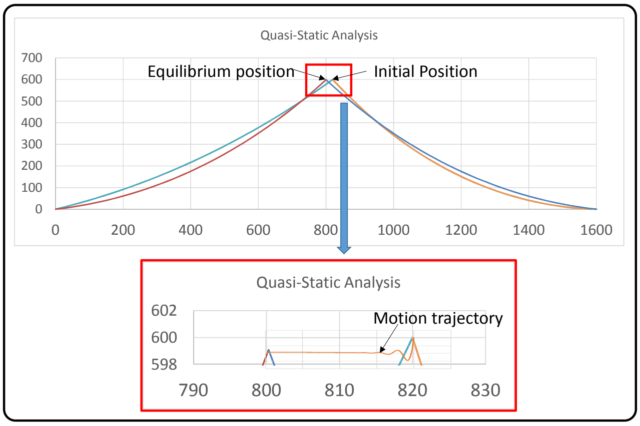


Figure 19. Simulation results of the quasi-static analysis

The graph shows the change of position of the floating body. The results shows that the restoring force caused by mooring line are successfully calculated by the developed solver.

7. Conclusions

There are few programs for multibody analysis of offshore operation, such as installation using multi-crane, considering non-linear mooring analysis. Therefore, it was aimed to develop a non-linear mooring analysis code, which can be applied to the floating multibody analysis code. In this study, the behavior of the coupled mooring system and floating body is analyzed. The equations motion are introduced for calculating mooring force connected with the multibody system. For formulating the equations of motion, the concept of the constrained force is applied for compact expression of it. The input and output data of the module for calculating mooring force is defined. The static analysis and quasi-static analysis are performed. For the analysis, equilibrium equation for elastic catenary mooring line is used by employing finite element method, and the C# solver is developed in this research. The static and quasi-static analysis results are validated by comparing with other research results. The solver for the dynamic analysis will be developed in future researches.

Acknowledgements

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