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Reexamination of Estimating Beta Coefficient as a Risk Measure in CAPM

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Abstract

This research examines the alternative ways of estimating the coefficient of non-diversifiable risk, namely beta coefficient, in Capital Asset Pricing Model (CAPM) introduced by Sharpe (1964) that is an essential element of assessing the value of diverse assets. The non-parametric methods used in this research are the robust Least Trimmed Square (LTS) and Maximum likelihood type of M-estimator (MM-estimator). The Jackknife, the resampling technique, is also employed to validate the results. According to finance literature and common practices, these coefficients have often been estimated using Ordinary Least Square (LS) regression method and monthly return data set. The empirical results of this research pointed out that the robust Least Trimmed Square (LTS) and Maximum likelihood type of M-estimator (MM-estimator) performed much better than Ordinary Least Square (LS) in terms of efficiency for large-cap stocks trading actively in the United States markets. Interestingly, the empirical results also showed that daily return data would give more accurate estimation than monthly return data in both Ordinary Least Square (LS) and robust Least Trimmed Square (LTS) and Maximum likelihood type of M-estimator (MM-estimator) regressions.

Keywords: Capital asset pricing model, robust regression, ordinary least square regression, and capital market theory.

JEL Classification Code: C13, C14, G12

1. Introduction

According to the Firm Foundation Theory of Stock Valuation, there are four factors that Malkiel (2003) stated in his book, "A Random Walk down Wall Street," that affect the stock price: expected growth rate, expected dividend payout, level of market interest rates, and degree of risk. The rational investors tend to pay more for a share when interest rates and degree of risk are low. In financial management, most investors are risk averse. These investors, given a fixed risk level, will choose investment portfolios which maximize return. Therefore, they will invest in riskier assets only if they expect to have positive risk premiums, the greater the risk, the larger the risk premium. Hence, risk estimation and analysis are the essential topics in finance. The total risk of an asset can be decomposed into systematic and non-systematic risks as follows:

$$\sigma_j^2 = \beta_j^2 \sigma_M^2 + \sigma^2(e_j),$$

where $\beta_j^2 \sigma_M^2$ represents the systematic risk (non-diversifiable or a beta risk) and $\sigma^2(e_j)$ is the non-systematic risk or firm specific risk (diversifiable) of security *j*. Nonsystematic risk is the portion of risk unique to an individual company such as strikes, lawsuits, losing a major contract, etc., and are not very important to investors since they can eliminate it through constructing an efficient and large portfolio (bad events in one firm will be offset by good events in another firm in a portfolio). In contrast, the systematic risk entails the portion of the return variation that depends upon a macro factor of an economy such as inflation, recession, high tax and interest rate, which reflects the risk of a general stock change. Hence, this type of risk cannot be eliminated by diversification, and is relevant and a concern to investors. Capital market theory states that

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investors can get only risk premium for assuming systematic risk. Therefore, being able to understand and forecast risk is very important with investors and financial analysts. The beta coefficient represents systematic risk standardized by the market portfolio risk according to the modern capital market theory.

In this research, we will explore the estimation of beta coefficient (β) through one of the most well-known theories in finance, Capital Asset Pricing Model (CAPM). In the past and in common practice, these coefficients are typically estimated using the Ordinary Least Square (LS) regression method and monthly return data in the finance and accounting literature. However, the estimation of these coefficients with Ordinary Least Square regression is known to be inefficient and unstable over time and therefore could be misleading for decision making.

This paper intends to explore the efficiency of beta estimation with alternative methods such as the robust Least Trimmed Squares (LTS) and Maximum Likelihood type M-estimator (MM-estimator) methods introduced by Rousseeuw and Leroy (2003) to see whether these methodologies could generate a more efficient and stable betas. We also examine whether daily return data would give more accurate estimation than monthly return data in both Least Square and robust regressions.

In Section 2, the literature review is presented. In Section 3, the data and empirical results of beta coefficients are calculated using daily and monthly return data, and their characteristics are examined. The normality of residuals of both daily and monthly return data also investigated. In Section 4, the Jackknife technique introduced by Turkey (1962) was used to examine the efficiency of the LS and robust MM estimators. In Section 5, the final conclusions of this research are provided. The Section 6 is for the references and the appendix. In the following, the literature review is presented.

2. Literature Review

To evaluate the relationship between the systematic risk and returns, the best and easiest to understand model that investors often turn to is the Capital Asset Pricing Model (CAPM), introduced by Sharpe (1964). For the security *i*= 1 ,2, ..., *n*, the regression model is as follows:

$$R_i(t) - R_f = \alpha_i + \beta_i\left[R_M(t) - R_f\right] + \varepsilon_i(t),$$

Where

t is the date (day, week, or month) of each pair of observations,

R_i is the return on the security *i*,

R_f is the risk free rate from U.S Treasury bill,

α_i is the security's expected excess return when the market excess return is zero (*αi* equals zero in an efficient market),

R_M is the return of the market index,

$\beta_i = Cov(R_i, R_M) \div \sigma^2{}_M$, the security's sensitivity to the market index,

ε_i is the random error term of the security *i* that has mean zero; it is the firm-specific surprise in the security return.

Researchers and practitioners have been using the ordinary Least Squares (LS) method to estimate this beta coefficient, assuming normal distribution of the security returns. Unfortunately, this assumption is questionable. Fama (1965) and McDonald & Nelson (1989) indicated that the distribution of the return data is not normal and, therefore, the beta estimator from the LS method is unreliable and inefficient since LS estimation gives excessive weight to outliers. Frequently, it is suggested that the data can be transformed using a log function to stabilize the variance. When a beta is calculated using a transformed data set, it is difficult to understand and interpret the new beta; the new beta cannot be considered to be the systematic risk any more. Blume (1971) also pointed out that predictability of beta for individual securities is very limited such that historical beta can only explain about 36 percent of the variation in the future estimated beta, leaving about 64 percent unexplained.

Other researchers attempted to estimate the beta coefficients under a non-normality assumption. Cornell and Dietrich (1978) proposed the Mean Absolute Deviation (MAD) since it gives less weight to outliers and could generate a more efficient estimation of beta compared to those by the LS approach. The results showed that the MAD approach did not necessarily produce a more efficient estimation of beta. Fong (1997) proposed a method called Generalized Student-t (GET) to estimate the beta. She claimed that the GET approach was able to handle well both skewness and excess kurtosis in the data set so that it was able to produce beta estimators that are much efficient than those by the LS approach in terms of mean square errors.

Alexander and Chervany (1980) proposed MAD approach to estimate a more stable beta. They used monthly return data to estimate beta of securities over different length of time period and found that the beta estimated over a period of 4-6 years was the most stable. The stability of betas was also significantly improved as the size of the portfolio increased to 10 securities while there was little improvement in beta estimation as the portfolio size was increased over 10 securities. Theobald (1981) also supported the fact that the stability of beta coefficient is a function of the length of estimation period.

Bowie and Bradfield (1998) proposed the robust estimators approach to estimate the beta coefficient of small-cap stocks. The sample size of small-cap stocks is usually small and its return distribution is non-normal. Since the LS estimator is sensitive to both outliers and non-normality of the sample observation, the estimated beta coefficients become unreliable. So they used robust estimators to estimate beta coefficients of the small-cap stocks and found that the robust estimators are always more reliable than the LS estimators, especially when normal assumption is violated. Shalit (2002) proposed the Gini estimator, a type of robust estimator, to estimate beta coefficients. The Gini estimator is non-parametric and does not require any assumption of return distribution. The author showed that the variance of the Gini estimator was less than the LS estimator for most of the time, and consequently, the Gini estimator was able to provide some more efficient and consistent beta coefficients than the LS estimator.

3. Data Description and Empirical Results

A sample of 50 individual securities is randomly selected from member securities of the S&P 500 index, and both their daily and monthly returns were collected over a period of the last five years. The daily and monthly returns on Treasury Bills were also collected and used as risk-free returns as Ross, Westerfield, and Jordan (2006) and Bodie, Kane, and Marcus (2008) suggested in their book. In the following, both daily and monthly returns were analyzed using LS and robust estimators and their outcomes were compared in terms of their efficiency between monthly and daily return data.

3.1. Beta estimates with robust MM and LS methods with daily data.

From the **Figure 1** below, the beta estimates of both LS and robust MM methods with daily data are presented. These two beta estimates of 18 companies are moving very closely to each other while the rest of the company betas are mixed. From the **Figure 2** below, the standard deviations of these beta coefficients are shown. It is clear that robust estimator provided much lower standard deviation than LS estimator did; 37 out of 50 firms showed significantly lower standard deviation for robust estimates. This result indicates that Robust MM estimator provided more consistent and efficient estimates than LS, which is expected from the data that does not have a normal distribution.

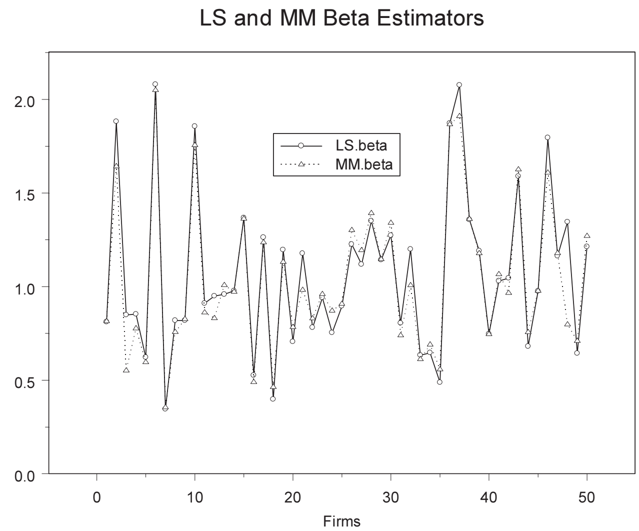

Figure 1. LS and MM beta coefficients with daily data.

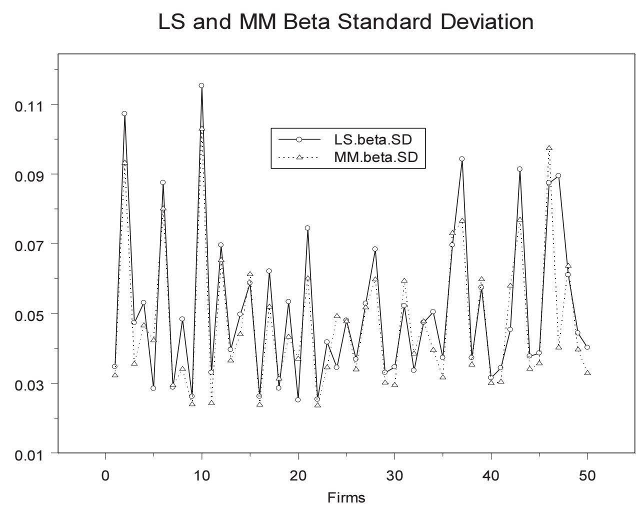

Figure 2. Standard deviation of LS and MM beta with daily data

3.2. Beta estimates with robust MM, LTS and LS methods with monthly vs. daily data.

Financial analysts frequently use the monthly return data to estimate the beta coefficient because monthly returns are more stable and tend to be normally distributed. Beta coefficients for 50 individual companies were calculated from the daily and monthly data using Robust MM approach, and presented in the **Figure 3** below. The result shows that 31 (62%) out of 50 companies have significantly lower beta coefficients than the monthly beta. The standard deviation of daily beta showned in the **Figure 4** below is much smaller in scale and very consistent for all of 50 different company stocks when compared to those of monthly beta. On the other hand, the beta coefficients generated from the monthly data tend to be overstated and very inconsistent.

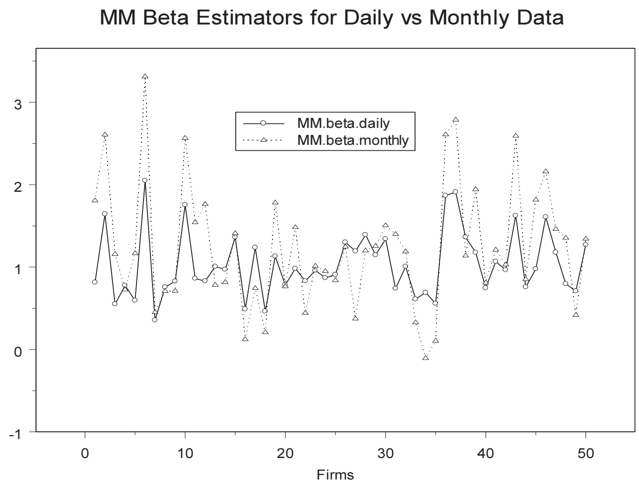

Figure 3. MM beta coefficients from daily and monthly data

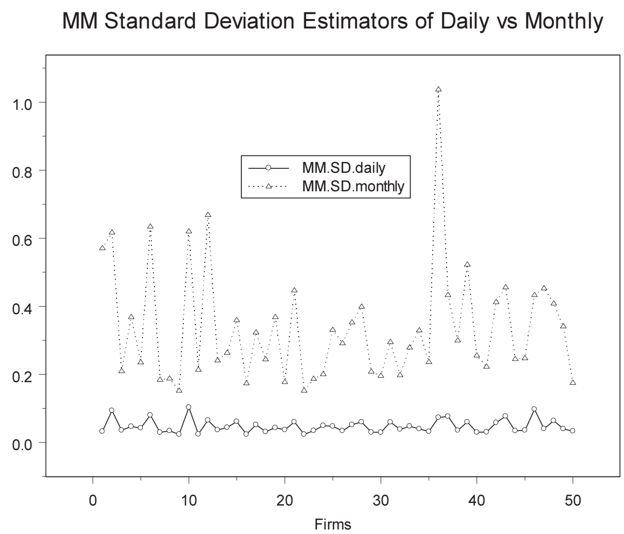

Figure 4. Standard deviation of MM beta from daily and monthly data

With LTS method, the results from the the **Figure 5** and **Figure 10** in the Appendix section below are similar as in the case of MM method; the result shows 35 out of 50 firms (70%) have significantly lower beta coefficients than the monthly beta. The standard deviation of daily beta is much smaller in scale and very consistent for all of 50 different company stocks when compared to those of monthly beta. Thus, the daily data is more consistent and reliable for estimation of beta coefficient.

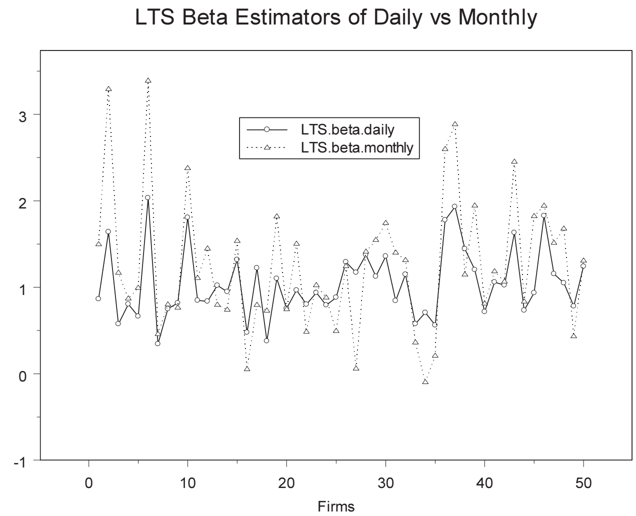

Figure 5 . LTS beta coefficients from daily and monthly data

Now with LS, there are similar results as in robust methods.

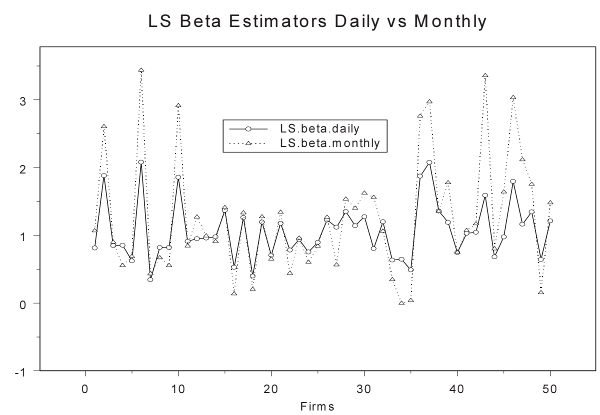

Figure 6. LS beta coefficients from daily and monthly data

From **Figure 6 and Figure 7**, we see that 32 out of 50 firms (64%) have daily beta estimators that are smaller than monthly data. The scale is from 2% to 52% (10 firms have daily beta estimators less than monthly beta estimators between 2\% - 10%; 6 firms between 10% - 20%; 6 firms between 20%-30%; 5 firms between 30% - 40%; 5 firms between 40% - 52%). There are 50 out of 50 firms (100%) in which daily beta standard deviation estimators are smaller than monthly. The scale is from 77% - 86%. Therefore, with monthly data, beta estimators tend to overestimate and there are more errors in beta estimators. Thus, the daily data is more consistent and reliable for estimation of beta coefficients of the firms.

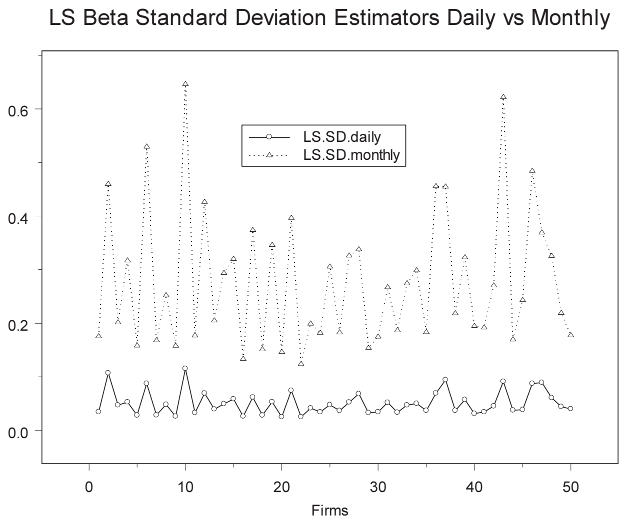

Figure 7. Standard deviation of LS beta from daily and monthly data

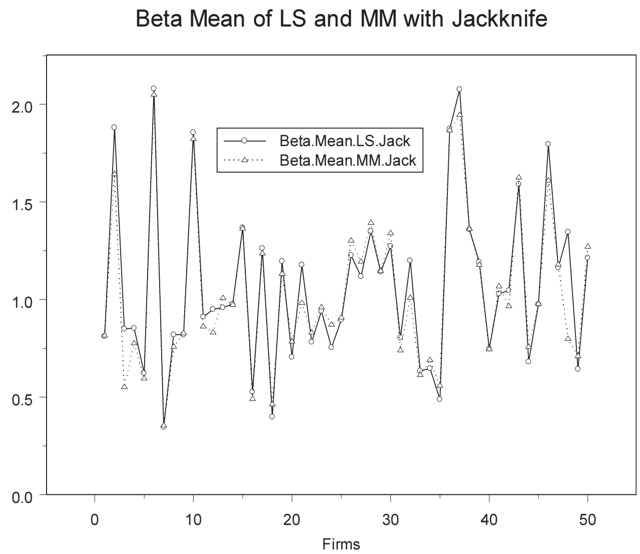

Figure 8. Means of LS and MM beta with Jackknife.

With all three MM, LTS, and LS methods, we can conclude that the daily returns data would give more accurate and efficient beta estimators than the monthly returns data. Therefore, daily returns of the data should be used in CAPM if available and it would be better off to use with robust such as MM instead of LS method.

4. Jackknife Methodology and Efficiency of Estimators

In this section, we assess LS and robust MM estimators' efficiency using the resampling technique, Jackknife with daily data. In Jackknife resampling, the beta coefficient of each firm is calculated for n possible times with one observation left out each time. If the Jackknife beta estimates are closely the same, then the estimator is considered as efficient. The estimator is considered inefficient if the Jackknife beta estimates exhibit high dispersion around the mean or there are many outliers. Therefore, we can assess the estimator's efficiency through two measurements: bias and the standard error (standard deviation) of the beta estimator. Bias measures the distance from the true value of beta and the sample mean of Jackknife beta estimates. The standard error measures the dispersion from the true value of beta, hence, it is considered as the primary efficiency.

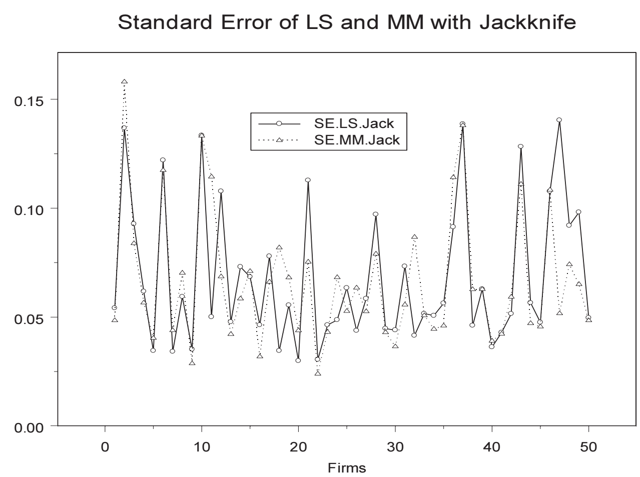

Figure 9. Standard error of LS and MM beta with Jackknife

From **Figure 8**, first we see that the beta mean estimators of both LS and MM are similar. This is consistent with the results in Figure 1 above. The beta standard error (standard deviation) estimators in **Figure 9** of both LS and MM have similar results as in **Figure 2**, where there is a majority of the firms (33 out of 50) with MM beta standard error estimators that are smaller than LS's estimators. With the results from Jackknife, we can conclude that MM estimators are more efficient than LS, which is consistent with results founded above.

5. Conclusions

The robust LTS and MM both work well in CAPM and outperformed the LS in estimation of beta coefficients in a heavy trading market. These results are expected and agreed with other studies. Applying the LS regression to estimate the systematic risk beta coefficient of the stocks in CAPM can give the closed estimators as the robust MM and LTS regressions, but LS estimators tend to have more errors and are not as accurate and not as consistent as the robust MM and LTS.

Even the monthly return data of the stocks are normally distributed, but the beta, error and beta standard deviation estimators are greater than daily return data for all three LS, MM, and LTS methods. The estimators from monthly return data are overestimated and not as consistent as those from daily data. Thus, the daily return data must be used in estimating the beta coefficient in CAPM.

Even though the robust MM and LTS estimators are more complicated and do not have as many good properties as the LS estimator, they are more consistent, efficient, and insensitive to outliers. Therefore, they would give the most reliable results for beta estimation in CAPM.

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