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How should the regulatory defaults be set?

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ABSTRACT

How to choose defaults in risk-informed regulations depends on the conservatism implicated in regulatory defaults. Without a universal agreement on the approaches dealing with the conservatism of defaults, however, the desirability of conservatism in regulatory risk analyses has long been controversial. The opponent views it as needlessly costly and irrational, and the proponent as a form of protection against possible omissions or underestimation of risks. Moreover, the inherent ambiguity of risk makes it difficult to set suitable defaults in terms of risk. This paper, the extension of the previous work [1], focuses on the effects of different levels of conservatism implicated in regulatory defaults on the estimates of risk. According to the postulated behaviors of regulated parties and the diversity of interests of regulators, in particular, various measures for evaluating the effect of conservatism in defaults are developed and their properties are explored. In addition, a simple decision model for setting regulatory defaults is formulated, based on the understanding of the effect of conservatism implicated in them. It can help decision makers evaluate the levels of safety likely to result from their regulatory policies.

1. Introduction

Most individuals and agencies in the world have always endeavored to avoid undesirable risks, or at least bring them under control. Despite these efforts, however, no attention has been paid to some risks yet. Moreover, new risks that are highly difficult to manage continue to emerge from the use of high technologies, such as chemicals, aircrafts, nuclear power, and so on. In seeking to control the risk issues, it is necessary to impose several types of regulations on those responsible for the risks, ensuring that they are the most effective ways to reduce risks, or to allocate limited resources to do so. Bier and Jang [1] insisted that the optimal balance between a relevant measure of benefit and cost should be produced in this regulatory process ideally. For example, in nuclear power plants, the existing deterministic regulations for component testing frequencies were largely established on an ad hoc basis before probabilistic safety analysis (PSA) techniques were available or widely used. However, a group of components that are subject to the same requirements through an ad hoc approach may differ widely in their importance to plant risk. Increased testing of the few risk-critical components can achieve significantly decreased risk for a small (even negligible) increase in cost, while decreased testing of the many less significant components can achieve substantially reduced compliance cost for a small (even negligible) increase in risk. These are thought of the typical examples of the risk-informed performance-based regulation that has the potential to achieve the simultaneous reduction of both risk and cost.

Bier and Jang [1] also pointed out that this approach has been slow to be adopted in practice, even though both regulators and regulated parties generally recognize the potential benefits of risk-informed regulation. For example, in the nuclear power area, the first nuclear plant PSA was published about a half century ago [2], but regulatory guidance for risk-informed regulation [3] was issued after spending nearly 2 decades attempting to formulate and apply an alternative approach, quantitative safety goals, with little concrete success [4]. The barriers to implementing risk-informed regulation are largely the same as the barriers to implementing quantitative safety goals [5], which are inherently attributable to ambiguities of risk. In spite of such ambiguities, 'defaults'[1] (also often called ‘requirements’, ‘acceptance criteria’, or ‘standards’ with the diversity of applications) are frequently used as important elements of the formal risk analysis and decision-making for the risk-informed regulation and application. However, the expression of

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[1] In the paper, ‘defaults’ are defined as officially approved modeling assumptions and parameter values of many uncertain and/or subjective quantities to be often specified by regulators and considered acceptable for use in risk analyses as input to regulatory decisions.

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regulatory defaults in terms of risk should be paid attention, because it is fundamentally liable to misinterpretation.

How should regulatory defaults be set? Bier and Jang [1] insisted that understanding of the effect of defaults should precede all others because stakeholder's interests conflict in this matter.

First, the question of whether or not regulatory defaults should be set conservatively has long been controversial [6]. The opponent views it as needlessly costly and irrational, and the proponent as a form of protection against possible omissions or underestimation of risks. Currently, agencies differ widely in their approaches to regulatory defaults, and the implications of these differences are not yet well understood. For example, in EPA risk assessment guidance for the Superfund program [7], the approved defaults for a variety of quantities are described as the "90th-percentile", "reasonable upper-bound", and "reasonable worst case". At the ASME standard for PSA [8], by contrast, defaults have generally been set at or near the mean of the nuclear power industry, to determine right priorities on risks. In view of the PSA quality, for example, the adoption of conservative assumptions in the PSA model is no longer welcomed for risk-informed regulation and application because they can cause the irrelevant priorities of risk-critical components, the so-called shadow effect [1].

Second, more importantly, regulators and regulated parties have systematically different goals or utility functions. In particular, regulators have a natural incentive (and in fact often a mandate) to seek large safety margins (*e.g.*, by ensuring that risks are estimated conservatively). Meanwhile, the costs of complying with regulations may be a secondary consideration for regulators. Regulated parties also have an incentive (in fact, a direct financial incentive) to ensure the safety of the businesses that they own and operate, but in their case this is balanced by a competing desire to minimize costs. Given the nuclear power industry is under increased competition owing to electric utility deregulation, cost minimization is likely to be an urgent matter. Therefore, a regulated party will generally have an incentive to ensure that the risks disclosed to regulators are not overestimated, in order to avoid additional burdensome regulation and the reduced operational flexibility that will likely result.

This paper, an extension of the previous work [1], focuses on the effects of different levels of conservatism implicated in regulatory defaults on the estimates of risk. According to the postulated behaviors of regulated parties and the diversity of interests of regulators, in particular, various measures for evaluating the effect of conservatism in defaults are developed and their properties are explored.

In the next two sections of the paper, several new measures and their mathematical formulations generalized from the previous one [1] are introduced to evaluate the effects of conservatism among regulatory defaults, based on the postulated behaviors of the regulated parties and the interests of regulators. In the subsequent section, a simple decision model to choose regulatory defaults is proposed as a way for solving the topic of 'how should the regulatory default be set?'. It can help decision makers evaluate the levels of safety likely to result from their regulatory policies. Finally, the conclusions and limitations of the proposed models are discussed.

2. Notations and general formulation

Fortunately, the effect of conservatism implicated in regulatory defaults is a topic that is amenable to a fairly rigorous mathematical analysis, using simple but plausible models of regulated party behavior. In particular, let X be the (uncertain) estimate of risk (or a risk-related parameter such as a component failure rate) that would result from a risk analysis performed using realistic parameter values and assumptions. Let us assume that the variability of X across the population of regulated parties is described by the probability function, $f_X(x)$. Furthermore, let D be the default value chosen for the same quantity. For example, if a regulated party elects to use the default rather than a realistic analysis, the same value D would be used by any regulated party in the population, regardless of its value of X. Thus, it is reasonable to assume that the risk estimate disclosed to regulators by a regulated party, Y, depends on the behaviors of regulated parties as follows.

$$Y = h(X, D) \tag{1}$$

where h is the function to represent the behaviors of regulated parties. Finally, the expectation of $T(=Y/X)$, $E[T]$, will be adopted as a simple measure to evaluate the effect of conservatism implicated in a particular regulatory default (D) on the estimates of risk.

3. Effects of conservatism in regulatory defaults

3.1. Measures of MGE and MGEE

First, Bier and Jang [1] assumed that the regulated party has perfect knowledge about its value of X (*e.g.*, it has already done a realistic risk analysis and is deciding whether to disclose the results to regulators). Thus, they suggested the risk estimate disclosed to regulators by a regulated party as follows.

$$Y = X \wedge D \tag{2}$$

where $X \wedge D$ represents the minimum of both quantities, $\{X, D\}$. In other words, it means that regulated parties will disclose realistic risk estimates when they are more favorable than the approved default, and will use the default value when that is more favorable. These simple assumptions will be relaxed in the succeeding subsections. For the sake of convenience, the expectation of T defined by Bier and Jang [1] will be called the maximum gross effect (MGE) to differentiate from other measures newly suggested in the paper.

MGE can be obtained in closed form for an arbitrary distribution of the regulated population as follows (refer to Appendix A for the details).

$$MGE = \int_0^1 \frac{D}{t} \cdot f_X\left(\frac{D}{t}\right) dt + F_X(D) \tag{3}$$

where F_X is the cumulative distribution function (CDF) of X. In addition, the variance of T can also be calculated as follows.

$$Var(T) = D \cdot \int_0^1 f_X\left(\frac{D}{t}\right) dt + F_X(D) - [E(T)]^2 \tag{4}$$

The MGE value ranges over (0,1), and means that the risk estimate disclosed by regulated party will be on average $[1 - MGE] \times 100\%$ lower than the real risk estimate. In addition, note that this degree of underestimation is an upper bound on the effect that might be observed in the real world, since the behavior of a regulated party assumes perfect gaming, *i.e.*, perfect choice of the minimum to disclose with the perfect knowledge about the value of X.

A preliminary analysis of this model has been undertaken for a wide variety of choices of the distribution $f_X(x)$, as shown in Table 1. The second column of Table 1 presents some MGE results for a few distributions. Here, some results for particular distributions such as uniform and exponential distributions were analytic results, while

Table 1
Underestimation of risks using mean value defaults[a] (MGE, MGEE).

Distribution	MGE	MGEE
Exponential	0.85[b]	0.20 ± 0.01
Weibull (shape parameter 2)	0.88 ± 0.02	0.40 ± 0.01
Weibull (shape parameter 3)	0.90 ± 0.02	0.51 ± 0.01
Weibull (shape parameter 5)	0.92 ± 0.01	0.66 ± 0.01
Uniform (lower bound = 0)	0.85[b]	0.505 ± 0.001
Lognormal (error factor = 3)	0.88 ± 0.02	0.24 ± 0.01
Lognormal (error factor = 10)	0.90 ± 0.02	0.10 ± 0.01
Lognormal (error factor = 30)	0.93 ± 0.01	0.06 ± 0.01
Lognormal (error factor = 100)	0.96 ± 0.01	0.07 ± 0.01

[a] Error bounds for simulation results are ± two standard errors.
[b] Mean values for analytic results (refer to Appendix A for the details).

others for less tractable distributions were based on a simulation. The results of this analysis suggest that if the default D is set equal to the expected value of the quantity of interest across the regulated population, then MGE will typically be between 0.85 and 0.96. In other words, the disclosed risk estimates will be on average 4%–15% lower than the results of a realistic risk analysis. Similar results were also obtained for other parameter values, and for the gamma and beta distributions. Bier and Jang [1] provide more discussions on the results.

More importantly, even if the estimate of the average risk is low by only about 15%, the most severe risks (*i.e.*, the largest values of X) will be underestimated by much more than this. The degree of underestimation at extreme risk, the so-called maximum gross effect of extreme (MGEE), can be defined as follows.

$$MGEE\left(X_{(n)}\right) = E\left[\frac{\min\left(X_{(n)}, D\right)}{X_{(n)}}\right] \tag{5}$$

where $X_{(n)}$ denotes the maximum value among observations taken from n facilities (in increasing order), *i.e.*, realistic risk estimate at the worst site. A measure of the MGEE type may be more important to regulatory matters because the degree of anticipated underestimation at the most severe risk would be much higher than that at the average risk. Considering the distribution of the largest value of X, the probability that all of n independent observations on a continuous variate are less than x is $[F_X(x)]^n$, which may be calculated approximately as $\exp(-n\cdot[1 - F_X(x)])$ by the first approximation in the Taylor expansion of $\ln F_X(x)$. The probability density function (PDF) of $X_{(n)}$ is given by $n\cdot[F_X(x)]^{n-1}\cdot f_X(x)$. Thus, the expectation of $T_{(n)}\left(= \frac{\min(X_{(n)},D)}{X_{(n)}}\right)$ is obtained in a closed form as follows.

$$MGEE\left(T_{(n)}\right) = n\cdot\int_0^1 \frac{D}{t}\cdot\left[F_X\left(\frac{D}{t}\right)\right]^{n-1}\cdot f_X\left(\frac{D}{t}\right)dt + [F_X(D)]^n \tag{6}$$

Without a loss of generality, we can define MGEE of $T_{(i)}\left(= \frac{\min(X_{(i)},D)}{X_{(i)}}\right)$ from equation (6), which can be delivered as follows.

$$MGEE\left(T_{(i)}\right) = \int_0^1 \frac{D}{t}\cdot f_{X_{(i)}}\left(\frac{D}{t}\right)dt + F_{X_{(i)}}(D) \tag{7}$$

In a hypothetical population of 100 nuclear power plants, the right-hand column of Table 1 shows that the risk at the worst plant can be underestimated by an order of magnitude. All of the results were based on a simulation due to less tractable distributions.

3.2. Measures of MPE and MPEE

MGE [1] measures the gross average on the degree of underestimation due to defaults. According to the circumstances, however, regulators may have an attribute to be more concerned about only the degree of pure underestimation of the regulated risks (*i.e.*, only the case of $X > D$), because they have a natural tendency to seek large safety margins as mentioned before. Moreover, if they have to set a new regulatory default, they may be concerned about the maximum pure underestimation on the risks disclosed to them by a regulated party in the future. Thus, the risk estimate disclosed to regulators by a regulated party will be simply defined as $Y = D$, given $X > D$. MGE can no longer be appropriate for reflecting such tendency of the regulators because it is the gross averaged measure of underestimation over the whole range of X. Measure of MGE also has a property in that the more left-skewed the distribution of a regulated population is (*e.g.*, a lognormal with a long tail), the less the degree of underestimation that may result.

Considering the diverse concerns of regulators on their regulatory problems, another measure, the so-called maximum pure effect (MPE) can be suggested, as follows.

$$MPE = E(T|T<1) = E\left(\frac{D}{X}|X>D\right) = \int_0^1 \frac{D}{t}\cdot f_X\left(\frac{D}{t}\right)dt \tag{8}$$

Note that MPE is defined as a conditional expectation and corresponds to the first term in the right hand side of equation (3), which is related to MGE. In other words, it means the pure effect of underestimation due to the default specified by regulators.

Preliminary simulation analyses of this model have been undertaken for lognormal distributions with an equal median (0.001) but different error factors (*e.g.*, 3, 10, 30, 100, respectively), as shown in Table 2. The results of the simulation analyses show that the magnitude of the maximum pure effect of an underestimation is in reverse order, compared with those of MGE. In other words, the more left-skewed with a long tail rightwards is the distribution of a regulated population, the more the degree of underestimation may result.

Similar to MPE, the conditional expectation of the order statistics of extreme risk, the so-called Maximum Pure Effect of Extreme (MPEE), can also be defined as the first term in the right-hand of equation (6), which is related to MGEE.

$$MPEE\left(T_{(n)}\right) = n\cdot\int_0^1 \frac{D}{t}\cdot\left[F_X\left(\frac{D}{t}\right)\right]^{n-1}\cdot f_X\left(\frac{D}{t}\right)dt \tag{9}$$

Table 2
Underestimation of risks using mean value defaults (MPE, MGE).

Distribution	MPE	MGE
Lognormal (error factor = 3)	0.66	0.88
Lognormal (error factor = 10)	0.50	0.90
Lognormal (error factor = 30)	0.47	0.93
Lognormal (error factor = 100)	0.44	0.96

3.3. Generalized gross effect

As mentioned in Section 3.1, the use of a rigid assumption on the regulated party behavior (*i.e.*, perfect choice of the minimum to disclose with the perfect knowledge about the value of X) yields the results likely to be the upper bounds on the effects of conservatism that might be observed from regulatory defaults in the real world, because they effectively assume perfect gaming. In practice, however, regulated parties will frequently have to decide whether to use realistic or default parameter values and assumptions in advance either due to the cost of performing a realistic risk analysis, or because of regulatory sanctions for failing to disclose the available risk results. In this case, a regulated party will have imperfect knowledge of the risk level that a realistic risk analysis would reveal. Thus, regulated parties will frequently have some non-zero probability of performing and disclosing the results of realistic risk analyses even when they are less favorable than the default, and correspondingly, of failing to perform realistic risk analyses even when they would have been more favorable than the default. Other factors may also contribute to non-zero disclosure probabilities for unfavorable risk estimates. For example, some companies may have a corporate policy of developing and disclosing realistic risk estimates as a way to encourage a strong safety culture among their employees, or as a way to build credibility with regulators. In addition, in some industries (such as nuclear power), companies have a substantial economic self-interest in knowing and controlling their risk levels, since these risks affect the value of the company's assets.

Considering the regulated party behaviors above, two kinds of probabilities can be simply defined as shown in the dichotomy of Table 3. In other words, p is defined for disclosure probability of unfavorable risk estimates (*i.e.*, probability of disclosing the results of realistic risk analyses even when they are less favorable than the default), and q for waiver probability of favorable risk estimates (*i.e.*, probability of failing to perform realistic risk analyses even when they would have been more favorable than the default). Thus, the risk estimate disclosed to regulators by a regulated party will be given by

$$Y = \|X, D\|_p \cdot I(X \rangle D) + \|D, X\|_q \cdot I(X \le D) \tag{10}$$

where I is the index function, and $\|A, B\|_C$ represents the function that choose $A(B)$ with the probability of $c(1-c)$. Note that equation (3) is a special case of equation (7) such that $p = q = 0$.

According to the general formulation in section 2, the expectation of T, so-called generalized gross effect (GGE), is defined as a new measure to evaluate the effect of conservatism implicated in regulatory defaults in the estimates of risk. GGE can be obtained in closed form for an arbitrary distribution of regulated population as follows.

Table 3
Dichotomy of regulated party behavior.

Situation	Disclosure	
	Realistic Risk Estimate (X)	Default (D)
$X > D$	p	$1-p$
$X \le D$	$1-q$	q

$$GGE = p\cdot(1-F_X(D)) + (1-q)\cdot F_X(D) + (1-p)\cdot\int_0^1 \frac{D}{t}\cdot f_X\left(\frac{D}{t}\right)dt + q\cdot\int_1^\infty \frac{D}{t}\cdot f_X\left(\frac{D}{t}\right)dt \tag{11}$$

In addition, the variance of T can also be calculated as follows.

$$Var(T) = p\cdot(1-F_X(D)) + (1-q)\cdot F_X(D) + (1-p)\cdot D\cdot\int_0^1 f_X\left(\frac{D}{t}\right)dt + q\cdot D\cdot\int_1^\infty f_X\left(\frac{D}{t}\right)dt - [E(T)]^2 \tag{12}$$

The range of GGE is $[0,\infty]$, dissimilar to MGE and MPE, because it covers the evaluation of the degree of underestimation as well as overestimation in risks disclosed by regulated parties. In addition, the degree of underestimation or overestimation depends on the values of both p and q. In other words, $GGE < 1.0$ in equation (11) means that the risk estimate disclosed by a regulated party will be underestimated by the degree of average $[1 - GGE] \times 100\%$ than the real risk estimate. Meanwhile, $GGE > 1.0$ presents the degree of overestimation on average $[GGE - 1] \times 100\%$. We can easily find that MGE is a special case of GGE such that it is obtained by setting $p = q = 0$ in equation (11).

According to the definitions of equation (10), the expectation of order statistics for extreme risk, so-called Generalized Gross Effect of Extreme (GGEE) can be delivered as follows.

$$GPEE\left(T_{(n)}\right) = n\cdot(1-p)\cdot D\int_p^1 \frac{t}{(t-p)^2}\cdot\left[F_X\left(\frac{1-p}{t-p}\cdot D\right)\right]^{n-1}\cdot f_X\left(\frac{1-p}{t-p}\cdot D\right)dt + n\cdot q\cdot D\int_1^\infty \frac{t}{\{t-(1-q)\}^2}\cdot\left[F_X\left(\frac{q}{t-(1-q)}\cdot D\right)\right]^{n-1}\cdot f_X\left(\frac{q}{t-(1-q)}\cdot D\right)dt \tag{13}$$

Without a loss of generality, GGEE of $T_{(i)}$ from equation (13) is as follows.

$$GPEE\left(T_{(i)}\right) = (1-p)\cdot D\int_p^1 \frac{t}{(t-p)^2}\cdot f_{X_{(i)}}\left(\frac{1-p}{t-p}\cdot D\right)dt + q\cdot D\int_1^\infty \frac{t}{\{t-(1-q)\}^2}\cdot f_{X_{(i)}}\left(\frac{q}{t-(1-q)}\cdot D\right)dt \tag{14}$$

4. Method for setting regulatory defaults

4.1. Basic approach

The problem of how defaults are chosen in risk-related regulatory matter totally depends on the effects of conservatism implicated in defaults on the estimates of risk. The results presented in Table 1 suggest that a systematic and substantial underestimation of the most severe risks may arise when defaults are set near the population means, particularly if the population exhibits significant heterogeneity. If more conservative defaults are therefore desirable, a simulation analysis of the model described in MGE (and more sophisticated variants, MGEE) can provide guidance on how conservatively the default D ought to be chosen in order to achieve a desired regulatory result. Namely, the question regulators wish to examine is "At what percentile of the distribution $f_X(x)$ should D be set if we wish to ensure that risk is underestimated by no more than $\alpha \times 100\%$ on average, and/or no more than $\beta \times 100\%$ at the worst site?" [1]. It can be formulated as follows.

$$D^* \equiv Max_D^{-1} F_X(D) \in \Omega, \text{ such that}$$

$$MGE \geq 1-\alpha \text{ and/or } MGEE \geq 1-\beta, \quad (15)$$

where D^* is the decision chosen from space Ω, the domain of X. The notation $D^* \equiv Max_D^{-1} F_X(D) \in \Omega$ means D^* is a value of D such that $F_X(D)$ is a maximum.

Note in inequality (15) that, according to the interests of regulators, they can replace MGE with either MPE or GGE suggested in the paper. In addition, instead of MGEE, regulators can use one of such measures as MPEE, GGEE etc.

4.2. Decision theoretic approach

In what we called a classical decision analysis, the goal is to seek the optimal decision D^* from the quantity of interest X, with our uncertainty expressed as probability distribution $f_X(x)$, the value parameters θ, and the domain parameter Ω. To obtain optimal decisions, a variety of decision criteria can be additionally introduced as an input to the decision analysis. Of them, the use of the maximum expected utility (MEU) or minimum expected loss (MEL) is the most popular in classical decision analyses. Including MEL as a decision criterion, a conceptual decision model can be formulated as follows [9].

$$Z(X, \theta, \Omega, MEL) \rightarrow D^* \quad (16)$$

Here, an optimal decision D^* of the model can be affected by uncertainty about the functional relationship Z, where Z incorporates the model structure being employed. The methods of operations research can basically provide a wide variety of methods of optimization, which produce an optimal solution D^* given the specified quantities, values of the parameters, and structure of the model.

If the classical decision model of equation (16) is applied to our problem of choosing defaults, a generalized decision model from the structure of equation (15) can be suggested with some constraints (*e.g.*, MGE and MGEE) as follows.

$$D^* \equiv Min_D^{-1} E[L(X,D)], \text{ such that}$$

$$MGE \geq 1-\alpha \text{ and/or } MGEE \geq 1-\beta, \quad (17)$$

where $L(X,D)$ is the loss function of the regulators (ultimately, the loss of public), $E[L(X,D)] \equiv \int_X L(x,D) \cdot f_X(x)dx$ denotes the expectation (over X) of the loss function for D, and finally D^* means the optimal decision (default) that minimizes the expected loss function.

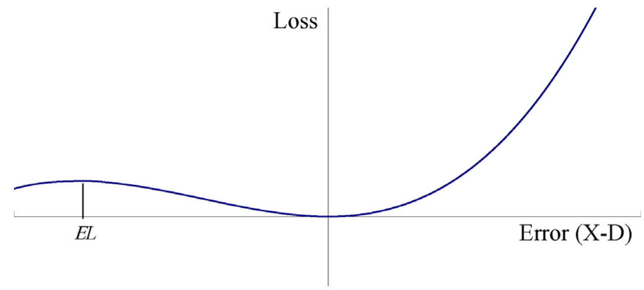

Fig. 1. Cubic loss function.

First, let us consider the loss functions appropriate in the paper. Asymmetric loss functions are required in the proposed decision model, since disclosing a default under the situation of $X > D$ (*i.e.*, when the results of the realistic risk analyses are less favorable than the default) generally brings a more severe loss than failing to perform realistic risk analyses under the situation of $X \leq D$ (*i.e.*, when the results of the realistic risk analyses would have been more favorable than the default). Therefore, a cubic loss function [9] can be primarily considered as a reasonable approximation for a wide variety of asymmetric and smooth functions in the present problem, which is given by

$$L(X,D) = a(X-D)^3 + b(X-D)^2, \quad (18)$$

where $a > 0$, $b > 0$, and it is plausible that the decision is constrained to be within the range, $(X-D) > EL\left(= -\frac{2b}{3a}\right)$. This cubic loss function is depicted in Fig. 1.

A bilinear loss function [9] is also applicable to the proposed decision model as follows.

$$L(X,D) = a(X-D) \cdot I(X > D) + b(X-D) \cdot I(X \leq D), \quad (19)$$

where $a > 0$, $b < 0$. This loss function is a simple asymmetric one, and is illustrated in Fig. 2.

The application of loss functions illustrated in equations (18) and (19) can bring us a more detailed, concrete formulation of the proposed decision problem. However, the preference and utility functions of both regulators and regulated parties need to be investigated in detail.

5. Conclusions

The topic of how the defaults are chosen depends on the effects of conservatism implicated in regulatory defaults on estimating the risks, particularly in risk-related regulations. Without any universal

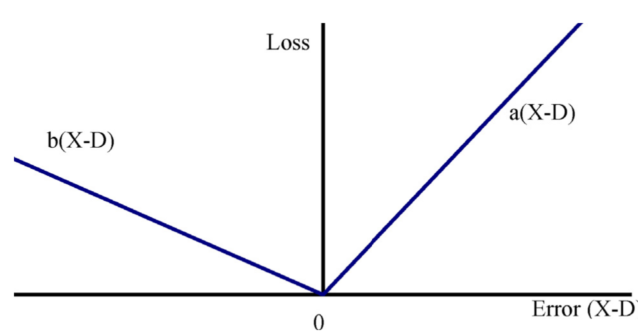

Fig. 2. Bilinear loss function.

agreement on the approaches dealing with the conservatism of defaults, the desirability of conservatism in regulatory risk analyses has long been controversial. The opponent views it as needlessly costly and irrational, and the proponent as a form of protection against possible omissions or underestimation of risks. Moreover, large heterogeneity for the quantity of risk in a regulated population makes it difficult to set suitable defaults.

First, this research focused on the effects of different levels of conservatism implicated in regulatory defaults on the estimates of risk. According to the postulated behaviors of the regulated parties and the diversity of interests of the regulators, such measures as MGE, MPE, GGE, MGEE, MPEE, GGEE, *etc.* Were developed, and their properties were explored. Second, new approaches to setting defaults in regulatory matters were proposed, based on the suggested measures. In summary, the proposed research can help decision makers evaluate the levels of safety likely to result from the current or future regulatory policies.

Further research areas are summarized as follows.

(i) New measures (*e.g.*, MPE, GPE, MPEE, GGEE, *etc.*) on the effect of conservatism implicated in the regulatory defaults on the estimates of risk need to be investigated in detail in the proposed research. Analytic solutions and/or simulation analyses for a wide variety of distributions are required to understand the characteristics of these measures.

(ii) More careful attention should be paid to a comparative study on the measures.

(iii) Investigation on the asymptotic properties of the maximum effect of an underestimation of extreme risk needs to be conducted in the future because it can grant us more robust solutions.

(iv) To resolve the problem of “How conservatively should defaults be chosen?”, the new decision-making procedures need to be explored including the preference and utility functions of both regulators and regulated parties.

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Appendix A. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.net.2018.07.006.

Appendix. A Proofs of the Results on T

By definition, the quantity of interest, X, is a positive variate and the cumulative distribution function (CDF) of X is known, i.e., $F_X(x) = P(X \le x)$. Then, the domain of $T(= Y/X)$ becomes $0 \le T \le 1$, there is the mass probability at $T = 1$ (i.e., $X \le D$), and the CDF of T for $0 \le T < 1$ can be derived by $P(T \le t) = P(D/X \le t) = P(D/t \le X) = 1 - F_X(D/t)$. Thus, CDF of T, $G_T(t)$, may be described below.

$$G_T(t) = \begin{cases} 0 & , T < 0 \\ 1 - F_X\left(\dfrac{D}{t}\right) & , 0 \le T < 1 \\ 1 & , T \ge 1 \end{cases} \quad \text{(A1)}$$

Reimann-Stieltjes integration leads from equation (A1) to the expectation of an arbitrary function of T as follows.

$$E[h(T)] = \int_0^1 h(t) \cdot dG_T(t) + h(1) \cdot \triangle G_T(1) \quad \text{(A2)}$$

where $\triangle G_T(1)$ means the mass probability at $T = 1$, and corresponds to $F_X(D)$. Substituting T for $h(T)$ in equation (A2), the expectation of T can be obtained in a closed form, as shown in equation (3). The variance of T in equation (4) can also be obtained using the similar method described above.

As an illustration, the analytical results obtained for uniform and exponential distributions in Table 1 can be derived from equations (3) and (4). If the quantity of interest across the regulated population follows a uniform distribution over $[a,b]$, the expectation and variance of T are given as follows.

$$E(T) = \frac{D}{b-a} \cdot \left\{ 1 + \ln\left(\frac{b}{D}\right) \right\} \quad \text{(A3)}$$

$$Var(T) = \frac{D}{b-a} \cdot \left(2 - \frac{D}{b}\right) - [E(T)]^2 \quad \text{(A4)}$$

Note that $D/b \le T \le 1$. If D is set to the mean of X, i.e., $(a+b)/2$, and $a = 0$, then the expectation and variance of T become 0.8466 and 0.0333, respectively, regardless of the value of b. This means that the risk estimate disclosed by the regulated party is a maximum of 15% lower than the realistic risk estimate. In other words, this presents the maximum effect that may be underestimated by the regulatory default of the mean value.

For a quantity of interest, X, which follows an exponential distribution with parameter λ, expectation and variance of T may be approximately calculated using a Taylor series expansion as follows.

$$E(T) = 1 - e^{-\xi} - \xi \cdot \left\{ A + \ln \xi + \sum_{n=1}^{\infty} \frac{(-\xi)^n}{n \cdot n!} \right\} \quad \text{(A5)}$$

$$Var(T) = 1 - (1-\xi) \cdot e^{-\xi} + \xi^2 \cdot \left\{ A + \ln \xi + \sum_{n=1}^{\infty} \frac{(-\xi)^n}{n \cdot n!} \right\} - [E(T)]^2 \quad \text{(A6)}$$

where $\xi = \lambda D$ and A stands for Euler's constant (0.57721 …). If the regulatory default is equal to the mean, $1/\lambda$, then the mean and variance of T are approximately given by 0.8515 and 0.05554, respectively, as shown in Table 1.

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