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A New Constraint Handling Method for Economic Dispatch

Xueping Li†, Canwei Xiao* and Zhigang Lu*

Abstract – For practical consideration, economic dispatch (ED) problems in power system have non-smooth cost functions with equality and inequality constraints that makes the problems complex constrained nonlinear optimization problems. This paper proposes a new constraint handling method for equality and inequality constraints which is employed to solve ED problems, where the incremental rate is employed to enhance the modification process. In order to prove the applicability of the proposed method, the study cases are tested based on the classical particle swarm optimization (PSO) and differential evolution (DE) algorithm. The proposed method is evaluated for ED problems using six different test systems: 6-, 15-, 20-, 38-, 110- and 140-generators system. Simulation results show that it can always find the satisfactory solutions while satisfying the constraints.

Keywords: Economic dispatch, Constraint handling, Incremental rate, Particle swarm optimization, Differential evolution.

1. Introduction

Economic dispatch (ED) is a complex problem in power system operation. The objective of ED problems is to minimize the generating cost under the premise of meeting load demand and operational constraints. Therefore, it is necessary to establish a complete and accurate mathematical model for dealing with ED problems in practice. There are some intelligence optimization algorithms for solving ED problems, e.g., simulated annealing (SA) [1], real-coded chemical reaction optimization (RCCRO) [2], differential evolution (DE) algorithm [3], mean-variance optimization (MVO) [4], particle swarm optimization (PSO) [5], biogeography based optimization (BBO) [6], teaching-learning based optimization (TLBO) [7], artificial bee colony algorithm (ABC) [8] and invasive weed optimization (IWO) [9]. However, premature convergence phenomenon often exists in these methods.

In order to overcome the limitation of heuristic intelligence optimization algorithms, a number of researchers have made various improvements to algorithms in recent years. Chaturvedi [10] introduced a new parameter automation strategy employed in PSO using time varying acceleration coefficients (PSO_TVAC) to avoid premature convergence. Thanushkodi [11] modeled and embedded the anti-predatory activity in PSO (APSO). Aiming at alleviating the shortcomings of the global PSO (GPSO), an orthogonal PSO (OPSO) in [12] divided the particles of the swarm into two groups and employed the orthogonal diagonalization process. A random drift PSO (RDPSO) algorithm was presented in [13] where the update equation of the particle's velocity was modified fundamentally. Niknam [14] integrated a new mutation with adaptive PSO (NAPSO) and tuned the inertia weight using fuzzy IF/THEN rules (FAPSO). Khamsawang [15] proposed a distributed Sobol approach based on PSO and tabu search algorithm (DSPSO-TSA), including the Sobol sequence to generate an inertia factor and a distributed process to reach the global solution rapidly. A memory based differential evolution (MBDE) algorithm was proposed in [16] with two swarm operators and a elitism operator. A differential evolution algorithm using self-adapting control parameters (SADE) was introduced in [17], where the mutation factor and crossover rate are self-adapting controlled in each generation. A modified differential evolution (MDE) algorithm was presented in [18] where a comprehensive trial vector generation strategy and the initial control parameters were studied. An improved differential evolution (IDE) algorithm including two mutation operators, a dynamical crossover rate and a population randomization, was introduced in [19], which improves the global searching ability. Bhattacharya [20] combined DE with BBO (DE/BBO), improving the searching ability and convergence speed of DE utilizing BBO algorithm effectively. A multi-strategy ensemble biogeography-based optimization (MsEBBO) based method was proposed in [21] to balance exploration and exploitation. A quasi-reflection oppositional BBO (OBBO) in [22] employed opposition-based learning along with BBO and used quasi-reflected numbers for population initialization and generation jumping. Simulated annealing with best move (SAB) and first move (SAF) based optimization in [23] has better performance in solving large scale ED problems. Bhattacharjee introduced an oppositional real coded chemical reaction optimization (ORCCRO) algorithm in [24] which improves the effectiveness, convergence speed

† Corresponding Author: Key Lab of Power Electronics for Energy Conservation and Motor Drive of Hebei Province, Yanshan University, Qinhuangdao, Hebei, 066004, China. (lixueping@ysu.edu.cn)

* Key Lab of Power Electronics for Energy Conservation and Motor Drive of Hebei Province, Yanshan University, Qinhuangdao, Hebei, 066004, China.(514393546@qq.com, zhglu@ysu.edu.cn)

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and robustness of solutions. An oppositional invasive weed optimization (OIWO) was advanced in [25] whose merit is high accuracy and good convergence characteristics and robustness. Kim used a mean-variance optimization algorithm with Kuhn-Tucker condition and swap process (KMVO) in [26] to improve the global minimum searching capability. A swarm based mean-variance mapping optimization (MVMOS) proposed in [27] applies the special mapping function for the mutation based on the mean and variance of n-best population. Serdar [28] used Iterated F-Race method in incremental artificial bee colony optimization (IABC) and incremental artificial bee colony optimization with local search algorithms (IABC-LS).

In practice, besides the influence of the algorithm, the equality and inequality constraints have great influence on the solution for the ED problem. So far there have been a large number of studies on constraint handling problems. Penalty method is one of the most commonly used methods to solve constraint handling problems. Homaifar [29] proposed an approach where the user defined several levels of violation, and a penalty coefficient is chosen for each level in such a way that the penalty coefficient increases as the user reached higher levels of violation. Penalty functions consisting of squared or absolute violations were used in [10] to reduce the fitness of the particle. Parsopoulos [30] introduced a penalty method where the penalty value was dynamically adjusted in the process of operation. There were also some combinations with other mathematical concepts for constraint handling purposes. A technique that handles constraints and objectives separately was proposed in [31] based on a co-evolutionary model and an analogy with a predator–prey model. Based on the discrete output constraint of generator, some discrete constraints of a generator were reconstructed into an inequality constraint according to the zero theorem of the function in [32]. A constraint treatment strategy applied to conventional PSO (CTPSO) in [33] combined chaotic sequences with conventional linearly decreasing inertia weights. A strategy in PSO dealing with equality constraints was presented in [34], where the variables from 1st to n-1th dimension were generated by the equations for modifying the velocity and position while the remaining variables of nth dimension were used to balance the equality constraints. A constraint handling mechanism in genetic algorithm with selecting the global best was proposed in [35], and a perturbation operator was defined to maintain the diversity of the population.

However, most existing constraint processing methods need to make complex improvements for the algorithm to obtain a better solution. Based on this situation, a new constraint handling method is proposed in this paper to solve ED problems. A modified repair process is used to facilitate the satisfaction of the equality constraint. This method takes the derivative of the objective function as the incremental rate and adjusts the output of the unit by sharing the unbalanced amount of equality constraint violation of the system. Without enhancing the algorithms, satisfactory results can be obtained using classical PSO and DE with proposed incremental rate method (PSO-IR and DE-IR).

This paper is organized as follows: Section 2 recalls the formulation of the ED problem. Section 3 presents the new constraint handling method. Six different test systems based on classical PSO and classical DE are conducted in Section 4. Section 5 outlines the conclusions.

2. Formulation of the Economic Dispatch Problem

The ED problem is to obtain the minimum value of the objective function of the total fuel cost under the condition of satisfying some equality and inequality constraints.

2.1 Objective function

The objective function of ED problem is to minimize the total fuel cost, which is formulated as follows:

$$F_C = \sum_{i=1}^{N} (a_i P_i^2 + b_i P_i + c_i) \tag{1}$$

where P_i is the power output of generator unit *i*; a_i, b_i, c_i are the cost coefficients of generator unit *i*; N is the total number of generators in the power system.

2.2 Constraints

2.2.1 System power balance

$$\sum_{i=1}^{N} P_i = P_D + P_{Loss} \tag{2}$$

where P_D is the total load demand; P_{Loss} is the transmission loss.

2.2.2 Transmission loss

Calculation of P_{Loss} using the **B**-matrix loss coefficients is expressed as a quadratic function:

$$P_{Loss} = \sum_{i=1}^{N}\sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{0i} P_i + B_{00} \tag{3}$$

where B_{ij}, B_{0i}, B_{00} are the **B**-matrix loss coefficients.

2.2.3 Generator capacity

$$P_i^{min} \le P_i \le P_i^{max} \tag{4}$$

where P_i^{min}, P_i^{max} are the lower and upper limit of generator unit *i*.

2.2.4 Ramp rate limits

$$\max(P_i^{min}, P_i^0 - DR_i) \le P_i \le \min(P_i^{max}, P_i^0 + UR_i) \quad (5)$$

where P_i^0 is the previous output power, UR_i and DR_i are the up-ramp and down-ramp limits of generator unit *i*.

2.2.5 Prohibited operating zones

$$P_i \in \begin{cases} P_i^{min} \le P_i \le P_{i,1}^l \\ P_{i,j-1}^u \le P_i \le P_{i,j}^l \quad j = 2,3,\ldots,pz_i \\ P_{i,pz_i}^u \le P_i \le P_i^{max} \end{cases} \quad (6)$$

where $P_{i,j}^u, P_{i,j}^l$ are the upper and lower limits of the prohibited operating zones of generator unit *i* as MW and pz_i is the prohibited operating zone number of generator unit *i*.

3. The New Constraint Handling Method

This method uses the incremental rate in the unbalanced amount sharing process to facilitate the satisfaction of the equality constraint. λ_i is defined as the incremental rate of unit *i* which is calculated using (7).

$$\lambda_i = 2a_i P_i + b_i \quad (7)$$

Its implementation consists of the following steps:

Step 1: Select the candidate solution which does not satisfy the equality constraint in the current iteration of the algorithm for solving the ED problem.

In the *k*th loop, the difference ΔP_k between total power output and total load demand is calculated using (8).

$$\Delta P_k = \sum_{i=1}^{N} P_{i,k} - P_D - P_{Loss} \quad (8)$$

where $P_{i,k}$ is the power output of unit *i* in the k th loop.

Step 2: Determine whether to increase or decrease the total power of the system based on the value of equality constraint violation and handle constraints. When $\Delta P_k \ne 0$, the incremental rate of the unit or its reciprocal is used as a regulation factor to share the unbalanced amount of equality constraint violation of the system. When $\Delta P_k = 0$, go to *Step 5* to deal with inequality constraints.

If $\Delta P_k > 0$, the regulation factor of unit i $R_{i,k}$, the adjusted ΔP_k and the adjusted $P_{i,k}$ are calculated using (9)-(13).

$$\Delta P_k = \begin{cases} \Delta P_k + (P_i^{min} - P_{i,k}), \quad \text{If } P_{i,k} \le P_i^{min} \\ \Delta P_k, \quad \text{Otherwise} \end{cases} \quad (9)$$

$$P_{i,k} = \begin{cases} P_i^{min}, \ \text{If } P_{i,k} \le P_i^{min} \\ P_{i,k}, \ \text{Otherwise} \end{cases} \quad (10)$$

The processing procedure is shown in (11) and (12) when the down-ramp rate limit is exceeded.

$$\Delta P_k = \begin{cases} \Delta P_k + \left(P_i^0 - P_{i,k} - DR_i\right), \\ \quad \text{If } P_i^0 - P_{i,k} \ge DR_i \\ \Delta P_k, \quad \text{Otherwise} \end{cases} \quad (11)$$

$$P_{i,k} = \begin{cases} P_i^0 - DR_i, \ \text{If } P_i^0 - P_{i,k} \ge DR_i \\ P_{i,k}, \ \text{Otherwise} \end{cases} \quad (12)$$

$$R_{i,k} = \begin{cases} 0, \ \text{If } P_{i,k} \le P_i^{min} \text{ or } P_i^0 - P_{i,k} \ge DR_i \\ \quad \text{or } P_{i,j}^l \le P_{i,k} \le P_{i,j}^u \\ \lambda_{i,k}, \ \text{Otherwise} \end{cases} \quad (13)$$

where $\lambda_{i,k}$ is the incremental rate of unit i in the k th loop.

In this situation, the system power output need to be reduced, here $R_{i,k}$ is the incremental rate of unit i. For one unit with large incremental rate, when its power increases, the system consumption rise fast, so the power of the unit should be reduced. As the power of each unit will be adjusted downward to varying degrees in this situation, so when $P_{i,k} > P_i^{max}$ or $P_{i,k} - P_i^0 > UR_i$, $P_{i,k}$ is likely to be adjusted to the appropriate value at *Step 4* and must be adjusted at *Step 5*.

If $\Delta P_k < 0$, related parameters can be calculated using (14)-(18):

$$\Delta P_k = \begin{cases} \Delta P_k - (P_{i,k} - P_i^{max}), \ \text{If } P_{i,k} \ge P_i^{max} \\ \Delta P_k, \ \text{Otherwise} \end{cases} \quad (14)$$

$$P_{i,k} = \begin{cases} P_i^{max}, \ \text{If } P_{i,k} \ge P_i^{max} \\ P_{i,k}, \ \text{Otherwise} \end{cases} \quad (15)$$

The processing procedure is shown in (16) and (17) when the up-ramp rate limit is exceeded.

$$\Delta P_k = \begin{cases} \Delta P_k - \left(P_{i,k} - P_i^0 - UR_i\right), \\ \quad \text{If } P_{i,k} - P_i^0 \ge UR_i \\ \Delta P_k, \quad \text{Otherwise} \end{cases} \quad (16)$$

$$P_{i,k} = \begin{cases} P_i^0 + UR_i, \ \text{If } P_{i,k} - P_i^0 \ge UR_i \\ P_{i,k}, \ \text{Otherwise} \end{cases} \quad (17)$$

$$R_{i,k} = \begin{cases} 0, \ \text{If } P_{i,k} \ge P_i^{max} \text{ or } P_{i,k} - P_i^0 \ge UR_i \\ \quad \text{or } P_{i,j}^l \le P_{i,k} \le P_{i,j}^u \\ \dfrac{1}{\lambda_{i,k}}, \ \text{Otherwise} \end{cases} \quad (18)$$

In this situation, the system power output need to be increased, here $R_{i,k}$ is the reciprocal of the incremental rate of unit i. For one unit with small incremental rate, when its power increases, the system consumption rise slowly, so the power should be raised. As the power of each unit will be adjusted upward to varying degrees, so when $P_{i,k} < P_i^{min}$ or $P_i^0 - P_{i,k} > DR_i$, $P_{i,k}$ is likely to be adjusted to the appropriate value at *Step 4* and must be adjusted at *Step 5*.

In the both situations mentioned above, if $P_{i,k}$ is in the prohibited operating zones, namely if $P_{i,j}^l \le P_{i,k} \le P_{i,j}^u$, $P_{i,k}$ will have the same possibility to be adjusted to the upper or lower boundary according to (19) and (20).

$$\Delta P_k = \begin{cases} \Delta P_k + P_{i,j}^l - P_{i,k} - \varepsilon, & \text{If } r < 0.5 \\ \Delta P_k - \left(P_{i,k} - \left(P_{i,j}^u + \varepsilon\right)\right), & \text{Otherwise} \end{cases} \tag{19}$$

$$P_{i,k} = \begin{cases} P_{i,j}^l - \varepsilon, & \text{If } r < 0.5 \\ P_{i,j}^u + \varepsilon, & \text{Otherwise} \end{cases} \tag{20}$$

where ε is set to 0.000001, *r* is set as a random number in [0,1].

Step 3: Calculate $\Delta P_{i,k}$ which is introduced as the unbalanced power sharing amount of unit i using (21).

$$\Delta P_{i,k} = \Delta P_k \times R_{i,k} / \sum_{j=1}^{k} R_{i,j} \tag{21}$$

Step 4: Calculate the output of unit i $P_{i,k+1}$ which is passed to the $k+1$th loop using (22).

$$P_{i,k+1} = P_{i,k} - \Delta P_{i,k} \tag{22}$$

When $\Delta P_k > 0$, $\lambda_{i,k}$ is larger, then $R_{i,k}$ is larger, the allocation of unit *i* in ΔP_k is more, namely $\Delta P_{i,k}$ is larger, and the final output decrease of unit *i* will be greater. When $\Delta P_k < 0$, similar to the above regulation, $\lambda_{i,k}$ is smaller, the final output increase of unit i will be greater. By constant loops, the unbalanced amount will gradually approach to 0 until the equality constraint is satisfied.

Step 5: If $\Delta P_k = 0$ that equality constraint is satisfied, the generator capacity limits and ramp rate limits will be dealt by (23) and (24).

$$P_{i,k} = \begin{cases} P_i^{max}, & \text{If } P_{i,k} > P_i^{max} \\ P_i^{min}, & \text{If } P_{i,k} < P_i^{min} \\ P_{i,k}, & \text{Otherwise} \end{cases} \tag{23}$$

$$P_{i,k} = \begin{cases} P_i^0 - DR_i, & \text{If } P_i^0 - P_{i,k} > DR_i \\ P_i^0 + UR_i, & \text{If } P_{i,k} - P_i^0 > UR_i \\ P_{i,k}, & \text{Otherwise} \end{cases} \tag{24}$$

Go to the $k+1$th loop until the equality and inequality constraints are satisfied.

The pseudo-code of the processing of constraints is shown in Fig. 1.

4. Numerical Case Study

4.1 Classical PSO

PSO algorithm was proposed by Kennedy and Eberh in 1995 [36]. In a d-dimensional search space with m particles, the velocity vectors and position vectors of the i th particle can be represented as $V_i = \left(v_{i1}, v_{i2}, \cdots, v_{id}\right)$ and $X_i = \left(x_{i1}, x_{i2}, \cdots, x_{id}\right)$. In each iteration, the best previous position of the i th particle is recorded as $pbest_i = \left(p_{i1}, p_{i2}, \cdots, p_{id}\right)$. If the g th particle is the best among all particles in the group so far, it is represented as $gbest_g = \left(p_{g1}, p_{g2}, \cdots, p_{gd}\right)$. The equations for modifying the velocity and position of each particle for fitness evaluation in the $k+1$th iteration are (25) and (26):

$$v_{id}^{k+1} = \omega \times v_{id}^k + c_1 \times rand_1 \times \left(pbest_{id} - x_{id}\right) + c_2 \times rand_2 \times \left(gbest_{gd} - x_{id}\right) \tag{25}$$

$$x_{id}^{k+1} = x_{id} + v_{id}^{k+1} \tag{26}$$

where c_1 and c_2 are acceleration coefficients. $rand_1$ and $rand_2$ are random numbers between 0 and 1, and ω is the inertia weight parameter. In general, ω is set according to (27):

$$\omega = \omega_{max} - \left(\omega_{max} - \omega_{min}\right) \times \frac{k}{iter} \tag{27}$$

where $iter$ is the maximum number of iterations.

Using the equations above, $gbest$ and $pbest$ values are updated with the number of iterations until the optimal solution is obtained.

4.2 Classical DE

DE algorithm, first introduced by Storn and Price [37], has emerged as a powerful stochastic search technique. Due to its simple structure, easy implementation, few control parameters and good search capability, DE has been frequently applied to various practical problems. In a d-dimensional search space with N_p candidate solutions generated randomly, new candidate solutions are generated by mutation and crossover using the information of the solutions in the current population. The mutation operator used in this paper is DE/rand/1 which is given by:

$$v_j^G = x_{j_1}^G + F \times \left(x_{j_2}^G - x_{j_3}^G\right) \tag{28}$$

where v_j is the mutant vector of the j th individual, and

```
(Step 1)
Suppose ε=1e-6; ΔP_k = Σ_{i=1}^{N} P_{i,k} − P_D − P_loss; Recal = 1
(Step 2)  while Recal = 1 do
  if ΔP_k > 0 then
  for i=1 to N do
    R_{i,k} = 2a_i P_{i,k} + b_i
      if P_{i,k} ≤ P_i^min then
        ΔP_k = ΔP_k + (P_i^min − P_{i,k});
        P_{i,k} = P_i^min ; R_{i,k} = 0
      end if
      if P_i^0 − P_{i,k} ≥ DR_i then
        ΔP_k = ΔP_k + (P_i^0 − P_{i,k} − DR_i)
        P_{i,k} = P_i^0 − DR_i ; R_{i,k} = 0
      end if
  end for
  else if ΔP_k < 0 then
  for i=1 to N do
    R_{i,k} = 1/(2a_i P_{i,k} + b_i)
      if P_{i,k} ≥ P_i^max then
        ΔP_k = ΔP_k − (P_{i,k} − P_i^max);
        P_{i,k} = P_i^max ; R_{i,k} = 0
      end if
      if P_{i,k} − P_i^0 ≥ UR_i then
        ΔP_k = ΔP_k − (P_{i,k} − P_i^0 − UR_i)
        P_{i,k} = P_i^0 + UR_i ; R_{i,k} = 0
      end if
  end for
  end if
  for i=1 to N do
    if P_{i,j}^l ≤ P_{i,k} ≤ P_{i,j}^u then
      if rand( )<0.5 then
      ΔP_k = ΔP_k + P_{i,j}^l − P_{i,k} − ε
      P_{i,k} = P_{i,j}^l − ε ; R_{i,k} = 0
      else
          ΔP_k = ΔP_k − (P_{i,k} − (P_{i,j}^u + ε))
          P_{i,k} = P_{i,j}^u + ε ; R_{i,k} = 0
          end if
      end if
  end for
(Step 3-4)
  P_{i,k+1} = P_{i,k} − ΔP_k × R_{i,k} / Σ_{j=1}^{k} R_{i,j}
  if abs(ΔP_k) > ε then
    Recal = 1
  else Recal = 0
  end if
(Step 5)  for i=1 to N do
    if P_{i,k} < P_i^min then
      P_{i,k} = P_i^min; Recal = 1
    else if P_{i,k} > P_i^max then
      P_{i,k} = P_i^max; Recal = 1
    end if
    if P_i^0 − P_{i,k} > DR_i then
      P_{i,k} = P_i^0 − DR_i; Recal = 1
    else if P_{i,k} − P_i^0 > UR_i then
      P_{i,k} = P_i^0 + UR_i; Recal = 1
    end if
  end for
  for i=1 to N do
    if P_{i,j}^l ≤ P_{i,k} ≤ P_{i,j}^u then
      Recal = 1
    end if
  end for
end while
```

Fig. 1. The pseudo-code of the processing of constraints

F is mutation factor. The three individuals j_1, j_2 and j_3 are selected randomly from the current population. In crossover process, the updating equation of the trail vector u_j^G is given by:

$$u_{j,i}^G = \begin{cases} v_{j,i}^G, & If\ rand_i \le CR\ or\ i = i_n \\ x_{j,i}^G, & Otherwise \end{cases}$$
$$j = 1,2,\cdots,N_p;\ i = 1,2,\cdots,n \quad (29)$$

Where CR is the crossover rate. $rand_i$ and i_n are random numbers with uniform distribution in the interval [0, 1] and [1, n] respectively, where i_n is an integer. Selection process only considers better solutions in each generation, which is represented as:

$$x_j^{G+1} = \begin{cases} u_j^G, & If\ f\left(u_j^G\right) < f\left(x_j^G\right) \\ x_j^G, & Otherwise \end{cases} \quad j = 1,2,\cdots,N_p \quad (30)$$

The three operations above are repeated until the maximum number of generations is reached or the terminating condition is met.

4.3 Case study

In order to evaluate the performance of the proposed method for ED problems, six study cases with one benchmark test system in each case study are tested based on classical PSO and DE. The benchmark systems are 6-, 15-, 20-, 38-, 110- and 140-generators system. All programs are coded and tested in MATLAB 7.1 and performed on a PC with an Intel Pentium CPU (3.2GHz) and 4.0GB RAM. 50 trials are conducted for each case. For all cases, $c_1 = 1.5$, $c_2 = 1.5$, $\omega_{max} = 0.5$, $\omega_{min} = 0.2$, $CR = 0.9$, $F = 0.5$.

MATLAB uses an interpreter that causes to degrade its performance, especially in executing iterative logic such as for-loop [38]. Hence, there is no comparability with the time used by IABC and IABC-LS [28] in test case 1 and test case 2 which simulate using C++.

4.3.1 Test case 1

This test case consists of 6 generating units where transmission loss, ramp rate limits and prohibited operating zones are considered. The system load demand is 1263 MW. The parameters of this system are obtained from [12].

Table 1. 6-generators test system: results of proposed method and previous literature

Methods	Generation costs ($/h)			Time
	Min.	Max.	Average	(s)
NAPSO [14]	15443.7657	15443.7657	15443.7657	NA
FAPSO [14]	15445.244	15451.63	15448.052	NA
OBBO [22]	15442.414	15442.419	15442.415	NA
IABC [28]	15441.108	15441.108	15441.108	0.012
IABC-LS [28]	15441.108	15441.108	15441.108	0.018
GPSO [12]	15442.8334	16103.3400	15458.4000	2.52
OPSO [12]	15442.8270	15443.9754	15443.1996	2.63
APSO [13]	15445.5109	15538.6016	15473.3164	NA
RDPSO [13]	15442.7575	15455.2936	15445.0245	NA
DSPSO-TSA [15]	15441.57	15446.22	15443.84	1.07
OBBO [22]	15442.414	15442.419	15442.415	NA
PSO-IR	**15442.7291**	**15442.7291**	**15442.7291**	**0.05**
DE-IR	**15442.7291**	**15442.7291**	**15442.7291**	**0.4**

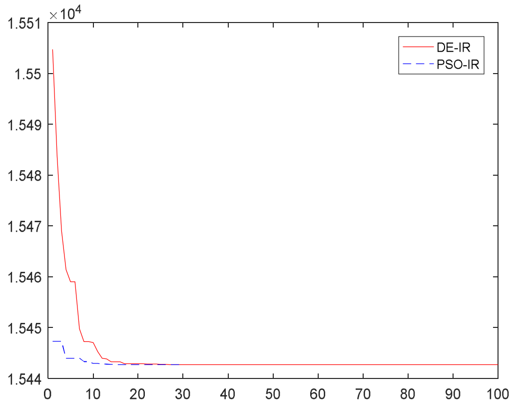

Fig. 2. Convergence of 6-generators system (P_D=1263MW)

In PSO-IR, the numbers of particles and iterations are set as $m = 10$ and $iter_{PSO} = 30$. In DE-IR, the numbers of populations and iterations are $N_p = 30$ and $iter_{DE} = 100$. The maximum, minimum and average cost value obtained by the proposed method and other methods from previous literature are listed in Table 1. The average values obtained by the proposed method are close to the best result in previous literature and better than most existing methods. The convergence characteristics in Fig. 2 are recorded for the run where the minimum cost value is obtained.

4.3.2 Test case 2

15 generating units at the demand of 2630 MW are comprised in this test case, which takes into account transmission loss, ramp rate limits and prohibited operating zones. The relevant data of this system are obtained from [12]. In PSO-IR, $m = 100$, $iter_{PSO} = 100$. In DE-IR, $N_p = 40$, $iter_{DE} = 200$. The experimental results are shown in Table 2. The average values obtained by proposed method are better than most methods except OBBO without ramp rate limits and OPSO with ramp rate limits. Besides, the results obtained by proposed method are relatively stable. Fig. 3 displays the convergence curves

Table 2. 15-generators test system: results of proposed method and previous literature

Methods	Generation costs($/h)			Time
	Min.	Max.	Average	(s)
Without ramp rate limits				
NAPSO [14]	32548.5859	32548.5904	32548.5869	NA
FAPSO [14]	32659.794	32676.07	32663.19	NA
OBBO [22]	32544.8670	32545.1749	32545.0101	NA
IABC [28]	32542.732	32632.486	32560.9885	0.090
IABC-LS [28]	32542.735	32574.8361	32548.3704	0.078
PSO-IR	**32548.0642**	**32548.1417**	**32548.0744**	**1.9**
DE-IR	**32548.0690**	**32548.1347**	**32548.0826**	**1.9**
With ramp rate limits				
GPSO [12]	32891.8329	33850.9528	33137.5549	3.59
OPSO [12]	32668.4863	32669.3005	32668.9205	4.38
APSO [13]	32687.9840	33359.6609	32948.0533	NA
RDPSO [13]	32666.1818	32934.3089	32739.7165	NA
DSPSO-TSA [15]	32715.06	32730.39	32724.63	2.30
PSO-IR	**32692.4628**	**32692.4646**	**32692.4632**	**2.8**
DE-IR	**32692.4628**	**32692.4667**	**32692.4630**	**2.3**

Table 3. 20-generators test system: results of proposed method and previous literature

Methods	Generation costs($/h)			Time
	Min.	Max.	Average	(s)
PSO(PD =2000MW) [7]	50760.8655	NA	NA	NA
TLBO(PD =2000MW) [7]	50531.7791	NA	NA	NA
PSO(PD =2500MW) [7]	60204.7180	NA	NA	NA
TLBO(PD =2500MW)[7]	60204.7175	NA	NA	NA
PSO-IR (PD=2000MW)	**50531.7791**	**50531.7792**	**50531.7791**	**9.7**
DE-IR (PD=2000MW)	**50531.7791**	**50531.7906**	**50531.7848**	**7.9**
PSO-IR (PD=2500MW)	**60152.5294**	**60152.5298**	**60152.5296**	**9.7**
DE-IR (PD=2500MW)	**60152.5294**	**60152.5295**	**60152.5294**	**7.9**

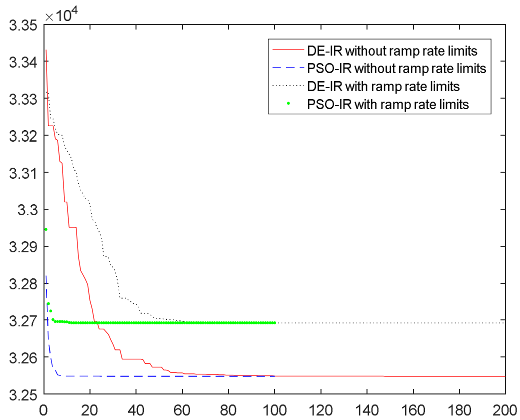

Fig. 3. Convergence of 15-generators system (P_D=2630MW)

Table 4. Output power of 20-generators test system using proposed method

Unit	PSO-IR	DE-IR
1	600	600
2	132.0784	132.1514
3	50	50
4	50	50
5	91.2695	91.3122
6	20	20
7	125	125
8	50	50
9	112.8476	112.8055
10	46.0196	46.0421
11	287.2407	287.2646
12	433.5590	433.5129
13	122.7960	122.8140
14	73.1820	73.1555
15	93.9297	93.9308
16	36.4282	36.4282
17	30	30
18	37.0249	37.0042
19	78.6244	78.5788
20	30	30
TP	**2500**	**2500**
TC	**60152.5294**	**60152.5294**

* TP: total power (MW), TC: total generation cost ($).

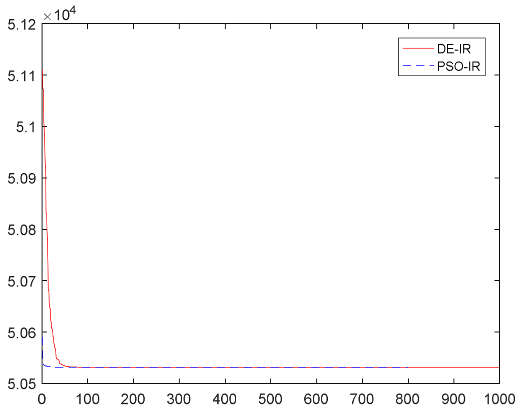

Fig. 4. Convergence of 20-generators system (P_D=2000MW)

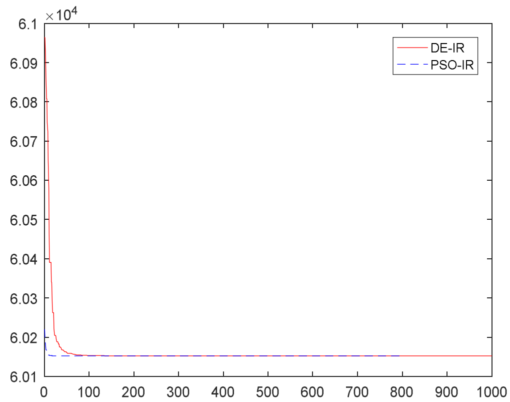

Fig. 5. Convergence of 20-generators system (P_D=2500MW)

of the solution obtained by the proposed method.

4.3.3 Test case 3

The system considered in this test case has 20 units and two load demands of 2000 MW and 2500 MW. The system data used are given in [7]. In PSO-IR, $m = 400$, $iter_{PSO} = 800$. In DE-IR, $N_p = 60$, $iter_{DE} = 1000$. The comparison of several different methods is shown in Table 3. PSO-IR and DE-IR give the same minimum generation cost at two different load demands and succeed to provide a significantly better cost value at the demand of 2500 MW. The convergence characteristics and the optimal solution obtained by the proposed method are shown in Fig. 4, Fig. 5 and Table 4.

4.3.4 Test case 4

In this test case, a 38-generators system at the demand of 6000 MW is utilized. The relevant data of this system are available in [19]. In PSO-IR, $m = 400$, $iter_{PSO} = 800$. In DE-IR, $N_p = 60$, $iter_{DE} = 1000$. Table 5 presents a comparison between the results obtained by the proposed

Table 5. 38-generators test system: results of proposed method and previous literature

Methods	Generation costs($/h)			Time (s)
	Min.	Max.	Average	
SPSO [10]	9543984.777	NA	NA	NA
PSO_Crazy [10]	9520024.601	NA	NA	NA
NewPSO [10]	9516448.312	NA	NA	NA
PSO_TVAC [10]	9500448.307	NA	NA	NA
BBO [20]	9417633.6376	NA	NA	NA
DE/BBO [20]	9417235.7864	NA	NA	NA
MsEBBO [21]	9417235.7757	NA	NA	NA
IDE [19]	9417235.7864	NA	NA	NA
PSO-IR	**9417235.8314**	**9417236.0692**	**9417235.9133**	**10.7**
DE-IR	**9417236.0676**	**9417322.2090**	**9417274.0645**	**13.4**

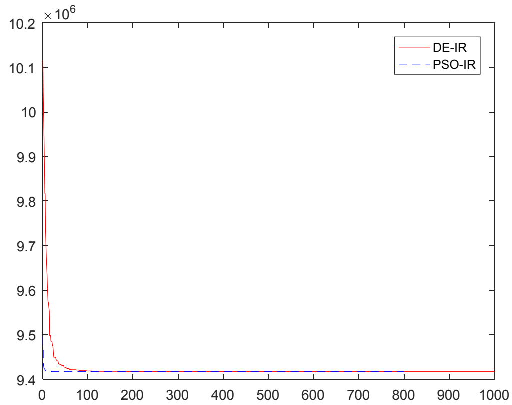

Fig. 6. Convergence of 38-generators system (P_D=6000 MW)

method and other methods from previous literature, where the results obtained by the proposed method are very close to the best result in previous literature. The convergence characteristics of the solution obtained by the proposed method are demonstrated in Fig. 6.

4.3.5 Test case 5

This test case involves a system with 110 generating units at the load demand of 15000 MW, whose characteristics are given in [25]. In PSO-IR, $m = 600$, $iter_{PSO} = 1000$. In DE-IR, $N_p = 60$, $iter_{DE} = 1000$. Table 6 provides a comparison between the results obtained by the proposed method and other methods from previous literature, where the proposed method presents the lowest minimum cost and computation time among all the results. Table 8 and Fig. 7 give the best solution and its convergence characteristics.

4.3.6 Test case 6

This test case contains 140 generating units at the total demand of 49342 MW, The relevant data of which are given in [26]. In PSO-IR, $m = 800$, $iter_{PSO} = 1000$. In DE-IR, N_p=60, $iter_{DE} = 1000$. The experimental results are shown in Table 7. The best results obtained by proposed method are more stable than those from previous literature, namely from MVMO^S. However, the computation time of proposed method is significantly reduced compared with that in MVMO^S. Fig. 8 presents the convergence curves of PSO-IR and DE-IR.

Table 6. 110-generators test system: results of proposed method and previous literature

Methods	Generation costs($/h)			Time (s)
	Min.	Max.	Average	
DE/BBO[24]	198,231.06	198,828.57	198,326.66	132
BBO[24]	198,241.166	199,102.59	198,413.45	115
ORCCRO[24]	198,016.29	198,016.89	198,016.32	45
SA [23]	198,352.6413	NA	201,595.19	NA
SAF [23]	207,380.5164	NA	207,813.37	NA
SAB [23]	206,912.9057	NA	207,764.73	NA
OIWO [25]	197,989.14	197,989.93	197,989.41	31
PSO-IR	197988.3030	198001.1340	197993.8500	28.8
DE-IR	197988.6591	197995.0583	197992.1345	33.6

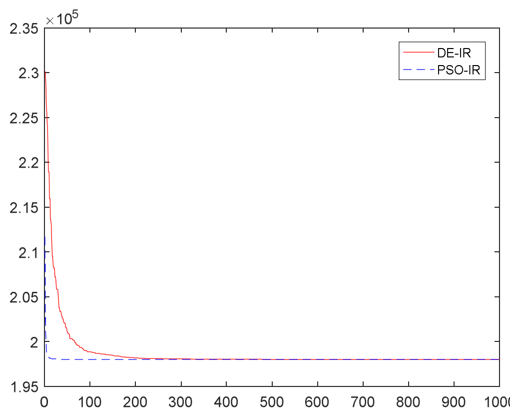

Fig. 7. Convergence of 110-generators system (P_D=15000 MW)

4.4 Discussions

Based on the above study of six different test cases, stable and satisfactory results can always be obtained without improving the algorithms. The proposed method gives the lowest minimum cost among the results of previous literature in case 3 and case 5. In addition, the proposed method is more effective when applied to the large-scale power system.

However, it should be noted that the method is not suitable for solving the ED problem with valve-point effects. When the objective function is convex, the stable point is a minimum point. But when the sine wave shaped curve is superimposed on the quadratic function, the consumption curve is not convex, instead the piecewise curve between valve points is approximate concave, therefore the stable point is the maximum point.

Table 7. 140-generators test system: results of proposed method and previous literature

Methods	Generation costs($/h)			Time (s)
	Min.	Max.	Average	
CTPSO [33]	1655685	1655685	1655685	50.1
CSPSO [33]	1655685	1655685	1655685	9.6
COPSO [33]	1655685	1655685	1655685	76.9
CCPSO [33]	1655685	1655685	1655685	42.9
KMVO [26]	1577607	1594251	1586547	NA
MVMOS [27]	1557461.803	1557481.743	1557481.743	105.39
PSO-IR	1557462.2098	1557467.4163	1557463.0007	43.7
DE-IR	1557463.0646	1557465.6140	1557464.3502	46.2

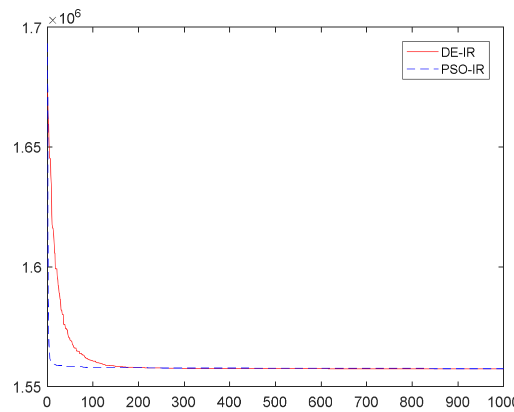

Fig. 8. Convergence of 140-generators system (P_D= 49342MW)

Table 8. Output power of 110-generators test system using proposed method

Unit	PSO-IR	DE-IR	Unit	PSO-IR	DE-IR	Unit	PSO-IR	DE-IR	Unit	PSO-IR	DE-IR
1	2.4000	2.4000	29	200	199.9917	57	25.2000	25.2090	85	324.9996	324.7644
2	2.4000	2.4000	30	100	99.9982	58	35	35.0061	86	440	439.9859
3	2.4000	2.4000	31	10	10	59	35	35	87	16.1825	10.1696
4	2.4000	2.4000	32	20	19.9938	60	45	45	88	20.0002	23.8049
5	2.4000	2.4000	33	79.5840	78.8268	61	45	45	89	84.5714	82.6911
6	4	4	34	250	249.9770	62	45	45	90	88.6817	90.6253
7	4	4	35	360	359.9978	63	185	184.9998	91	57.7003	55.8578
8	4	4	36	400	399.9998	64	185	184.9998	92	99.1110	99.8014
9	4	4	37	40	39.9801	65	185	184.9759	93	440	439.9995
10	63.9115	64.5586	38	70	69.9978	66	185	184.9857	94	500	500
11	61.6710	63.8309	39	100	99.9875	67	70	70	95	600	599.9973
12	35.6619	35.7199	40	120	119.8408	68	70	70	96	472.4868	471.2172
13	56.7668	56.6176	41	157.5694	158.9195	69	70	70	97	3.6000	3.6000
14	25	25	42	220	219.9891	70	360	360	98	3.6000	3.6000
15	25	25	43	440	439.9997	71	400	400	99	4.4000	4.4000
16	25	25	44	560	559.9939	72	400	399.9998	100	4.4000	4.4000
17	155	154.9979	45	660	659.9989	73	105.7656	105.3408	101	10	10.0081
18	155	155	46	615.8793	616.8339	74	192.3092	191.8142	102	10	10
19	155	154.9984	47	5.4000	5.4000	75	90	90	103	20	20
20	155	154.9998	48	5.4000	5.4000	76	50	49.9994	104	20	20
21	68.9000	68.9000	49	8.4000	8.4000	77	160	160.0171	105	40	40
22	68.9000	68.9000	50	8.4000	8.4000	78	296.4018	297.9784	106	40	40
23	68.9000	68.9000	51	8.4000	8.4001	79	174.9039	173.1956	107	50	50
24	350	349.9902	52	12	12	80	98.7420	100.3494	108	30	30
25	400	399.9998	53	12	12	81	10	10	109	40	40
26	400	399.9986	54	12	12	82	12	12.0014	110	20	20
27	500	499.9998	55	12	12	83	20	20.2546	**TP**	**15000**	**15000**
28	500	499.9984	56	25.2000	25.2301	84	200	199.9841	**TC**	**197988.303**	**197988.659**

5. Conclusion

In this paper, the incremental rate is considered to solve ED problems. This method uses the incremental rate of the unit to share the unbalanced amount of equality constraint violation of the system. The applicability of proposed method is proved based on classical PSO and classical DE. The study cases of six different test systems demonstrate that the proposed method is able to quickly and efficiently solve the constraint handling ED problem. Moreover, the proposed method is more stable than most existing methods and more effective when applied to the large-scale power system. Future research can be oriented to considering other constraints and making improvements to the algorithm in order to better solve ED problems.

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Xueping Li He received the M.Eng. degree in Electrical engineering from Yanshan University, Qinhuangdao, China, in 2009, the Ph.D. degree in Power electronics and power transmission from Yanshan University, Qinhuangdao, China, in 2013. His research interests include economic operation and control of power system, power grid state estimation, inspired optimization algorithms and their application on various optimization problems.

Canwei Xiao She received the B.Eng. degree Communication Engineering from Changsha University of Science & Technology, Changsha, China, in 2015. She is currently pursuing the M.Eng degree in the Department of electric power systems and automation at the Yanshan University, Qinhuangdao, China. Her research interests include optimization algorithm, economic operation and control of power system.

Zhigang Lu He received the BSc degree in automatic control form the Xi'an Jiaotong University of Xi'an, Shaanxi, China, in 1985, the M.S. degree in systems engineering form the Xi'an Jiaotong University of Xi'an, Shanxi, China, in 1988, and the Ph.D. degree in electric power systems and automation form the North China Electric Power University of Beijing, China, in 1998. He is the dean of school of electrical engineering Yanshan University of Qinhuangdao, Hebei, China. His current research is engaged in electric power systems and automation. His current research is mainly engaged in multi-fault repair in power system, economic operation and control of power system, power grid state estimation, power system planning, inspired optimization algorithms and their application on various optimization problems.