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INSERTION PROPERTY OF NONZERO POWERS AT ZERO PRODUCTS

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Abstract. This article concerns a ring property which is seated between IFP and IPFP rings. We study the insertion property of nonzero powers at zero products, introducing the concept of strongly IPFP ring. The structure of strongly IPFP rings is investigated in relation with nearly seated ring properties and ring extensions.

1. Strongly IPFP rings

Throughout this note every ring is an associative ring with identity unless otherwise stated. Let R be a ring. $R[x]$ denotes the polynomial ring with an indeterminate x over R. Use $J(R)$ for the Jacobson radical of R. Use $U(R)$ for the group of all units in R. Denote the n by n full (resp., upper triangular) matrix ring over R by $\mathrm{Mat}_n(R)$ (resp., $T_n(R)$). $D_n(R)$ denotes the subring $\{(a_{ij}) \in T_n(R) \mid a_{11} = \cdots = a_{nn}\}$ of $T_n(R)$. Use E_{ij} for the matrix with (i, j)-entry 1 and elsewhere 0. Let $\mathbb{Z}$ ($\mathbb{Z}_n$) denote the ring of integers (modulo n). For $f(x) \in R[x]$, $C_{f(x)}$ denotes the set of all coefficients of $f(x)$.

A ring is usually called *reduced* if it has no nonzero nilpotent elements. Due to Bell [4], a ring R is said to satisfy the *Insertion-of-Factors-Property* (simply, be an *IFP* ring) if $ab = 0$ implies $aRb = 0$ for $a, b \in R$. This property is easily shown to be a generalization of both reduced rings and commutative rings. Shin [13] used the term *SI* for IFP rings; while Narbonne [11] used *semicommutative* in place of IFP rings. A ring (possibly without identity) is usually called *Abelian* if every idempotent is central. It is also easily checked that every IFP ring is Abelian.

Following Cheon et al. [5], a ring R is said to be an *IPFP* ring provided that for every $r \in R$ there exists $n = n(r) \geq 1$, depending on r, such that $ar^nb = 0$ whenever $ab = 0$ for $a, b \in R$. This is a proper generalization of IFP rings as we see in [5, Example 1.2]. IPFP rings are Abelian by [5, Lemma 1.2(1)]. We next consider a special case of IPFP rings.

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Definition 1.1. A ring R is said to be *strongly IPFP* provided that for every $0 \neq r \in R$ there exists $n = n(r) \geq 1$, depending on r, such that $r^n \neq 0$ and $ar^nb = 0$ whenever $ab = 0$ for $a, b \in R$.

The class of strongly IPFP rings is closed under subrings obviously. We will use this fact freely. In what follows, we see that strongly IPFP rings are seated between IFP rings and IPFP rings.

Example 1.2. (1) Let A be a finite Abelian ring, and $R = D_n(A)$ for $n \geq 4$. Then R is IPFP by [5, Lemma 1.3(2)] and [8, Lemma 2]. We claim that R is not strongly IPFP.

Consider the matrices E_{12}, E_{23} and E_{34}. Then $E_{12}E_{34} = 0$ and $E_{12}E_{23}E_{34} = E_{14} \neq 0$. But $E_{23}^2 = 0$, and so there cannot exist $k \geq 1$ such that

$$E_{23}^k \neq 0 \text{ and } E_{12}E_{23}^kE_{34} = 0.$$

Thus R is not strongly IPFP.

(2) We use the ring and argument in [9, Example 2]. Let

$$A = \mathbb{Z}_2\langle a_0, a_1, a_2, b_0, b_1, b_2, c\rangle$$

be the free algebra with noncommuting indeterminates $a_0, a_1, a_2, b_0, b_1, b_2, c$ over $\mathbb{Z}_2$. Let B the set of all polynomials with zero constant terms in A. Next, consider the ideal I of A generated by

$$a_0b_0,\ a_0b_1 + a_1b_0,\ a_0b_2 + a_1b_1 + a_2b_0,\ a_1b_2 + a_2b_1,\ a_2b_2,$$
$$a_0rb_0,\ a_2rb_2,\ (a_0 + a_1 + a_2)r(b_0 + b_1 + b_2),\ r_1r_2r_3r_4$$

where $r \in A$ and $r_1, r_2, r_3, r_4 \in B$. Then clearly $B^4 \in I$. Set $R = A/I$. Identify $a_0, a_1, a_2, b_0, b_1, b_2, c$ with their images in R for simplicity. Let H_n be the set of all linear combinations of monomials in B of degree n over $\mathbb{Z}_2$. Clearly H_n is finite for any n. Note that the ideal I is *homogeneous* (i.e., if $\sum_{i=1}^{3} r_i \in I$ with $r_i \in H_i$, then every r_i is contained in I).

Now consider the polynomial ring $R[x]$. Then $R[x]$ is not IFP by the argument in [9, Example 2]. We will show that $R[x]$ is strongly IPFP. Every polynomial $f(x)$ in $R[x]$ can be expressed by

$$f(x) = \alpha(x) + f_0(x) \text{ with } \alpha(x) \in \mathbb{Z}_2[x] \text{ and } f_0(x) \in B[x].$$

Since $B[x]$ is an ideal of $R[x]$ satisfying $B[x]^4 = 0$, we have $B[x] \subseteq J(R[x])$. But $R[x]/B[x]$ is isomorphic to $\mathbb{Z}_2[x]$. Suppose that $\alpha(x) + f_0(x)$ is a unit. Say that $[\alpha(x) + f_0(x)][\beta(x) + g_0(x)] = 1$ for $\beta(x) \in \mathbb{Z}_2[x]$ and $g_0(x) \in B[x]$. Then

$$1 - \alpha(x)\beta(x) \in B[x] \subseteq J(R[x]),$$

whence $\alpha(x)\beta(x)$ is a unit in $R[x]$. However $\mathbb{Z}_2[x]$ is a domain, and so $\alpha(x)\beta(x) = 1$. This implies $\alpha(x) = 1 = \beta(x)$, entailing

$$U(R[x]) = \{1 + p(x) \mid p(x) \in B[x]\}.$$

This result yields $J(R[x]) = B[x]$.

Suppose that $f(x)g(x) = 0$ for nonzero polynomials $f(x), g(x) \in R[x]$. Then $f(x), g(x) \in B[x]$ because $R[x]/B[x]$ is isomorphic to $\mathbb{Z}_2[x]$. Consider a nonzero polynomial $h(x)$ in $R[x]$. Then $h(x) = \gamma(x) + h_0(x)$ for $\gamma(x) \in \mathbb{Z}_2[x]$ and $h_0(x) \in B[x]$. If $h_0(x) = 0$, then

$$f(x)h(x)g(x) = f(x)\gamma(x)g(x) = \gamma(x)f(x)g(x) = 0.$$

So assume $h_0(x) \neq 0$. If $h_0(x) \in H_m[x]$ with $m \geq 2$, then

$$\begin{aligned} f(x)h(x)g(x) &= f(x)(\gamma(x) + h_0(x))g(x) = \gamma(x)f(x)g(x) + f(x)h_0(x)g(x) \\ &= f(x)h_0(x)g(x) = 0 \end{aligned}$$

because $f(x)h_0(x)g(x) \in I[x]$. So assume $h_0(x) \in H_1[x]$. Then every coefficient of $h_0(x)$ is contained in the set $C = \{a_0, a_1, a_2, b_0, b_1, b_2, c\}$, say $h_0(x) = ux^k + vx^{k+1} + \cdots + wx^l$ with $u, v, \ldots, w \in C$. This implies that

$$h_0(x)^2 = u^2x^{2k} + (uv + vu)x^{2k+1} + \cdots + w^2x2l \neq 0$$

because $u^2 \neq 0$. We now have

$$h(x)^2 = (\gamma(x) + h_0(x))^2 = \gamma(x)^2 + h_0(x)^2 \neq 0;$$

and

$$\begin{aligned} f(x)h(x)^2g(x) &= f(x)(\gamma(x)^2 + h_0(x)^2)g(x) \\ &= \gamma(x)^2f(x)g(x) + f(x)h_0(x)^2g(x) = f(x)h_0(x)^2g(x) = 0 \end{aligned}$$

because $f(x)h_0(x)^2g(x) \in I[x]$. Therefore $R[x]$ is a strongly IPFP ring but not IFP.

Recall that IPFP rings are Abelian, and so strongly IPFP rings are also Abelian. But there exist Abelian rings but not IPFP by [5, Example 1.4]. We observe next some conditions under which the properties of strong IPFP, IFP, and IPFP are equivalent.

Based on Armendariz [3, Lemma 1], Rege and Chhawchharia [12] called a ring R *Armendariz* if $ab = 0$ for all $a \in C_{f(x)}$ and $b \in C_{g(x)}$ whenever $f(x)g(x) = 0$ for $f(x), g(x) \in R[x]$. Reduced rings are Armendariz by [3, Lemma 1], and Armendariz rings are Abelian by [9, Corollary 8].

The concepts of Armendariz and IFP are independent of each other by the [9, Examples 2 and 14] and [12, Proposition 4.6]. Moreover there exists an Armendariz ring that is not IPFP. The construction follows Antoine [2, Example 4.8]. Let F be a field and $A = F\langle a, b\rangle$ be the free algebra with noncommuting indeterminates over F. Consider the ideal I of A generated by b^2 and set $R = A/I$. Identify a, b with their images in R for simplicity. R is Armendariz by [2, Example 4.8]. But R is not IPFP since $b^2 = 0$ and $ba^nb \neq 0$ for any $n \geq 1$.

Proposition 1.3. *Let R be an Armendariz ring. Then the following conditions are equivalent*:

(1) *R is IPFP*;

(2) *R is strongly IPFP*;
(3) *R is IFP.*

Proof. It suffices to prove (1) implying (3). Let R be an IPFP ring and suppose that $ab = 0$ for $a, b \in R$. Then for every $r \in R$ there exists $n \geq 1$ such that $ar^n b = 0$. But since R is Armendariz, we have $arb = 0$ by [9, Lemma 7(1)]. Thus R is IFP. □

The rings in Example 1.2 are not Armendariz. In fact, the ring R in (1) is not Armendariz by [10, Example 3]. For the ring R in (2), consider $f(x) = a_0 + a_1 x + a_2 x^2$ and $g(x) = b_0 + b_1 x + b_2 x^2$ in $R[x]$. Then $f(x)g(x) = 0$ but $a_0 b_1 \neq 0$. So R is not Armendariz.

Consider next other sorts of conditions under which strongly IPFP, IPFP, and IFP are equivalent. First let R be an Abelian ring that is an algebra over a field F of characteristic zero. Then, by help of [5, Theorem 1.7], the following conditions are equivalent: (i) R is a reduced ring; (ii) $D_3(R)$ is strongly IPFP; (iii) $D_3(R)$ is IFP; (iv) $D_3(R)$ is IPFP.

Following [7], a ring R is called (*von Neumann*) *regular* if for each $a \in R$ there exists $b \in R$ such that $a = aba$. For a regular ring R, we can obtain the following equivalences by help of [7, Theorem 3.2] and the fact that (strongly) IPFP rings are Abelian: (i) R is strongly IPFP; (ii) R is IFP; (iii) R is IPFP; (iv) R is Abelian; (v) R is reduced.

We see a sort of IPFP ring which is of similar structure to the polynomial ring $R[x]$ in Example 1.2(2). Note that $R[x]$ is a strongly IPFP ring such that $J(R[x])^4 = 0$ and $J(R[x])^3 \neq 0$.

Example 1.4. Let R be a local ring such that $R/J(R)$ is a finite field and $J(R)^3 = 0$. We show that R is a strongly IPFP ring. The argument is almost similar to the proof of [5, Proposition 1.1(1)], but we write here for completeness. Let $ab = 0$ for $a, b \in R$. Then $a, b \in J(R)$ clearly. If $r \in J(R)$, then $arb = 0$ because $J(R)^3 = 0$. So assume $r \notin J(R)$, i.e., $r \in U(R)$. Then $r = u + j$ with $u \in U(R)$ and $j \in J(R)$. Since $R/J(R)$ is a finite field, $r^k + J(R) = (r + J(R))^k = (u + J(R))^k = 1 + J(R)$ for some $k \geq 1$ by the proof of [9, Proposition 16]. This yields $r^k = 1 + c$ for some $c \in J(R)$, whence we have $ar^k b = a(1 + c)b = ab + acb = 0$. Here $r \in U(R)$ implies $r^k \neq 0$. Thus R is strongly IPFP.

As in [5, Proposition 1.1(2)], let K be a finite field and A be the free algebra generated by a set T of noncommuting indeterminates over K. Let I be the ideal of A generated by abc for all $a, b, c \in T$. Then A/I is a strongly IPFP ring by the preceding argument.

We obtain an information for strongly IPFP polynomial rings by help of Proposition 1.3.

Theorem 1.5. *Let R be an Armendariz ring. Then the following conditions are equivalent*:

(1) *R is IPFP*;
(2) *R is strongly IPFP*;
(3) *R is IFP*;
(4) *$R[x]$ is IPFP*;
(5) *$R[x]$ is strongly IPFP*;
(6) *$R[x]$ is IFP.*

Proof. The equivalence of the conditions (1), (2), and (3) is shown by Proposition 1.3. The equivalence of the conditions (3) and (6) is shown by [12, Proposition 4.6]. The remainder of the proof is shown by the fact that the classes of IFP, IPFP, and strongly IPFP rings are closed under subrings. □

If R is an Armendariz ring, then $R[x]$ is also Armendariz by [1, Theorem 2]. So the equivalence of the conditions (4), (5), and (6) in Theorem 1.5 is also shown by Proposition 1.3.

Recall that if R is a regular ring, then the following conditions are equivalent: (i) R is strongly IPFP; (ii) R is IFP; (iii) R is IPFP; (iv) R is Abelian; (v) R is reduced. So we have another information for polynomial rings over strongly IPFP rings.

Theorem 1.6. *Let R be a regular ring. Then the following conditions are equivalent*:
(1) *R is (strongly) IPFP*;
(2) *R is reduced*;
(3) *R is Abelian*;
(4) *$R[x]$ is reduced*;
(5) *$R[x]$ is Abelian*;
(6) *$R[x]$ is (strongly) IPFP*;
(7) *$R[x]$ is an Armendariz ring.*

Proof. Reduced rings are Armendariz by [3, Lemma 1], and (4) implying (7) follows. The equivalence of the conditions (2) and (4) is obvious. The equivalence of the conditions (3) and (5) is proved by [10, Lemma 8]. The remainder of the proof is shown by the arguments above. □

2. Extensions of strongly IPFP rings

In this section we study the structure of strongly IPFP rings being concerned with some sorts of ring extensions which have important roles in ring theory.

Use $\oplus$ to denote the direct sum of rings. Let R be an algebra over a commutative ring S. Following [6], the *Dorroh extension* of R by S is the Abelian group $D = R \oplus S$ with multiplication given by $(r_1, s_1)(r_2, s_2) = (r_1r_2 + s_1r_2 + s_2r_1, s_1s_2)$, where $r_i \in R$ and $s_i \in S$.

Proposition 2.1. *Let R be an algebra over a commutative domain S. Then R is strongly IPFP if and only if so is the Dorroh extension D of R by S.*

Proof. Note that $s \in S$ is identified with $s1 \in R$. We extend the proof of [5, Proposition 2.1]. Suppose that R is strongly IPFP. Let $(r_1, s_1)(r_2, s_2) = 0$ for $(r_1, s_1), (r_2, s_2) \in D$. Then $r_1r_2 + s_1r_2 + s_2r_1 = 0$ and $s_1s_2 = 0$. Since S is a domain, $s_1 = 0$ or $s_2 = 0$.

Suppose $s_1 = 0$. Then $r_1r_2+s_2r_1 = 0$ and $r_1(r_2+s_2) = 0$. Let $0 \neq (r, s) \in D$. If $r + s = 0$, then

$$(r_1, 0)(r, s)(r_2, s_2) = (r_1(r + s), 0)(r_2, s_2) = (0, 0)(r_2, s_2) = 0.$$

Assume $r + s \neq 0$. Since R is strongly IPFP, there exists $n \geq 1$ such that $(r + s)^n \neq 0$ and $r_1(r + s)^n(r_2 + s_2) = 0$. Here if $s = 0$, then $r^n \neq 0$ (hence $(r, 0)^n = (r^n, 0) \neq 0$); and if $s \neq 0$, then $(r, s)^n = (r', s^n) \neq 0$, with $r' \in R$, because S is a domain. Thus $(r, s)^n \neq 0$ in any case, and this result is equivalent to

$$(r_1, 0)(r, s)^n(r_2, s_2) = (r_1(r + s)^n(r_2 + s_2), 0) = 0.$$

When $s_2 = 0$ we also obtain that $(r_1, s_1)(r, s)^m(r_2, 0) = 0$ for some $m \geq 1$, via a similar method (i.e., use $(r_1 + s_1)(r + s)^nr_2 = 0$). Therefore D is strongly IPFP. The converse is obvious. □

Consider next the case of Laurent polynomial ring. An element u of a ring R is usually called *right regular* if $ur = 0$ for $r \in R$ implies $r = 0$. A *left regular* element is defined analogously, and a *regular* element means both left and right regular.

Proposition 2.2. *Let R be a ring and M be a multiplicatively closed subset of R which consists of central regular elements. Then R is a strongly IPFP ring if and only if so is $M^{-1}R$.*

Proof. Let R be a strongly IPFP ring. We apply the proof of [5, Proposition 2.2]. Suppose that $AB = 0$ for $A = u^{-1}a, B = v^{-1}b \in M^{-1}R$. Then $ab = 0$. Let $0 \neq s^{-1}r \in M^{-1}R$. Then $r \neq 0$. Since R is strongly IPFP, there exists $n \geq 1$ such that $r^n \neq 0$ and $ar^nb = 0$. From this result, we obtain the following implications:

$$ar^nb = 0 \Rightarrow u^{-1}(s^{-1})^nv^{-1}(ar^nb) = 0 \Rightarrow A((s^{-1})^nr^n)B = A(s^{-1}r)^nB = 0.$$

Therefore $M^{-1}R$ is strongly IPFP. The converse is obvious. □

The ring of *Laurent polynomials* in x, coefficients in a ring R, consists of all formal sums $\sum_{i=k}^{n} r_ix^i$ with the usual addition and multiplication, where $r_i \in R$ and k, n are (possibly negative) integers. We denote this ring by $R[x; x^{-1}]$.

Corollary 2.3. *Let R be a ring. $R[x]$ is a strongly IPFP ring if and only if $R[x; x^{-1}]$ is a strongly IPFP ring.*

Proof. Let $M = \{1, x, x^2, \ldots\}$. Then M is clearly a multiplicatively closed subset of central regular elements in $R[x]$ and $R[x; x^{-1}] = M^{-1}R[x]$. So we have the proof by Proposition 2.2. □

Let R be a ring and I be a proper ideal of R. When the factor ring R/I is strongly IPFP, it is natural to consider some properties of I under which R may be strongly IPFP. As an example, one may ask whether R is strongly IPFP when I is strongly IPFP as a ring without identity. But the answer is negative by the following.

Example 2.4. (1) Let A be a non-reduced strongly IPFP ring that is an algebra over a field F of characteristic zero (e.g., $D_3(\mathbb{Z})$). Then $R = D_3(A)$ is not strongly IPFP by help of [5, Theorem 1.7]. Let $I = \{(a_{ij}) \in R \mid a_{ii} = 0 \text{ for all } i\}$. Then I is an ideal of R such that R/I is isomorphic to A, entailing that R/I is strongly IPFP. Moreover I is an IFP ring without identity because $I^3 = 0$.

(2) Let F be a division ring and $R = T_2(F)$. Then R is not strongly IPFP because it is non-Abelian. In the following argument we follow the computation of [5, Example 2.10(2)]. $I_1 = \left(\begin{smallmatrix} F & F \\ 0 & 0 \end{smallmatrix}\right)$, $I_2 = \left(\begin{smallmatrix} 0 & F \\ 0 & F \end{smallmatrix}\right)$, and $I_3 = \left(\begin{smallmatrix} 0 & F \\ 0 & 0 \end{smallmatrix}\right)$ are all nonzero proper ideals of R. Then each I_j is IFP as a ring without identity, by [8, Example 5]. Note that both R/I_1 and R/I_2 are isomorphic to F; and $R/I_3 \cong F \oplus F$ is a reduced ring. Summarizing, both R/I_j and I_j are strongly IPFP for all $j = 1, 2, 3$.

But if I is reduced as a ring, then we have an affirmative answer as follows, which is similar to [5, Proposition 2.11].

Proposition 2.5. *Let R be a ring and I be a proper ideal of R such that R/I is a strongly IPFP ring. If I is reduced as a ring without identity, then R is strongly IPFP.*

Proof. We apply the proofs of [9, Theorem 6] and [5, Proposition 2.11]. Assume that I is a reduced ring without identity. Let $ab = 0$ for $a, b \in R$. Then $(bIa)^2 = bIabIa = 0$. But since I is reduced and $bIa \subseteq I$, we have $bIa = 0$. We use the condition of I being reduced freely.

Let $r \in R \backslash I$, i.e., $r + I \neq 0$. Then since R/I is strongly IPFP, there exists $n \geq 1$ such that $r^n \notin I$ and $ar^nb \in I$. Next, from $bIa = 0$, we get $(ar^nbI)^2 = ar^nbIar^nbI = ar^n(bIa)r^nbI = 0$, whence $ar^nbI = 0$. Since $ar^nb \in I$, we have $(ar^nb)^2 \subseteq (ar^nb)I = 0$. This implies $ar^nb = 0$.

Let $0 \neq r \in I$. From $bIa = 0$, we also obtain

$$(aIbI)^2 = aIbIaIbI = aI(bIa)IbI = 0,$$

entailing $aIbI = 0$. So $(aIb)^2 = (aIb)(aIb) \subseteq (aIb)I = 0$, and $aIb = 0$ follows. This yields $arb = 0$. Therefore R is strongly IPFP. □

We apply Proposition 2.5 to the following construction. By this we can obtain extended strongly IPFP rings from any given strongly IPFP rings.

Example 2.6. The construction follows [5, Example 2.12(2)]. Let A be a strongly IPFP ring which is an algebra over a commutative reduced ring S. Set $R = A \oplus S$, and consider the ideal $B = \{0\} \oplus S$ of R. Then B is a reduced ring such that R/B is isomorphic to A. So R is strongly IPFP by Proposition 2.5.

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