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Original Article

# Efficiency criteria for optimization of separation cascades for uranium enrichment

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ABSTRACT

As it is known, uranium enrichment is carried out on industrial scale by means of multistage separation facilities, i.e., separation cascades in which gas centrifuges (GCs) are connected in series and parallel. Design and construction of these facilities require significant investment. So, the problem of calculation and optimization of cascade working parameters is still relevant today. At the same time, in many cases, the minimum unit cost of a product is related to the cascade having the smallest possible number of separation elements/GCs. Also, in theoretical studies, it is often acceptable to apply as an efficiency criterion the minimum total flow to supply cascade stages instead of the abovementioned minimum unit cost or the number of separation elements. In this article, cascades with working parameter of a single GC changing from stage to stage are optimized by two of the abovementioned performance criteria and are compared. The results obtained allow us to make a conclusion about their differences.


## Introduction

Despite the upcoming transition to fast-breeder reactors for fuel consumption of the $^{238}U$ isotope as well as the increasing use of blended fuels (such as uranium–plutonium), both existing and soon-to-be implemented nuclear power reactors will remain in use for some time to come. As such, there will be ongoing demand for uranium enrichment technology that produces low enriched uranium (i.e., up to 5% of $^{235}U$).

At an industrial level, multistage separation facilities (cascades), consisting of series–parallel combinations of GCs, are used for uranium enrichment [1]. The design and construction of such separation plants requires significant financial investment that substantially affects the cost of electricity generated by nuclear power plants. As such, determining the most effective cascade to produce enriched uranium is important.

The optimum cascade refers to an installation that obtains the required amount of enriched uranium at the minimum cost (i.e., per unit of a product) [2,3]. Numerous studies have considered cascades that correspond to the minimum specified cost per smallest possible number of GCs [3,4,5]. The “cost-per-number” ratio is important because the capital (and subsequent operating) costs for the design and construction of a cascade are proportional to the total number of GCs in the installation [6,7]. Furthermore, for “fine separation” (i.e., overall separation factor for separation element is close to unity) and symmetrical work of a GC[1] [2], there is a correlation between optimum cascades and an ideal (i.e., nonmixing) cascade [7]. Under the same symmetric work conditions of a single GC, where the overall separation factor, $q$, does not exceed 5, the total flow in the optimum and ideal cascades are almost identical [8,9].

In classical cascade theory, when an overall separation factor $q$ is constant over all cascade stages, the total number of separation elements in a cascade of arbitrary configuration can be obtained by the following relationship [10]:

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[1] $\alpha = \beta = \sqrt{q}$, where q is the overall separation factor and α and β are the separation factors of enriching and depleting parts of a GC, respectively.

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$$\sum_s Z_s = \Delta U_{id}/\eta\delta U_{el}, \quad (1)$$

where, $Z_s$ is the number of GCs at the $s^{th}$ stage of the cascade; $\Delta U_{id}$ is the separative power of an ideal cascade; $\delta U_{el}$ is the separative power of a single GC; and $\eta$ is the separative efficiency of the cascade. Here, equality $\eta = 1$ corresponds to an ideal cascade. For a case in which the external conditions are specified, i.e., a case with target component concentration values given for the product and waste flows and with feed flow rate values given for the cascade, each value of the feed flow, $g$, to a single GC, results in a corresponding value to the total flow in the cascade. To find the minimum cost of a product, the separative power of a single GC, $(\delta U_{el} \rightarrow \delta U_{\max})$, is maximized. In this case, the minimum number of GCs needed to fulfill a separation task can be estimated by the formula:

$$\left(\sum_s Z_s\right)_{min} = \Delta U_{id}/\delta U_{\max}. \quad (2)$$

Now, the working parameters of a single GC corresponding to its maximum separative power can simply be adjusted to find the minimum number of separation elements in a cascade. As mentioned above, for a general case, the overall separation factor of a cascade stage/separation element and its separative power are functions of the feed flow rate (and performance) and the cut (i.e., ratio of product and feed flow rate). Here, the optimum cascade ceases to be identical to the ideal cascade, such that the number of separation elements obtained by the two efficiency criteria for the minimal number of GCs and the minimal total flow do not match [4]. Therefore, for the expected result to satisfy the minimum cost criteria, an optimization should be performed using only the criterion for the minimum number of GCs in a cascade [4].

The authors note that the minimum total flow is not an independent criterion. As such, if the overall separation factor of a single separation unit, i.e., GC, is dependent only on the feed flow rate to the machine and all GCs in the cascade perform the same at all cascade stages, then the total flow is directly proportional to the total number of machines. Here, execution of these conditions simultaneously provides the minimum number of separation elements and the minimum total flow for such a cascade [7]. Hence, for a special case, when the working parameters of separation elements are identical, using total flow in the cascade as the efficiency criterion is justified. Otherwise, when the overall separation factors at the cascade stages are different and there is no directly proportionate (i.e., corresponding) value between the total flow and the total number of separation elements, and the minimum values of these two functions should not be the same.

Previous research on cascade optimization (with variable overall separation factors over cascade stages) obtained results based on the criteria of minimum number of GCs [4] and the minimum total flow in a cascade [11]. However, when total flow was used as an optimization criterion, the possible error in the total number of GCs remained. This article aims to establish an efficient and reliable method to estimate the magnitude of this error. It also examines the extent to which the optimal parameters of a cascade with variable overall separation factors differ under the two efficiency criteria.

## Approach to research

### *Theoretical background*

This paper considers a symmetrical countercurrent cascade, which has one ingoing and two outgoing flows, as shown in Fig. 1. The ingoing flow is a feed flow, $F$, and both outgoing flows are product flows, $P$. The flows are enriched in a target component and a waste flow, $W$, that is depleted in the same target component. The flows $F$, $P$, and $W$ along with the corresponding concentrations of the target component (i.e., $C_F$, $C_P$, and $C_W$), are the external parameters of the cascade. This type of cascade is widely used for isotope separation.

Under stationary conditions and ignoring working substance losses at the cascade stages, the external parameters of the cascade should satisfy the material balance equations:

$$F = P + W, \\ FC_F = PC_P + WC_W. \quad (3)$$

Let the cascade stages be numbered sequentially from the waste end to the product end. Assume that the external feed flow of the cascade, $F$, is fed to the entrance of stage $f$. The parameters $f$ and $N$ are denoted as the cascade design parameters. The internal parameters (i.e., $L_s$, $L'_s$, $L''_s$, $C_s$, $C'_s$, $C''_s$) of an arbitrary stage with the number $s$ in the stationary operation regime are bound by the balance equations for the working substance, both in total and for each component, as per study by Palkin [3] and Borisevich et al. [1].

For the enriching section of the cascade:

$$\theta_s L_s - (1-\theta_{s+1})L_{s+1} = P, \quad (4)$$

$$\theta_s L_s C'_s - (1-\theta_{s+1})L_{s+1}C''_{s+1} = PC_P. \quad (5)$$

For the depleting section of the cascade:

$$\theta_s L_s - (1-\theta_{s+1})L_{s+1} = -W, \quad (6)$$

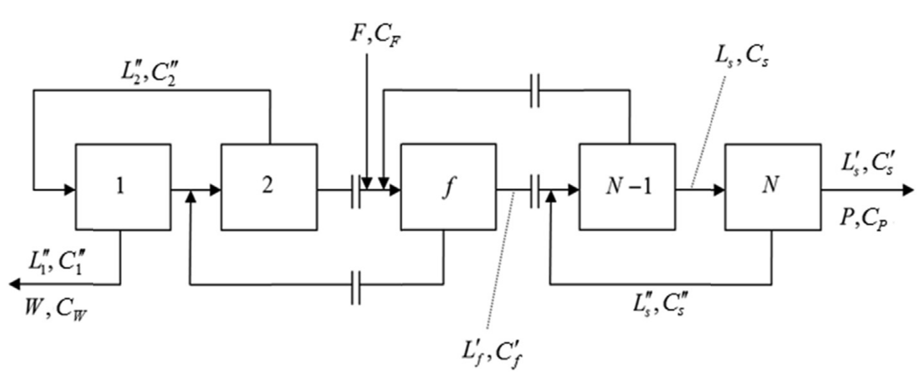


**Fig. 1.** Schematic drawing of separation stages connected in a symmetrical countercurrent cascade.

$$\theta_s L_s C'_s - (1-\theta_{s+1}) L_{s+1} C''_{s+1} = -W C_W, \tag{7}$$

where, $L_s$, $L'_s$, and $L''_s$, are the entering, head, and tail flows of stage $s$, respectively, and $C_s$, $C'_s$, and $C''_s$ are the corresponding concentrations of the target components in these flows. $\theta_s$ is a cut of stage $s$, defined as $\theta_s = L'_s/L_s$.

The external and internal parameters of the cascade are associated with the following boundary conditions.

$$L'_N = \theta_N L_N = P, \tag{8}$$

$$L''_1 = (1-\theta_1) L_1 = W, \tag{9}$$

$$C'_N = C_P, \tag{10}$$

$$C''_1 = C_W. \tag{11}$$

To calculate the parameters of the cascade, specify the separation characteristics of its stages, which are expressed in the form of the following dependency.

$$q_s = q_s(\theta_s, g_s), \tag{12}$$

where, $q_s$ is the overall separation factor at stage $s$ and $g_s$ is a feed flow rate to a single separation element/GC at stage $s$. This kind of dependency (for the overall separation factor) is observed for various types of GC [11].

## Optimization problem

The cascade optimization problem is formulated as a definition of its internal parameters that minimize the total number of GCs for the specified external parameters, where ($\psi = \sum_s Z_s \to \min$) satisfies the conditions $C'_N = C_P$ and $C''_1 = C_W$.

To obtain the desired relationships, assume that the number of GCs at the first stage of a cascade, $Z_1$, is defined as an implicit function of the values of $Z_2, \ldots, Z_N$ and $L'_2, \ldots L''_N$. Here, the extremum condition for the $\psi$ function is also applied. Hereinafter, $Z_1, Z_2, \ldots, Z_N$ are the numbers of GCs at the corresponding stages.

For the $s$th stage, $C'_s$ and $C''_s$ are calculated as follows [3].

$$C'_s = \frac{q_s C''_s}{1 + (q_s - 1) C''_s}, \tag{13}$$

$$C''_1 = C_W; \quad C''_s = C'_{s-1}\left(1 - \frac{\tau'_s}{L''_s}\right) + \frac{\tau'_{us}}{L''_s}. \tag{14}$$

Here $\tau'_s$ is the transit flow of a mixture and $\tau'_{us}$ is a target component in the waste end direction of the cascade. Also, $\tau'_s = W$ and $\tau'_{us} = W C_W$ for s < f and $\tau'_s = -P$ and $\tau'_{us} = -P C_P$ for $f < s < N$.

To make a closed system for the equations, the following information is required: dependency of the overall separation factor, $q_s$, on the feed flow rate to a single GC, $g_s$, and a cut $\theta_s$ at each stage of the cascade. Both of these may differ from stage to stage. The current research uses definiteness for the dependence, per study by Palkin [4]:

$$q = \exp\left(a_0 + a_1\theta - a_2\theta^2\right) g^{-a_3}, \tag{15}$$

where, the values of the coefficients in Eq. (15) are $a_0 = 1.2$, $a_1 = 1.8$, $a_2 = 2.2$, and $a_3 = 0.4$; also $g$ (mg/s of $UF_6$) is the feed flow rate to a single GC.

The separative power of one GC for C << 1, which corresponds to the case for natural uranium enrichment, was calculated by Yamamoto and Kanagawa [12]:

$$\delta U = g \cdot [\ln(1 + \theta(q-1)) - \theta \ln q]. \tag{16}$$

Optimization of separation cascade parameters is achieved by using the two abovementioned efficiency criteria, i.e., the minimum total number of GCs in a cascade, hereinafter referred to as Criterion 1 (“C1”), and the minimum total flow in the same cascade, hereinafter referred to as Criterion 2 (“C2”).

Mathematically, the optimization problem is formulated as follows.

For C1:

Minimize the objective function $\psi(L''_2; \theta_2, \theta_3, \ldots, \theta_N; Z_1, Z_2, \ldots, Z_N)$.

Subject to $C_F$, $C_P$, $C_W$, $P$; $N$; $f$.

For C2:

Minimize the objective function $L(L''_2; \theta_2, \theta_3, \ldots, \theta_N; g_1, g_2, \ldots, g_N)$.

Subject to $C_F$, $C_P$, $C_W$, $P$; $N$; $f$,where, $L \equiv \sum_{s=1}^{N} L_s$, with given values of ($L''_2$; $\theta_2, \theta_3, \ldots, \theta_N$; $Z_1, Z_2, \ldots, Z_N$) or ($L''_2$; $\theta_2, \theta_3, \ldots, \theta_N$; $g_1, g_2, \ldots, g_N$), $\theta_1$, $C_s, C'_s, C''_s$ $(s = \overline{1,N})$ (except $C''_s$), and $L_s, L'_s, L''_s$ $(s = \overline{1,N})$ (except $L''_1, L''_2$) can be computed using the relationships from Eqs. (3)–(14). Now, $\psi$ and $L$ can be easily obtained.

The ultimate goal of optimization is to compare the total number of GCs from two optimum cascades with the same specified external conditions and then further optimize them by using two different efficiency criteria. When C1 is used, the entering flow to stage $s$ is calculated by $L_s = g_s Z_s$; when C2 is used for optimization, the total number of the GCs in the cascade is calculated by dividing the total feed flow rate at each cascade stage by a feed flow rate to a single machine, i.e., $\sum_s Z_s = \sum_s L_s/g_s$. The obtained result is rounded to the nearest integer of the separation elements.

## Optimization techniques

Two different optimization techniques are applied to solve the problem. The first technique (“T1”) was developed at MEPhI and the second technique (“T2”) was developed at Tsinghua University. The simultaneous development of two techniques reduces the possibility of calculation errors because each calculation result (as shown below) is derived by two independent scientific groups each using a different optimization technique.

The technique T1 is reduced to the parameter variations listed above (i.e., $L''_2$, $\theta_s$ and $Z_s$ or $g_s$) with the calculations that follow and are used for all other cascade parameters. Next, the optimal parameter is determined by comparing the parameters of each cascade for the given efficiency criterion. This also determines the minimum value of the objective function while simultaneously satisfying the specified external parameters. This method is implemented per the Hooke–Jeeves optimization technique [13].

The cascade parameters in each iteration of the optimization procedure are calculated in the following manner. First, the $F$ and $W$ flow rate values are obtained from the system of equations for the four given external parameters, $C_F$, $C_P$, $C_W$, and $P$. Next, transit flows in the cascade are determined and the flow over the cascade stages is calculated by:

$$\begin{cases} L'_s = L''_{s+1} - \tau'_{s+1}, & s = \overline{1, N-1} \\ L_s = L'_s + L''_s, & s = \overline{1, N} \end{cases}. \tag{17}$$

Now, with the overall separation factors at the cascade stages, $qs$, defined by Eq. (15), the concentrations in the outgoing flows at the cascade stages are obtained by Eqs. (13)–(14). This procedure allows us to make the mathematical formulation for the cascade optimization problem more compact. The problem can be treated by defining internal cascade parameters that minimize the total number of GCs (or total flow in a cascade) for the specified external parameters and designated parameters of the cascade. This is equivalent to obtaining the minimum value of $\sum_{s=1}^{N} Z_s$ on a set of valid values of $L_2'', \theta_1, .., \theta_N, Z_1, .., Z_N$ (or obtaining the minimum value of $\sum_{s=1}^{N} L_s$ on a set of valid values of $L_2'', \theta_1, .., \theta_N, g_1, .., g_N$) that satisfy the conditions for the specified concentration, $C_P$, $C_W$, and the product flow rate, $P$, from the cascade.

Optimization under T2 is executed as follows.

For C1:

Minimize $\psi(\theta_1, \theta_2, \ldots, \theta_{N-1}; g_1, g_2, \ldots, g_N)$.

Subject to $\left|C_1'' - C_W\right| \le 10^{-6}$, $\left|C_N' - C_P\right| \le 10^{-7}$.

For C2:

Minimize $L(\theta_1, \theta_2, \ldots, \theta_{N-1}; g_1, g_2, \ldots, g_N)$.

Subject to $\left|C_1'' - C_W\right| \le 10^{-6}$, $\left|C_N' - C_P\right| \le 10^{-7}$.

The parameters $\theta_N$ and $L_s (s = \overline{1, N})$ can be calculated from Eqs. (4), (6), (8), and (9) with specified $\theta_s (s = \overline{1, N-1})$. Then, with $\theta_s (s = \overline{1, N-1})$ from Eq. (15), $q_s(\theta_s, g_s)$, $C_s'$, and $C_s''$ can be obtained using the $Q$-iteration [14]. The number of machines/GCs, $Z_s = L_s / g_s$, is calculated by rounding its value to the nearest integer. Now, $\psi$ is minimized with regard to C1 and $L$ is minimized with regard to C2.

Minimization is based on the Non-linear Programming Method with a Quadratic Lagrange function, i.e., the sequential quadratic programming method [15]. This approach ensures that there are solutions to the optimization problems, with constraints in the form of equalities and inequalities.

## Results and discussion

To verify usage compliance of computer codes in the two techniques, the results are compared with those obtained in the study by Palkin [4]. The verification includes the calculations and the optimizations of the uranium enrichment cascades. As per the study by Palkin [4], the calculation parameter set is established as follows. Total number of stages in the cascade, N = 5; stage number for feed flow to the cascade, $f = 2$; $^{235}U$ component concentrations in the feed, product, and waste flows, $C_F = 0.711\%$, $C_P = 3.0\%$, and $C_W = 0.3\%$, respectively; and product flow from the cascade, $P = 1$ g/s. The overall separation factor varies across the cascade stages to satisfy Eq. (15). A comparison of Table 1 [4] and Table 2 (present research), each of which employ both techniques (i.e., T1 and T2), shows that the relative differences of all cascade parameters in both cases are not more than 5%. This confirms the individual correctness of each technique as well as the mutual consistency.

After verification, techniques T1 and T2 are applied to the optimization of the same cascade by minimizing the total flow. This calculation determines the difference in the total number of GCs in the cascade optimized by the two different efficiency criteria.

The total flow of the working substance in a cascade is monotonically dependent on the stage productivity (or on the stage feed flow rate). Here, the feed flow rates ($L_s$, cascade stage and $g_s$, a single GC) are associated with the quantity of GCs in the stage, $Z_s$, connected in parallel through the relationship $L_s = Z_s \cdot g_s$. The $L_s$ function is defined on the set of $Z_s > 0, g_s > 0$. Consequently, its partial derivatives, with respect to variables $Z_s$ and $g_s$, never arrive at zero in this area; also, the total flow of the cascade has no extrema in these variables. In particular, for fixed values of $Z_s$, the total flow is a monotonic function of $g_s$. Hence, a decrease in $g_s$ results in a decrease to the total flow of the cascade. The authors note that, in some cases, feed flow rate (e.g., $g_s$ to a GC) may be limited by technological considerations. In such cases, as a result of the optimization procedure for obtaining the minimum total flow, the lowest total flow is always achieved at the lowest possible value of $g_s$ for all cascade stages. This is confirmed by the calculation results for cascade parameters optimized in the total flow, as listed in Table 3.

Technique T1 results are verified by T2. For the present research, the limiting condition, $5\ \text{mg/s} \le g_s \le 9\ \text{mg/s}$, is introduced while varying the value of $g_s$. The chosen dependence of the GC separative power on its feed flow rate, $g_s$, has its maximum value in this interval [11]. Data analysis shows that for cascade parameter optimization for the total flow, the minimum total flow in the cascade (as expected) corresponds to the minimum possible feed flow rate to a single GC (i.e., 5 mg/s), as listed in Table 3. However, in this case, the quantity of GCs in the total flow of the

**Table 1**
Cascade parameters optimized by minimum number of GCs from the study by Palkin [4].

| Stage number, $s$ | Quantity of GCs, $Z \cdot 10^3$ | Concentration, % | | Parameters | | |
|---|---|---|---|---|---|---|
| | | Product | Waste | g, mg/s | $\theta$ | $q$ |
| 1 | 1.50 | 0.69 | 0.30 | 6.4 | 0.42 | 2.29 |
| 2 | 2.43 | 1.02 | 0.46 | 6.8 | 0.42 | 2.23 |
| 3 | 1.40 | 1.48 | 0.69 | 7.2 | 0.41 | 2.18 |
| 4 | 0.73 | 2.12 | 1.01 | 7.6 | 0.42 | 2.14 |
| 5 | 0.29 | 2.99 | 1.44 | 8.1 | 0.43 | 2.08 |
| | $\sum_{s=1}^{N} Z_s = 6350$ | | | | | |

GC, gas centrifuge.

**Table 2**
Cascade parameters obtained by different optimization techniques for minimum number of GCs criterion.

| $s$ | Quantity of GCs, $Z \cdot 10^3$ | $L$, g/s | Concentrations, % | | Stage parameter | | | $Z \cdot \delta U$, g/s | $\delta U/g$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | Product | Waste | g, mg/s | $\theta$ | $q$ | | |
| Technique 1 | | | | | | | | | |
| 1 | 1.52 | 9.53 | 0.69 | 0.30 | 6.25 | 0.415 | 2.303 | 0.821 | 0.086 |
| 2 | 2.40 | 16.60 | 1.02 | 0.46 | 6.91 | 0.425 | 2.212 | 1.296 | 0.078 |
| 3 | 1.42 | 10.27 | 1.48 | 0.69 | 7.23 | 0.409 | 2.174 | 0.766 | 0.074 |
| 4 | 0.71 | 5.52 | 2.12 | 1.01 | 7.76 | 0.418 | 2.113 | 0.383 | 0.069 |
| 5 | 0.29 | 2.30 | 2.99 | 1.44 | 7.80 | 0.430 | 2.107 | 0.158 | 0.068 |
| | $\sum_{s=1}^{N} Z_s = 6352$, $\sum_{s=1}^{N} L_s = 44.24$ g/s | | | | | | | | |
| Technique 2 | | | | | | | | | |
| 1 | 1.42 | 9.57 | 0.67 | 0.30 | 6.74 | 0.418 | 2.236 | 0.767 | 0.080 |
| 2 | 2.40 | 16.59 | 1.00 | 0.45 | 6.90 | 0.423 | 2.214 | 1.298 | 0.078 |
| 3 | 1.44 | 10.24 | 1.45 | 0.67 | 7.11 | 0.412 | 2.189 | 0.777 | 0.076 |
| 4 | 0.78 | 5.52 | 2.10 | 0.97 | 7.12 | 0.417 | 2.188 | 0.419 | 0.076 |
| 5 | 0.32 | 2.30 | 3.00 | 1.41 | 7.31 | 0.435 | 2.162 | 0.169 | 0.074 |
| | $\sum_{s=1}^{N} Z_s = 6355$, $\sum_{s=1}^{N} L_s = 44.22$ g/s | | | | | | | | |

GC, gas centrifuge.

**Table 3**
Cascade parameters optimized in the total flow.

| $s$ | Number of GCs, $Z \cdot 10^3$ | $G$, g/s | Concentration, % | | GCs parameters | | | $Z \cdot \delta U$, g/s | $\delta U/g$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | Product | Waste | g, mg/s | $\theta$ | $q$ | | |
| Technique 1 | | | | | | | | | |
| 1 | 1.70 | 8.51 | 0.30 | 0.75 | 5.0 | 0.346 | 2.498 | 0.858 | 0.100 |
| 2 | 2.58 | 12.94 | 0.45 | 1.13 | 5.0 | 0.342 | 2.495 | 1.298 | 0.100 |
| 3 | 1.38 | 6.94 | 0.58 | 1.42 | 5.0 | 0.506 | 2.468 | 0.683 | 0.098 |
| 4 | 0.99 | 4.99 | 0.79 | 1.93 | 5.0 | 0.496 | 2.478 | 0.498 | 0.099 |
| 5 | 0.49 | 2.46 | 1.21 | 3.00 | 5.0 | 0.401 | 2.519 | 0.259 | 0.105 |
| | $\sum_{s=1}^{N} Z_s = 7171$, $\sum_{s=1}^{N} L_s = 35.86$ g/s | | | | | | | | |
| Technique 2 | | | | | | | | | |
| 1 | 1.74 | 8.68 | 0.30 | 0.75 | 5.0 | 0.358 | 2.506 | 0.888 | 0.102 |
| 2 | 2.77 | 13.85 | 0.46 | 1.15 | 5.0 | 0.374 | 2.513 | 1.437 | 0.104 |
| 3 | 1.38 | 6.89 | 0.71 | 1.76 | 5.0 | 0.394 | 2.519 | 0.723 | 0.105 |
| 4 | 0.62 | 3.11 | 1.04 | 2.57 | 5.0 | 0.448 | 2.512 | 0.325 | 0.105 |
| 5 | 0.28 | 1.39 | 1.49 | 3.00 | 5.0 | 0.718 | 2.044 | 0.065 | 0.046 |
| | $\sum_{s=1}^{N} Z_s = 6784$, $\sum_{s=1}^{N} L_s = 33.92$ | | | | | | | | |

GC, gas centrifuge.

optimal cascade is 13% greater, despite a 19% lower total flow than that of the optimal cascade in the total number of GCs. Here, optimization of cascade parameters in the total flow occurs when the overall separation factor of a cascade stage depends only on $\theta_s$ but the value of $g_s$ remains fixed for all stages. Based on this case, the question arises: is there an optimal value of $g_s$ that allows the parameters obtained under the given conditions (i.e., optimization for the minimum total flow) to simultaneously satisfy the criterion of the minimum quantity of GCs? This question is answered by calculating the optimal cascade parameters in the total flow that are obtained for different values of $g_s$ in the range of 5 mg/s $\leq g_s \leq$ 9 mg/s. Next, the total quantity of GCs is calculated for each case. Figs. 2 and 3 show the GC dependences of the total flow and the total quantity in a cascade on the feed flow rate, $g_s$, to a single GC.

The optimal total flow value in the cascade monotonically increases with the feed flow increase to a GC. Also, the dependence of the GC quantity in the cascade has a minimum value that corresponds to the optimal value of the feed flow rate to a GC machine when the maximal separative power of a single machine occurs for the adopted relationship, $q_s = q_s(\theta_s, g_s)$ [11].

Fig. 3 shows that the minimum value of the function predominantly coincides with the minimum number of GCs in the cascade that are optimized by the criterion $\sum_{s=1}^{N} Z_s \rightarrow \min$, as listed in Table 2. The resulting discrepancy is such that for the minimum number of GCs per optimization, $g_s$ (as an optimization parameter) varies and its optimum value ranges from 6.3 to 7.8 mg/s, with a mean of approximately 7 mg/s. Thus, when cascade optimization has a constant, $g_s$, its optimum value is approximately equal to the average value of $g_s$. However, cascade optimization for a total flow results in the same number of GCs only in a case in which the feed flow rate to a GC, $g_s$, is equal to its maximum value across all cascade stages. If the feed flow value variations move away from point $g_s = g_{opt}$, then deviation in the minimum number of GCs increases. For example, when $g_s$ deviates from the optimum value within 10%, the difference in the number of GCs is approximately 2% (i.e., about 130 GCs), which is significant from a practical standpoint.

For a physical interpretation of the results, an additional efficiency criterion is employed to compare the maximum possible exploitation of the cascade at each cascade stage. Here, the efficiency criterion, $\chi$, reveals the deviation of the realizable separative performance from its theoretical maximum. Such criterion by analogy with [16] is written as follows:

$$\chi = \sum_{s=1}^{N} \frac{\delta U_{\max} - \delta U_s}{\delta U_{\max}}, \quad (18)$$

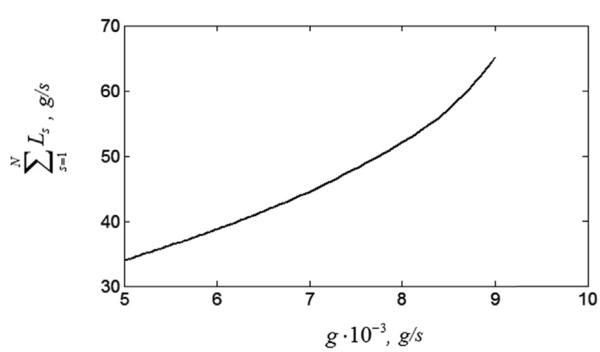


**Fig. 2.** Dependence of optimum total flow in cascade on feed flow rate to a single GC.
GC, gas centrifuge.

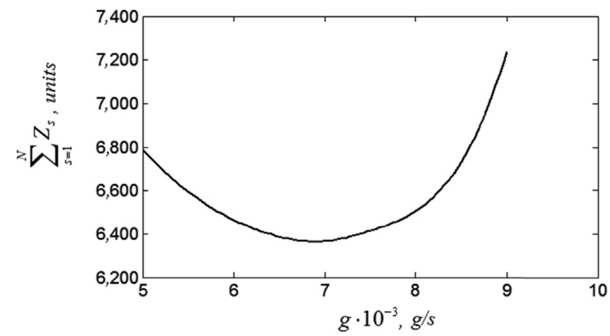


**Fig. 3.** Dependence of total number of GCs in cascades optimized in total flow on feed flow rate to a single GC.
GC, gas centrifuge.

where, $\delta U_{\max}$ is the maximum separative power of a cascade stage and $\delta U_s$ is the realized separative power at the $s$th stage, which can be calculated by Eq. (16).

Fig. 4 shows the dependence of criterion $\chi$ on the feed flow rate to the cascade stage, $g_s$. Here, (as expected) the minimum value of $\chi$ is reached as $g_s$ approaches 7 mg/s. In this case, all stages of the cascade are operated at maximum efficiency.

The results support the following deduction. For a case in which the overall separation factors vary across the cascade stages, an efficiency criterion should use the total number of GCs in the cascade to avoid possible deviations. This deduction is substantiated by the following facts. For a case in which a feed flow rate varies across the cascade stages under optimization by efficiency criterion $\sum_{s=1}^{N} Z_s \rightarrow \min$, there is a unique set of cascade parameters that provides the minimum number of separation elements to solve the separation problem of optimization. If optimization is performed using the total flow as the efficiency criterion, then there are multiple solutions that relate to the range and to the law of change according to the $g_s$ parameter. Among the solutions, there is only one that corresponds to the minimum number of GCs in the cascade.

The authors note that the obtained results should be qualitatively validated for other possible dependencies, with $q_s = q_s(\theta_s, g_s)$ having a single maximum for the variables $\theta_s, g_s$ [11].

## Conclusions

Using two efficiency criteria (i.e., minimum number of GCs in the cascade and minimum total flow in the cascade), this paper compares the parameters in a cascade as it changes from stage to

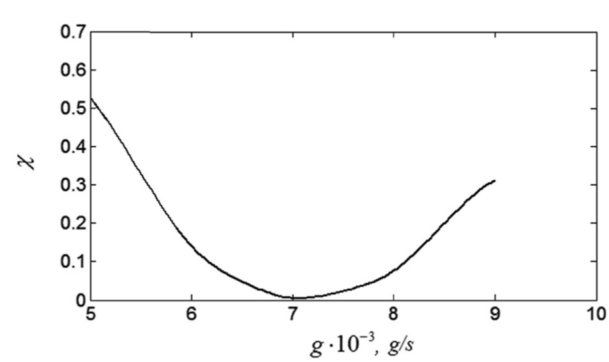


**Fig. 4.** Dependence of $\chi$ criterion efficiency for cascade optimized in total flow on feed flow rate to a single GC.
GC, gas centrifuge.

stage for an overall separation factor of a single GC. The results show that the parameters of these optimized cascades are significantly different. Here, the error in the total number of GCs in a cascade while optimizing the total flow is so large that it should be neglected in practice. The results also show that, according to the limitations imposed on the feed flow rate to a single GC, the optimization problem has an infinite number of solutions when minimizing the total flow in a cascade. However, only one of these solutions corresponds to the solution found by minimizing the total number of GCs in a cascade. In conclusion, the results of the present study show that, because it provides an unambiguous solution to the optimization problem, “total number of GCs” (rather than total flow) should be used as the efficiency criterion for the optimal design of an industrial separation plant for uranium enrichment with overall variable separation factors for GCs.

## Conflicts of interest

The authors whose names are listed above certify that they have no affiliations with or involvement in any organization or entity with any financial or non-financial interest in the subject matter discussed in the article.

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